

SOURCE CONDUCTANCE SCALING FOR HIGH FREQUENCY SUPERCONDUCTING QUASIPARTICLE RECEIVERS

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ABSTRACT

It has been suggested that the optimum source conductance G_s for the superconductor-insulator-superconductor (SIS) quasiparticle mixer should have a $1/f$ dependence. This would imply that the critical current density of SIS junctions used for mixing should increase as frequency *squared*, a stringent constraint on the design of submillimeter SIS mixers, rather than in simple proportion to frequency as previously believed. We have used Tucker's quantum theory of mixing for extensive numerical calculations to determine G_s for an optimized SIS receiver. We find that G_s is very roughly independent of frequency (except for the best junctions at low frequency), and discuss the implications of our results for the design of submillimeter SIS mixers.

INTRODUCTION

Superconductor-insulator-superconductor (SIS) quasiparticle mixers [1] are now firmly established as the most sensitive receiving devices in the vicinity of 100 to 200 GHz. Their behavior is well described by Tucker's quantum theory of mixing [2]. There are now many publications which show excellent agreement between the theory's predictions of a mixer's conversion properties and experimental results, especially at 100 GHz, and the theory also appears to be successful in predicting the noise temperature of the most sensitive SIS mixers.

Given the success of the Tucker theory at 100 GHz, it is desirable to know the predicted performance of SIS mixers at higher frequencies, where there are fewer experimental results but many experiments underway. A large step in this direction was taken by Kerr and Pan [3], who developed a "design procedure" for SIS mixers, really a

set of rules for scaling a successful and reasonably understood low-frequency SIS mixer design to higher frequency. Their argument was carried further and ratified in Ref. [4]. Kerr and Pan concluded that the critical current density of SIS junctions used for mixing should increase as frequency *squared*, rather than in simple proportion to frequency as previously believed. This result presents a stringent constraint on the design of submillimeter SIS mixers, implying that high frequency SIS mixers are much more difficult to realize than had previously been appreciated. This widely quoted conclusion certainly is influencing the design of the current generation of submillimeter SIS mixers.

Kerr and Pan based their analysis on the " $\omega R_N C = 4$ rule": the best SIS mixer performance appears to be obtained when the characteristic parameter $\omega R_N C$ is near 4, where ω is the LO frequency and R_N is the normal state resistance and C the capacitance of the SIS junction. As first advanced in Ref. [5] and more recently discussed in Ref. [6], all SIS mixer experiments exhibiting infinite available gain have $\omega R_N C \geq 4$, while $\omega R_N C < 1$ has always resulted in considerable conversion loss. (To our knowledge this correlation still holds to date.) Presumably, good mixer conversion requires the reduction of harmonic conversion effects by the relatively large capacitance. Indeed, computer simulations show that harmonic conversion becomes significant for $\omega R_N C < 4$ [7]. On the other hand, unnecessarily large capacitance entails greater difficulty in tuning and narrower bandwidth.

The damping time $R_N C$ of an SIS junction varies in inverse proportion to its critical current density, j_c . Therefore j_c must increase proportional to frequency to maintain a constant $\omega R_N C$, and this alone requires an inconveniently large j_c for submillimeter SIS mixers. However, Kerr and Pan rightly note that while the $\omega R_N C = 4$ rule may be valid for 100 GHz SIS mixers, there is no reason to expect that the optimum $\omega R_N C$ is independent of frequency. In particular, their calculations indicate that the quantity $G_S R_N$, the mixer source conductance normalized to R_N , "should have a $1/f$ dependence for mixers in the quantum-limited regime." This immediately implies that j_c should increase as frequency *squared*.

CALCULATIONS

It is not feasible to optimize an SIS mixer by maximizing the calculated conversion gain. There is no unique optimum bias point: the quantum theory of mixing predicts infinite gain for high quality SIS junctions over a wide range of parameter values. Such high gain is unrealistic and undesirable. Kerr and Pan avoid this difficulty by positing a set of requirements, including unity gain and moderately well matched input ($V_{SWR} \leq 2$), for optimum SIS design. We take a different approach.

