ON THE FRACTAL STRUCTURE OF MOLECULAR CLOUDS

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Molecular clouds show structure on all scales observed and many attempts have been made to characterize the structure which is linked to important aspects of star formation. Being generated predominantly by the turbulent internal cloud motion, the structure of molecular clouds is likely to be random to a large degree. As the information on the structural properties of random function is basically contained in its power spectrum, it is obviously important to study the power spectrum of observed cloud images. For HI clouds (Green 1993), the azimuthally averaged power spectra derived directly from interferometer observations are well characterized by a power law with a spectral index close to 2.8.

A very powerful tool to study the power spectrum of a random time series has been introduced by Allan (1966) within the context of characterizing the stability of atomic clocks. For a random time series, the Allan variance on a given time scale T is basically the variance of the differences of subsequent averages over time T. In Fourier space, the Allan variance turns out to be the weighted average of the power spectrum of the random function with the weighting $\sin(\pi fT)^4/(\pi fT)^2$. This filtering function peaks approximately at $1/(\pi T)$. If the random function is purely white noise, i.e., has a flat power spectrum, the Allan variance goes as $\sigma_A^2 \propto 1/T$; if the random function has drift behaviour, i.e., $< (f(t+T) - f(t))^2 > \propto T^{2H}$, the power spectrum is $PS \propto f^{-\beta}$ with $\beta = 1 + 2H$ and the Allan variance is $\sigma_A^2 \propto T^{\beta-1}$ for $0 < \beta < 3$, and $\sigma_A^2 \propto T^2$ for $\sigma > 3$. Note that the discrimination at $\beta = 3$ implies that the Allan variance actually contains information different from the power spectrum itself.

Extension to higher dimensions is pretty straightforward. For the analysis of a 2-dim spatial structure, such as a molecular cloud image, we have to take the difference between adjacent averages of size L. With no azimuthal direction preferred, this is naturally done in cylindrical symmetry, i.e., using a function with a value of $+1$ on a circle of diameter L enclosed

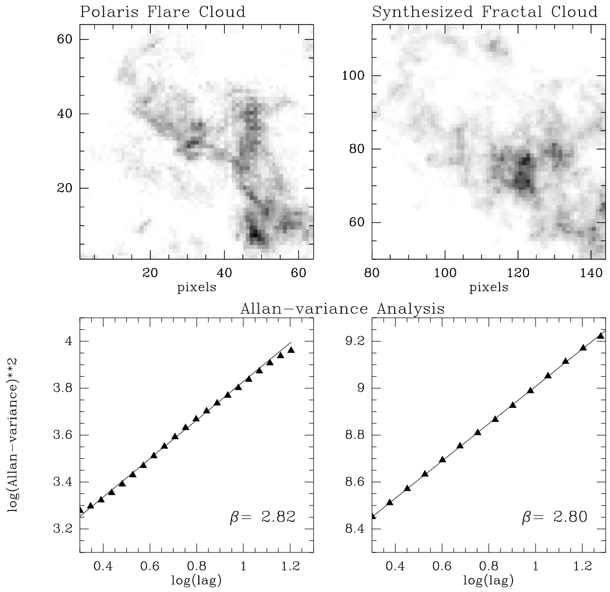

Figure 1. Allan variance of (left) an observed image of the Polaris flare cloud (Heithausen et al. 1990) showing a power law power spectrum with a spectral index $\beta = 2.8$ and (right) the same for a synthesized fBm-image.

by a region with value $-1/8$ (for equal weight of the two areas) on a ring of outer radius $3L/2$ as the spatial filter function. One then calculates the 2-dim Allan variance as the average of the 2-dim power spectrum weighted with the corresponding power spectrum of the 2-dim filter function. Figure 1 (left) shows the Allan variance as defined above for an integrated intensity image of the Polaris Flare to closely follow a power law with a power law spectral index of $\beta = 2.8$, similar to values for other observed molecular cloud images.

It is well established within the work that has been done over the last years in analysing and synthesizing fractal images (e.g., in order to generate artificial landscapes for computer animations, etc.) that such structures can easily be generated as so-called "fractal Brownian motion" (fBm)-images. These are generated by specifying a power law power spectrum with a given index $\beta = E + 2H$, where E is the Euclidian dimension of the image and H ranges between 0 and 1, and completely random phases of the image. Such structures can be shown to have a drift behaviour (autocorrelation function)$< (f(\rightarrow r + \rightarrow D) - f(\rightarrow r))^2 > \propto D^{2H}$, and their iso-intensity contours follow an area/perimeter relation with a fractal dimension $d_H = (E-1) + 1 - H$ (for a short introduction, see the book by Peitgen & Saupe, 1988). Figure 1 (right) shows that such a structure actually gives a good representation of the structure observed for molecular cloud images. In fact, eye inspection of a series of synthesized fBm images covering the full range of $H = 0...1$ allow to identify the spectral index for observed molecular clouds to be close to 2.8 (i.e., H around 0.4). This value is in good agreement with the value derived above from the Allan variance analysis, and with the range of 1.4–1.6 derived for the area/perimeter fractal dimension.

Observed clouds have a power law clump mass spectrum $dN/dM \propto M^{-\alpha}$, with α in the range 1.5 to 1.7 for most clouds (Stutzki & Güsten 1990; Williams et al. 1994). Although the concept of dividing the observed structure into discrete subentities seems to be almost orthogonal to analysing the structure in terms of fractal dimensions or power spectra, it can be shown that α is connected to the power law spectral index for the 2-dim projected image by $\beta = 6 - 2\alpha$. The derivation assumes that the clumps have, on average, constant column density (following one of Larson's relation). Note that the regime of observed values for α and β is consistent with the above relation.

It can be shown (see Stutzki et al. 1995) that the 3-dim structure resulting in the observed 2-dim image with a power law power spectrum and random phases also has a power law spectrum with identical spectral index, i.e., $\beta = 2.8$. In addition, under the reasonable assumption that the 3-dim structure has no correlations introducing some preferential 3-dim shapes that are hidden in the 2-dim projection, the 3-dim structure should also have random phases. This is a very important result, because it implies that the 3-dim structure is completely dominated by surface. A 3-dim fBm structure is limited to the range $\beta = E + 2H = 3...5$, corresponding to $H = 0...1$. The respective surface/volume relation for the fractal structure of the iso-density surfaces (the 3-dim analogon to the perimeter/area relation for the iso-intensity contours), then has a fractal dimension in the range $d_H = E + 1 - H = 4...3$, i.e., $\log(A) \propto \log(V)^{d_H/3}$. The extreme value of $H = 1$, i.e., $d_H = 3$, already corresponds to the surface increasing proportionally to the volume. The value of $\beta = 2.8$ for a 3-dim structure derived above then implies an even faster increase of surface with volume.

The result that the 3-dim cloud structure is dominated by surface is in accordance with the fact that even ^{12}CO, though being a completely optically thick tracer, measures cloud mass, as well as the recently emerging view, that most line emission from molecular cloud tracers is largely dominated by surface effects. From the results presented, it seems very important to try to understand the derived value for the power law power spectrum index as a unique structure parameter from theoretical arguments on the turbulent structure of molecular clouds. For a full understanding of the 3-dim cloud structure, a link to the velocity structure is crucial. Velocity structure becomes very important as a tool to tell overlapping 3-dim spatial structures apart, and to keep the emission of individual clumps from mutual shielding in the radiative transfer.

References

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