THEORETICAL AND PRACTICAL LIMITS ON THE RF BANDWIDTH OF SIS MIXERS*

A. R. Kerr
National Radio Astronomy Observatory**
Charlottesville, VA 22903

ABSTRACT

Most millimeter-wave mixers have an RF equivalent circuit that falls into one of the following categories: (i) Parallel R-C (the junction) connected to a resistive source with no broadband matching circuit. (ii) Parallel R-C, and possibly series L, followed by a broadband matching circuit. (iii) Parallel R-C-L, and possibly series L, followed by a broadband matching circuit. This paper examines the maximum (RF) bandwidths achievable by SIS mixers in each of these categories.

Fundamental limitations on the matching bandwidth between a resistive source and a capacitive load were derived by Bode in 1945. In 1950, Fano developed a more general theory which included capacitive devices with series inductance. To use the work of Bode and Fano to determine the useful bandwidth of a practical mixer, it is necessary first to know the range of (complex) source impedances within which acceptable performance is obtained. From SIS mixer theory it is found that acceptable performance results when the magnitude of the source reflection coefficient ρ, relative to the optimum source impedance, is less than some value. For Nb/Al-Al_2O_3/Nb SIS mixers in the 70-350 GHz range, $|\rho| \leq 0.4$ appears appropriate.

It is found that the inductance of a series array of junctions only limits the bandwidth if it exceeds a critical value. Otherwise the inductance must actually be augmented to achieve the theoretical maximum bandwidth.

---

*A more extensive version of this paper has been accepted for publication in *IEEE Transactions on Microwave Theory and Techniques*.

**The National Radio Astronomy Observatory is operated by Associated Universities, Inc. under cooperative agreement with the National Science Foundation.

Introduction

As SIS mixers achieve lower noise over wider frequency bands, it is interesting to examine the factors that limit their ultimate bandwidth. These can be divided into two categories: theoretical limits, and limits imposed by practical constraints such as conductor dimensions and tolerances. The term *bandwidth* here refers to the frequency range over which the LO can be tuned while maintaining satisfactory mixer performance without re-tuning the RF or IF circuits of the mixer. We assume here a low IF, so the junction sees the same embedding impedance in the upper and lower sidebands.

The performance of an SIS mixer, operating as a mixer, with given LO frequency, LO amplitude, and bias voltage, is governed by the embedding impedances seen by the junction at the sideband and intermediate frequencies, $f_{LO} \pm f_{IF}$ and f_{IF}. The first step in determining the maximum theoretical bandwidth is therefore to determine the desired IF load impedance and the acceptable range of RF embedding impedances. With a suitable normalizing impedance (characteristic impedance), the acceptable range of RF embedding impedances can be expressed as a maximum allowable reflection coefficient. This allows us to make use of the famous integral equations relating match and bandwidth, developed over 40 years ago by Bode [1] and Fano [2].

The Range of Acceptable Embedding Impedances

The performance of a typical 230 GHz double sideband $Nb/Al\text{-}Al_2O_3/Nb$ SIS receiver is shown in Fig. 1. The contours are plotted on Smith charts of RF embedding *admittance* (*i.e.*, in the $(-\rho)$-plane). The embedding admittance includes the capacitance of the junction, and is assumed equal in the upper and lower sidebands. The mixer gain and receiver noise temperature are shown as single-sideband (SSB) quantities. The receiver includes an IF amplifier with $T_{IF} = 4$ K, and an IF isolator at 4 K. Admittances in the diagram are normalized to the optimum source conductance $1/R_{S,opt}$ where [3,4]

$$R_{S,opt} = \left(\frac{R_N}{2.4}\right)\left(\frac{f_{GHz}}{100}\right)^{0.72}. \quad (1)$$

R_N is the normal resistance of the junction (or array of junctions). The IF load impedance Z_{IF} is fixed and equal to $R_{S,opt}$, and the pumping parameter $\alpha = eV_{LO}/\hbar\omega_{LO} = 1.2$, with the DC bias voltage at the mid-point of the first photon step. For $Nb/Al\text{-}Al_2O_3/Nb$ SIS mixers operating up to at least 60% of the gap frequency, this set of operating conditions has been found to result in well behaved receivers with noise temperatures close to their minimum, and with contour plots similar in character to those in Fig. 1.