We use the quantum mixer theory for extensive numerical calculations, to determine the minimum value of the SSB (single sideband) noise temperature T_R of an SIS receiver, subject to reasonable experimental constraints. Thus our calculation involves a trade-off between minimizing the mixer noise temperature and maximizing the mixer conversion gain, which is mediated by the noise temperature of the IF amplifier T_{IF} . Full details will appear elsewhere. For our current purpose we make the following approximations: We consider DSB (double sideband) operation in the three-frequency low-IF approximation, which should be a fairly good representation of most well-designed experimental mixers. We do not include any interference from the Josephson effect, although this is likely to be a problem for experiments at the higher frequencies. In addition, we ignore all reactances. Taken together, these approximations are equivalent to assuming 1) that the geometrical capacitance of the SIS junction is large enough to both short out the LO harmonics and their sidebands and to eliminate Josephson interference, 2) that the capacitance is itself resonated by a relatively broadband external tuning circuit, so that the intrinsic junction nonlinearity is presented with a resistive embedding impedance at all relevant frequencies, and 3) that the quantum susceptance has no significant effect. This third assumption is controversial. It has recently been argued that the quantum susceptance is a central element of the behavior of SIS mixers [8]. Nevertheless, we believe that this nonlinear reactance has little effect on the performance of an optimized SIS receiver, though it may affect the optimum bias point. This question will be addressed in further research.

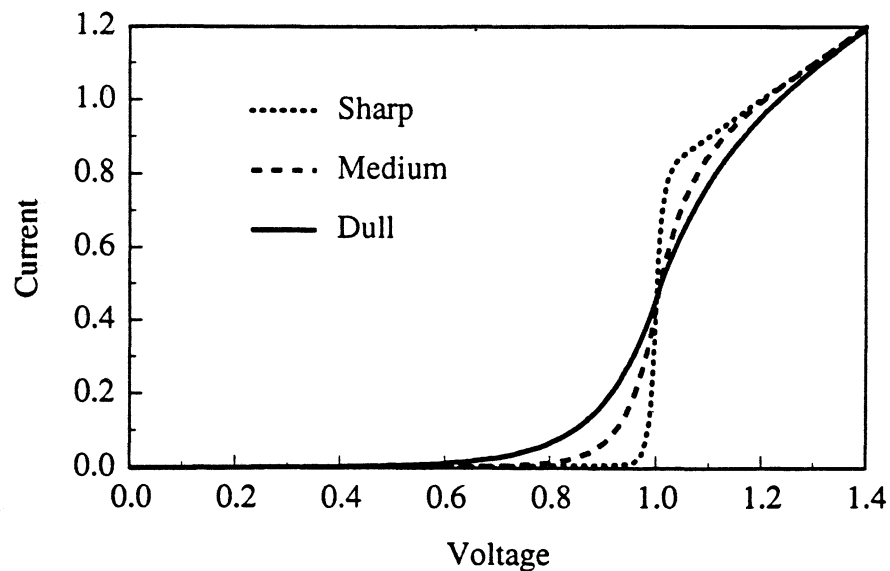


Fig. 1. Three synthetic normalized I-V characteristics used for these calculations.

The equations employed in the calculation of T_R are taken from Ref. [1] and will not be reproduced here. For convenience we assume zero physical temperature; the only serious effect of this is to ignore the thermal noise from the IF termination which is reflected from the mixer back into the IF amplifier. For real SIS receivers this can be an important contribution to the total noise. We require a reasonable input match: in particular we require that both the signal reflection gain *and also the signal-to-image conversion gain* be $\leq 1/4$ (which corresponds to VSWR ≤ 3). We find that this constraint completely eliminates every instance of high conversion gain. What remains is a distinct solution with stable moderate realistic conversion gain and low mixer noise. Moreover we find that our quantitative results are extremely insensitive to the level of returned signal or image power allowed. These topics are discussed at length in Ref. [9].