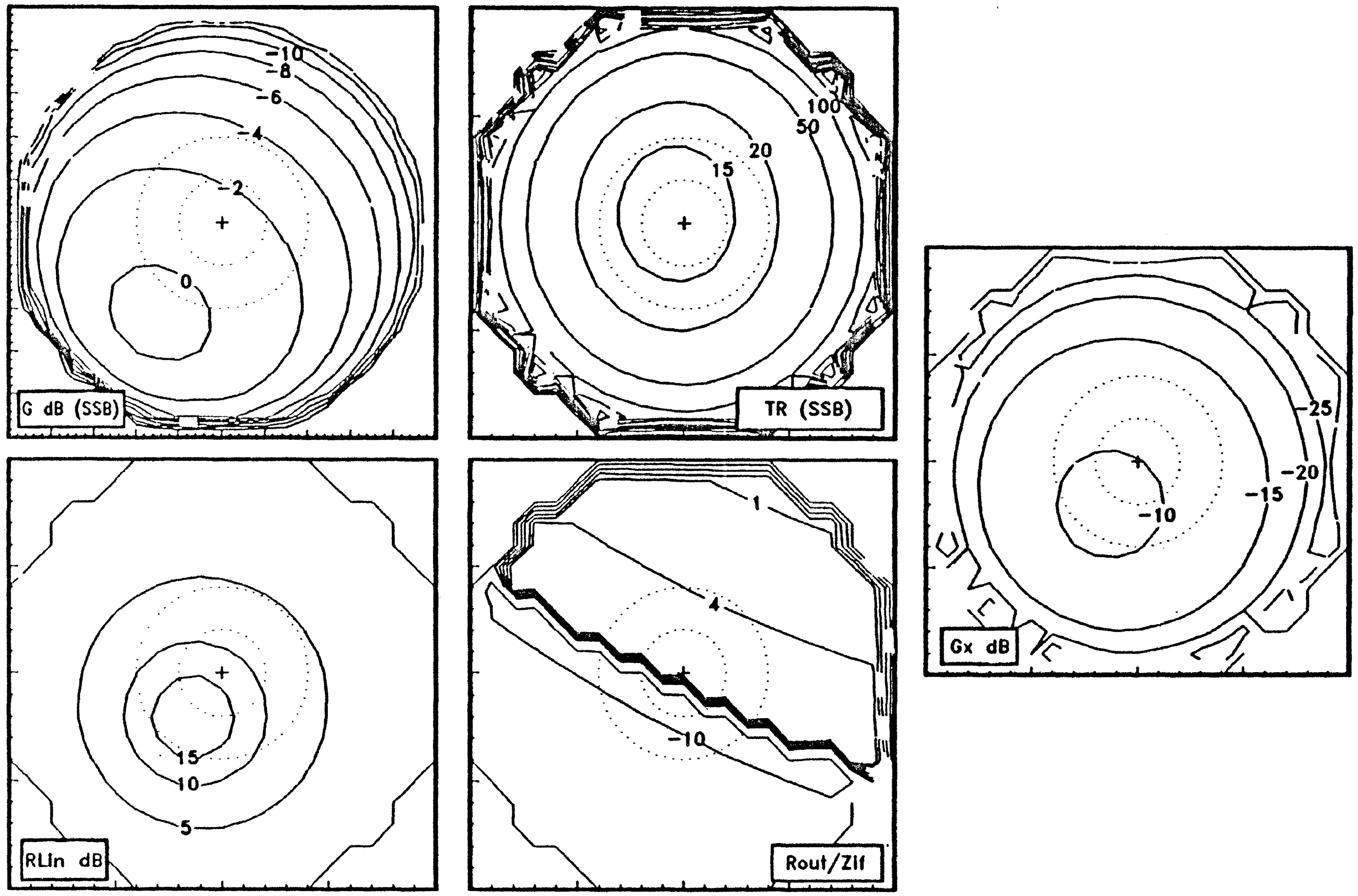

Fig. 1. Contour plots of mixer gain, receiver noise temperature, input return loss, IF VSWR, and signal-to-image conversion gain, for a 230 GHz SIS receiver using typical $Nb/Al\text{-}Al_2O_3/Nb$ junctions. The contours are plotted on Smith charts of RF source admittance (*i.e.*, in the $(-\rho)$-plane). The dotted circles are $|\rho| = 0.2$ and $|\rho| = 0.4$. The receiver includes an IF amplifier with $T_{IF} = 4$ K, and an IF isolator at 4 K. The mixer gain and receiver noise temperature are shown as SSB quantities. (See text for further details.)

The dotted circles, $|\rho| = 0.2$ and 0.4 in the figure, indicate regions of embedding admittance within which two possible levels of acceptable performance are obtained – see Table I. Note that the choice of $R_{S,opt}$ as given by eq. (1), and of $Z_{IF} = R_{S,opt}$, are used here to elucidate the bandwidth theory in the following section but are not crucial to it; other values could equally well be used, as, for example, those given in [5].

TABLE I

	$\lvert\rho\rvert \le 0.2$	$\lvert\rho\rvert \le 0.4$
Mixer conversion gain (SSB)	-0.5 → -1.5 dB	+0.5 → -4.0 dB
Receiver noise temp (SSB)	12 → 15 K	12 → 20 K
Input return loss	> 8 dB	> 5 dB
Signal-to-image conv. loss	9 → 11 dB	9 → 13 dB
Output VSWR (R_{out}/Z_{IF})	-17 → +6	-5 → +3