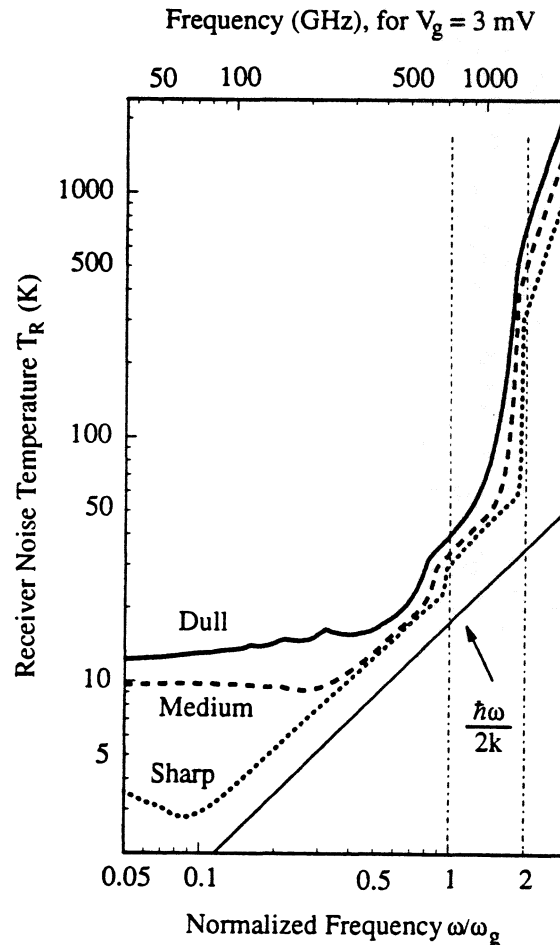


Fig. 2. The SSB noise temperature of a DSB SIS receiver optimized at each frequency, calculated for the three I-V curves of Fig. 1, $G_L = 0.3/R_N$, $T_{IF} = 3$ K, and $V_g = 3$ mV.

We have performed these calculations for each frequency for a wide range of parameters, but only a few of the results can be presented here. The illustrations given in this paper use the synthetic SIS junction I-V curves depicted in Fig. 1. The "sharp" curve corresponds to the best experimental SIS I-V curves, the "medium" curve corresponds to a good quality junction, and the "dull" curve corresponds to a moderate quality junction. We normalize voltages to the energy gap voltage V_g , conductances to the normal state resistance R_N , and frequencies to the energy gap frequency $\omega_g \equiv eV_g/\hbar$.

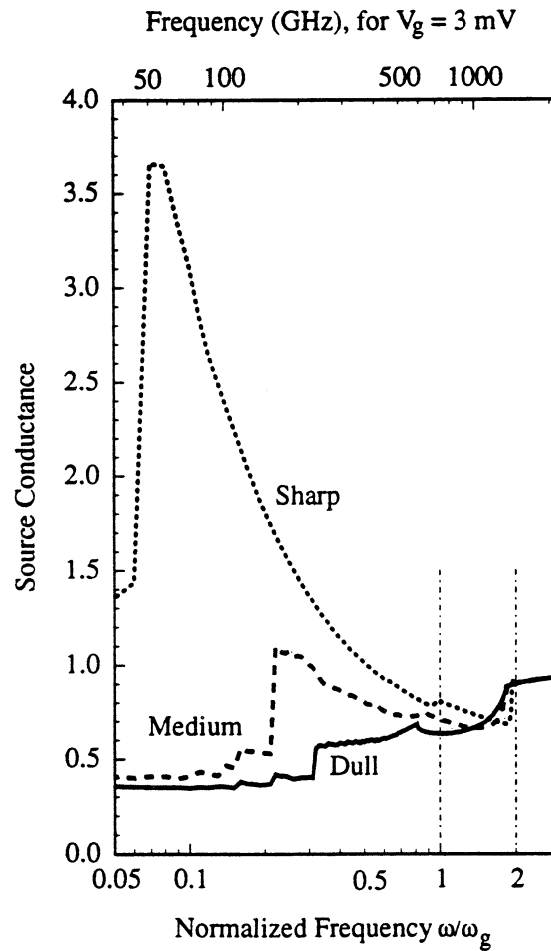


Fig. 3. The normalized source conductance $G_S R_N$ required to optimize the receiver of Fig. 2, calculated for the three I-V curves of Fig. 1.

RESULTS

Figure 2 shows the minimum theoretical SSB noise temperature of a DSB SIS receiver with IF load conductance $G_L = 0.3/R_N$, $T_{IF} = 3$ K, and $V_g = 3$ mV, for the three I-V curves of Fig. 1. Figure 3 shows the optimum value of the normalized source conductance $G_S R_N$ required to achieve the minimum T_R . At lower frequencies (below the vertical rise in each curve) the mixer is biased on higher number photon steps and G_S is relatively constant as expected for classical behavior. On the first photon step, however, the behavior of G_S is quite different. At the lowest frequencies on the first step G_S is strongly dependent on the I-V curve quality; for high quality junctions the optimum G_S is quite large. As the frequency increases, the optimum G_S gradually changes to approach a value ≈ 0.7 , for all three I-V curves at frequencies near $2\omega_g$.