Equivalent Circuit of the Practical SIS Junction

The simplest equivalent circuit of an SIS junction is shown in Fig. 2(a). Here C is the geometrical capacitance of the junction, and R represents the resistance of the tunneling barrier.

In most applications the connection between the SIS junction and the next element in the RF circuit will have some series inductance, as shown in Fig. 2(b), which might be expected to limit the ultimate bandwidth of the circuit. This inductance could be the inductance of a junction across a waveguide or the series inductance of an array of junctions.

Some SIS mixer designs have used a parallel inductance connected directly across the junction terminals [4,6-11] to resonate the junction capacitance, as in Fig. 2(c). In that case also, the inductance of the connection to the next RF circuit element may restrict the ultimate bandwidth of the mixer.

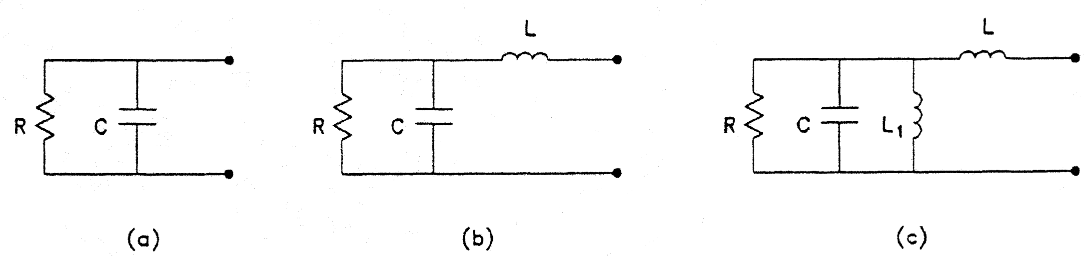

Fig. 2. Equivalent circuits of: (a) a capacitive device, (b) a capacitive device with series inductance, and (c) an inductively tuned device with series inductance.

The theoretical limits on the match-bandwidth of these three circuits will now be examined. It is assumed that each circuit is connected to a resistive source through a lossless matching circuit of arbitrary complexity.