Figure 3 clearly shows that the optimum G_S does not have a $1/f$ dependence. To emphasize this point, in Fig. 4 we plot the quantity $G_S \omega$ vs. ω for the data of Fig. 3. The $1/f$ dependence predicted by Kerr and Pan [3] would give horizontal lines in Fig. 4, and horizontal lines are nowhere seen. Rather, the optimum G_S for the sharp curve is given by the empirical formula $G_S = 1/2 + 0.25/\omega$ for bias points on the first photon step. This behavior is quite widespread. For instance, Fig. 5 shows the the optimum G_S computed for SIS receivers with various values of T_{IF} , for the sharp I-V curve. The same empirical formula also works well when we consider different values of load conductance, I-V curves with considerable leakage current, etc.

In order to better understand the behavior of the optimum G_S , in Fig. 6 we compare it with all of the important "input" conductances in our calculations. It is seen that even though G_S is determined by a trade-off between the gain and the shot noise, the optimum G_S is quite close to that which minimizes the shot noise, G_{shot} , but far from that which maximizes the gain, G'_S . This surprising result can be explained by examination of the equations of the SIS mixer. On one hand, the dependence of the conversion gain upon G_S is given by a simple impedance matching formula which has its minimum at $G_S = |G'_S|$; a fairly large mismatch therefore results in only a small decrease in gain. On the other hand, the mixer noise is minimized by the exact cancellation of the correlated components of the shot noise at the IF and the signal and image frequencies, which occurs at $G_S = G_{shot}$. If G_S strays from this value the shot noise grows rapidly. The optimum G_S is also far from the signal input conductance, G_S^0 , but never more than a factor of three lest the signal reflection gain become too large.

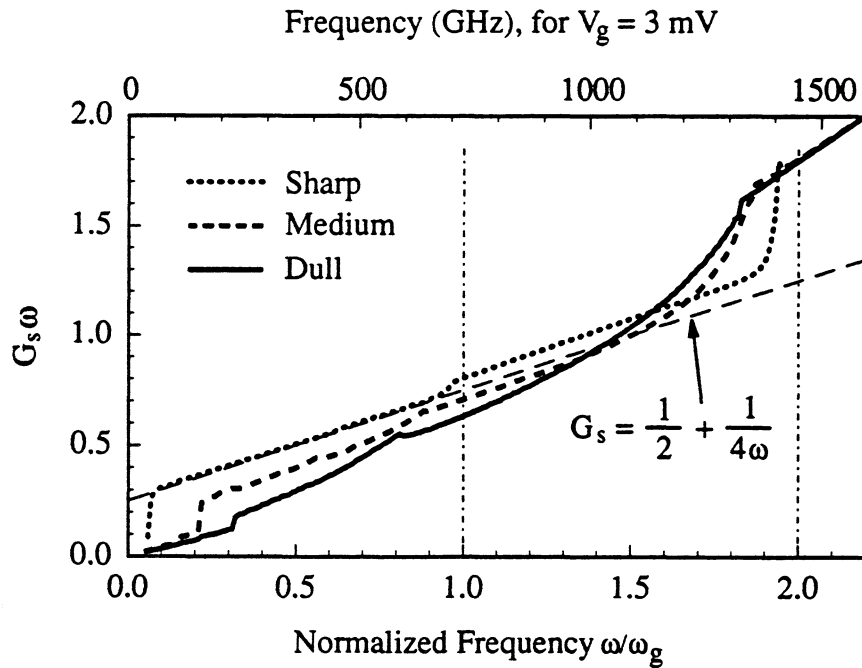


Fig. 4. The data of Fig. 3 are multiplied by ω and replotted (in normalized units), and are compared to an empirical formula.

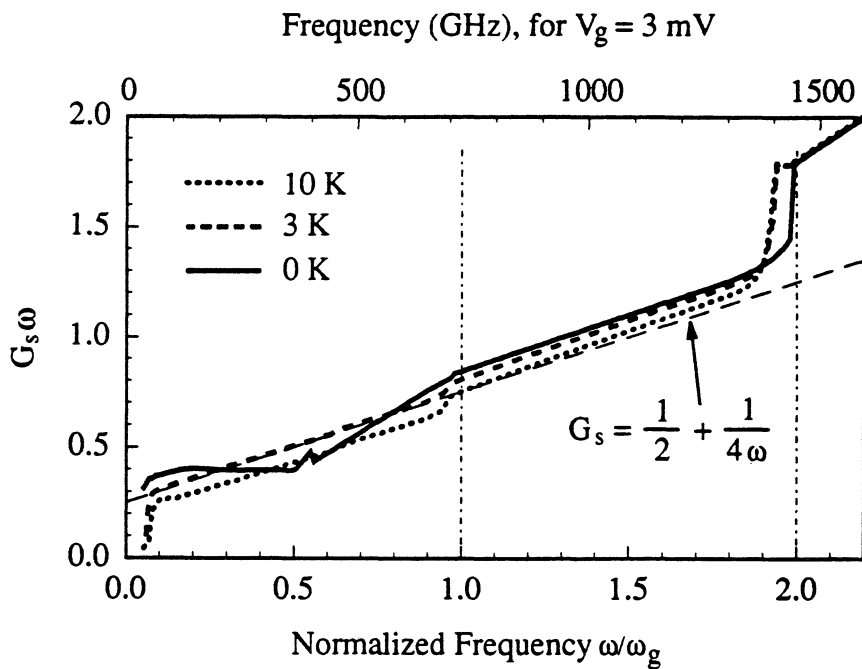


Fig. 5. The optimum source conductance G_s of an SIS receiver whose IF amplifier noise temperature $T_{IF} = 10 \text{ K}$, 3 K , and 0 K , respectively, using the "sharp" I-V curve of Fig. 1, $G_L = 0.3/R_N$, and $V_g = 3 \text{ mV}$.

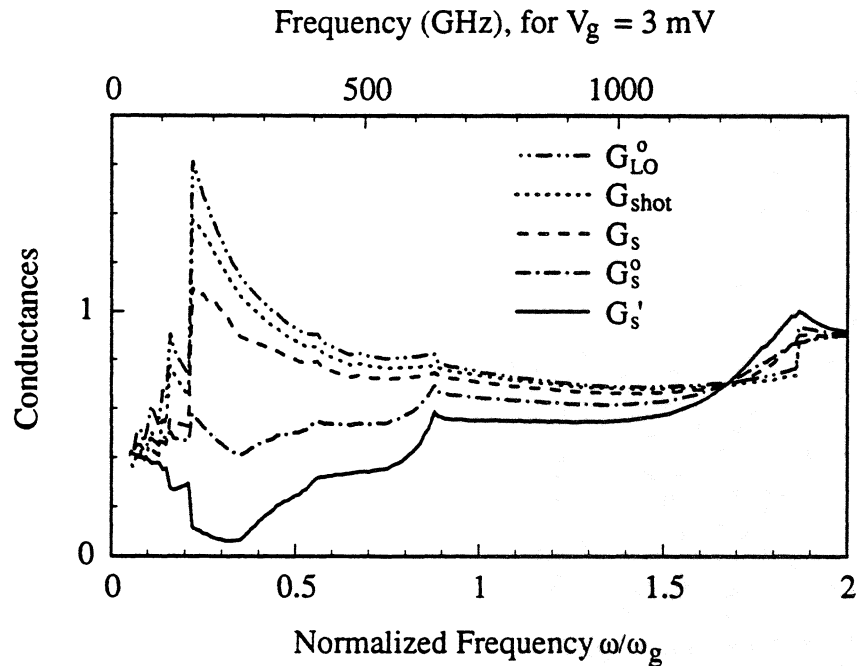


Fig. 6. The optimum source conductance G_s of an SIS receiver using the "medium" I-V curve of Fig. 1, $G_L = 0.3/R_N$, $T_{IF} = 3$ K, and $V_g = 3$ mV, compared to various "input" conductances: G_{LO}^o and G_s^o are the input conductances of the mixer at the LO and the signal frequencies, respectively, G_{shot} is the value of G_s which would minimize the shot noise of the mixer, and G_s' is the value of G_s which would maximize the gain of the mixer.