Bode's Theory for the Circuit of Fig. 2(a)

In 1945 Bode at Bell Labs [1] showed that for the simple circuit of Fig. 2(a), connected via a lossless matching network to a resistive source, the reflection coefficient ρ is constrained by the integral equation:

$$\int_0^\infty \ln\left|\frac{1}{\rho(\omega)}\right| . d\omega \leq \frac{\pi}{RC} . \tag{2}$$

This equation is derived solely from the conditions for physical realizability of the lossless matching circuit. Inspection of eq. (2) indicates that the lowest value of the upper bound of $|\rho|$ (ρ_a in Fig. 3) within the frequency band $\omega_1 \leq \omega \leq \omega_2$ is achieved when $|\rho| = \rho_a$ within that band, and $|\rho| = 1$ at all other frequencies. The optimum $|\rho(\omega)|$ therefore coincides with the solid curve in Fig. 3. The integral in eq. (2) is then simply evaluated, giving the well known result:

$$\ln\left[\frac{1}{\rho_{a,min}}\right] = \frac{\pi}{RC}(\omega_2 - \omega_1) . \tag{3}$$

Analysis of the Circuit of Fig. 2(b)

In 1950, Fano at MIT published a broadband matching theory that applied to a wider range of circuits. For the RCL circuit of Fig. 2(b), physical realizability requires two integral equations to be satisfied:

$$\int_0^\infty \ln\left|\frac{1}{\rho_1}\right| . d\omega = \pi\left(\frac{1}{RC} - \Sigma\lambda_{ri}\right) , \tag{4}$$

$$\int_0^\infty \omega^2 \ln\left|\frac{1}{\rho_1}\right| . d\omega = -\frac{\pi}{3}\left(\frac{1}{R^3C^3} - \frac{3}{RLC^2} - \Sigma\lambda_{ri}^3\right) . \tag{5}$$

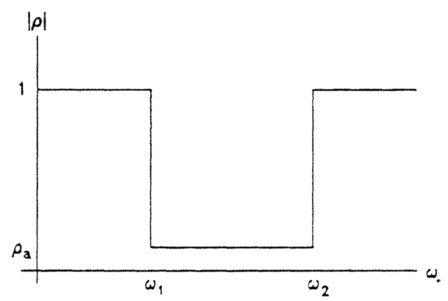

Fig. 3. $|\rho|$ is required to lie below the solid curve. The minimum possible value of ρ_a, in the given frequency band ω_1 and ω_2, is to be determined.

Here the λ_{ri} are the zeros of the reflection coefficient of the embedding circuit that lie in the right half of the complex frequency (s) plane. Fano solved these equations to find the lowest possible reflection coefficient $|\rho_{a,min}|$ for a given bandwidth, but only for the low-pass case ($\omega_1 = 0$ in Fig. 3). We have extended his analysis to the band-pass case, with the result:

$$K^3 - \frac{6}{bQ_C}K^2 + \left[\frac{12}{b^2}\left(1 + \frac{1}{Q_C^2}\right) + 1\right]K - \frac{24}{b^3 Q_L Q_C^2} = 0\ , \qquad (6)$$

where $K = (2/\pi)\ln(1/\rho_{a,min})$, the fractional bandwidth $b = (\omega_b - \omega_a)/\omega_0$, $Q_C = \omega_0 RC$, and $Q_L = \omega_0 L/R$. Equation (6) is a cubic in K and b, and can be solved analytically.

Analysis of the Circuit of Fig. 2(c)

Here Fano's theory is used with the standard low-pass to band-pass mapping, $\omega \rightarrow \omega_x(\omega/\omega_x - \omega_x/\omega)$, giving

$$K^3 - \frac{6}{bQ_C}K^2 + \left(\frac{12}{b^2 Q_C^2} + 4\right)K - \frac{24}{b^3 Q_L Q_C^2} = 0 \qquad (7)$$

where, as above, $K = (2/\pi)\ln(1/\rho_{a,min})$, the fractional bandwidth $b = (\omega_2 - \omega_1)/\omega_0$, $Q_C = \omega_0 RC$, and $Q_L = \omega_0 L/R$. Again, this equation is cubic in K and b.