Note in Fig. 6 that G_{shot} , and thus the optimum G_s , follows closely the input conductance at the LO frequency, G_{LO}^o . This is exactly as predicted by the simple photodiode theory of SIS mixing [10], which reproduces the the equations of the quantum theory of mixing in the limit of small LO voltage amplitude (small α). It is surprising that G_{shot} follows G_{LO}^o so closely for the relatively large α of our simulations. In any case this enables us to explain the empirical formula $G_s = 1/2 + 0.25/\omega$. In the limit of small α , G_{LO}^o is the slope of the chord connecting the photon point $I_{dc}(V_0 - \hbar\omega/e)$ to the photon point $I_{dc}(V_0 + \hbar\omega/e)$ on the unpumped dc I-V curve. Therefore, using the preferred value for the optimum dc bias voltage $V_0 = 0.9$ for the sharp I-V curve, this gives $G_{LO}^o = 1/2 + 0.35/\omega$ in the small α limit. G_s follows but is slightly less than G_{LO}^o (Fig. 6) and so is very well approximated by the empirical formula.

DISCUSSION

The results presented here are for particular parameter values, but they are quite general and representative of our more extensive calculations. In disagreement with Ref. [3], we find that $G_s R_N$ is very roughly independent of frequency (except for the best junctions at low frequency). This means that there is no reason to suppose that it is advantageous to increase j_c as frequency squared in the design of high frequency SIS mixers.

Why do our results differ from Ref. [3]? It is likely that this disagreement arises because in Ref. [3] the gain was fixed to unity, whereas we find that the mixer gain of an optimized SIS receiver falls off roughly as $1/\omega$ for bias points on the first photon step (Fig. 7). Note in Fig. 7 that we find conversion gain as high as 8 dB in the vicinity of 100 GHz, but in our solution the mixer is operating far from instability [9] with low noise and quite low returned signal and image power.

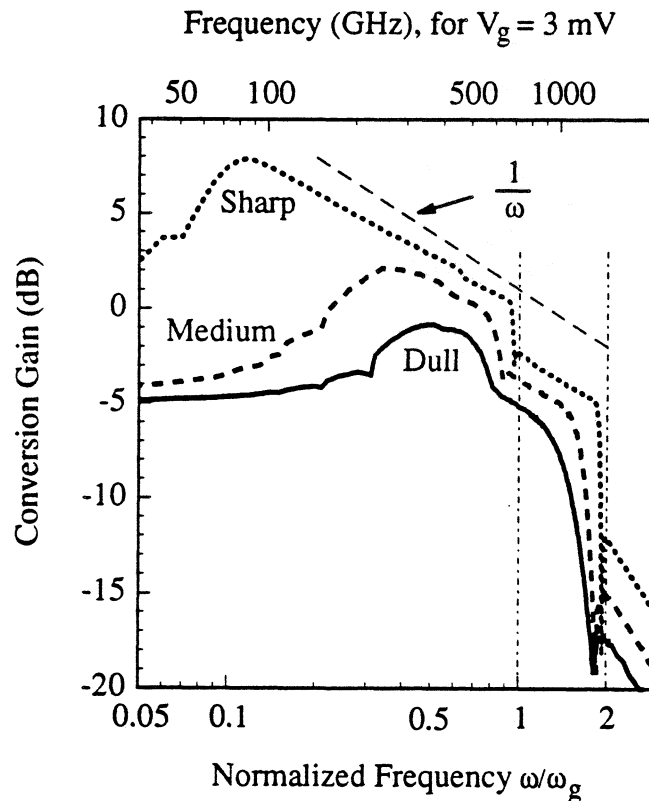


Fig. 7. The IF conversion gain corresponding to the three curves of Fig. 2.

Nevertheless, we agree with Ref. [3] that the " $\omega R_N C = 4$ rule" should be modified for submillimeter SIS mixers. Harmonic conversion effects should become much less important as the frequency is increased, because the SIS junction presents a much weaker nonlinearity for harmonic frequencies above ω_g , especially so for frequencies above $2\omega_g$. This implies that the beneficial effects of the capacitance are reduced as the frequency is increased, and smaller values of $\omega R_N C$ can be tolerated. Since it is more difficult to resonate the capacitance at high frequency, smaller values of $\omega R_N C$ are desirable. However, small area and high critical current SIS junctions are difficult to fabricate, and usually entail undesirable consequences such as inferior junction quality, poorer yield, etc. Therefore, the choice of $\omega R_N C$ for submillimeter SIS mixers will at best be an informed compromise.

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