Results and Discussion

For the circuits of Fig. 2, connected to a resistive source through a lossless matching network of arbitrary complexity, the above equations give the minimum possible value of $|\rho|$ that can be obtained over a fractional bandwidth b. In this section we choose the case of $\omega RC = 4$ and examine $|\rho_{a,min}|$ and b for several values of $\omega L/R$.

Results for the RCL circuit of Fig. 2(b) and the RCL_1L circuit of Fig. 2(c) are shown in Figs. 4(a) and (b) (solid curves). Also shown in Fig. 4 are: (i) the Bode limit for the case of L = 0 (*i.e.*, Fig. 2(a)) – dashed curve; (ii) $|\rho_{a,min}|$ for a parallel RC circuit tuned by a parallel inductor and connected to a source resistance R_S chosen to maximize the bandwidth for each value of $|\rho_{a,min}|$, but with no additional matching (*i.e.*, Fig. 2(c) with L = 0) – dotted curve; and (iii) $|\rho|$ for the RC device of Fig. 2(a) connected directly to a source of resistance R with no matching circuit – short horizontal line.

It is clear from Fig. 4 that for values of L less than some value L_B (whose value depends on the desired value of $|\rho_{a,min}|$ and the other elements of the equivalent circuit) the bandwidth is not limited by L, and the full Bode limit (dashed curve) is theoretically attainable. For such a circuit, the first element of the optimum matching network is an additional series inductance; in this case the inductance in Figs. 2(b) or

(c) can be regarded simply as part of the first element of the optimum matching network that would be used with the RC circuit of Fig. 2(a).

Mixers are not generally matched when connected to their optimum RF embedding impedance. In using these results to determine the maximum bandwidth of a mixer it is appropriate therefore to set the device resistance R equal to the optimum RF source resistance, $R_{S,opt}$, determined from mixer theory. The reflection coefficient of the embedding circuit is thus normalized to $R_{S,opt}$. Equations (3), (6), and (7) are then used to determine the bandwidth within which $|\rho| \leq |\rho_{max}|$, where $|\rho_{max}|$ is also determined from mixer theory to give acceptable receiver performance. The appropriate values of Q_C and Q_L are therefore $\omega R_{S,opt}C$ and $\omega R_{S,opt}/L$, respectively.

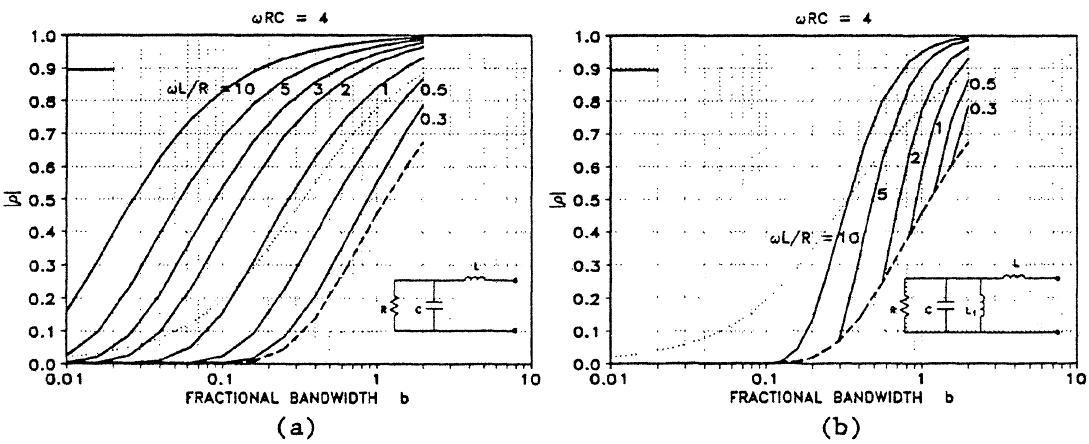

Fig. 4. The lowest upper bound $|\rho_{a,min}|$ on the reflection coefficient *vs* fractional bandwidth b, with $\omega_0RC = 4$ and various values of ω_0L/R. The Bode limit for the simple RC device is indicated by the dashed curves. The dotted curves are for a parallel RC device tuned by a parallel inductor but with no additional matching. The short horizontal lines are for a parallel RC device connected to a source resistance R with no matching.

An Example

It is interesting to see how closely the theoretical bandwidth limit can be approached in a practical SIS mixer with a matching circuit of moderate complexity. As an example, we choose a 250 GHz mixer with a coplanar array of four SIS junctions, shown in Fig. 5, and a matching circuit consisting of a series capacitor and four transmission lines in series, similar, except for the capacitor, to the tuning circuit discussed in [12]. The complete circuit is shown in Fig. 6. Note that the electrical length of the array of junctions is small, so the coplanar line sections can be regarded as lumped inductances, and the equivalent circuit of Fig. 2(b) is applicable.

The capacitor C_A was initially adjusted to make the impedance of the SIS array real at the center frequency. Lines 1, 2, and 3 were set to a quarter wavelength, and the fourth line set to half a wavelength. The microwave circuit design program MMICAD [13] was then used to optimize the elements of the matching network to give $|\rho| \leq 0.4$ over the widest possible bandwidth.

With the requirement, stated above, that the IF load $Z_{IF} = R_{S,opt}$, $R_{S,opt}$ was chosen for convenience as 50 ohms, corresponding to a normal resistance $R_N = 62$ ohms (from eq. (1)). The quantity $\omega R_N C$ was chosen as 5.0, corresponding to $\omega R_{S,opt} C = 4.0$. The resulting embedding admittance is shown in Fig. 7 on an admittance Smith chart ((-ρ)-plane). The optimized values of the elements in the matching network are: $C_A = 75$ fF, $Z_{01} = 2.48\ \Omega$, $Z_{02} = 2.34\ \Omega$, $Z_{03} = 16.5\ \Omega$, $Z_{04} = 61.2\ \Omega$, $l_1 = 0.265\ \lambda_{g0}$, $l_2 = 0.246\ \lambda_{g0}$, $l_3 = 0.254\ \lambda_{g0}$, and $l_4 = 0.516\ \lambda_{g0}$.

The magnitude of the reflection coefficient $|\rho| \leq 0.4$ from 206-296 GHz, giving a fractional bandwidth b = 0.36. For comparison, the Fano bandwidth limit for the same mixer can be deduced from Fig. 4(a), using $\omega RC = 4$ and $\omega L/R = 0.4$, and is $b_{Fano} = 0.55$. The Bode limit for SIS junctions with $\omega RC = 4$ is $b_{Bode} = 0.85$. The bandwidth for an inductively shunted junction (*i.e.*, with a parallel tuning inductor and no other matching elements) and the same value of ωRC, is $b_{isj} = 0.24$. These numbers are listed in Table II.

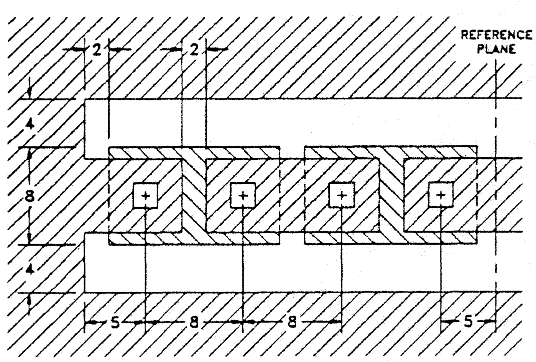

Fig. 5. Series array of four SIS junctions in coplanar waveguide. The substrate is fused quartz, with $\epsilon_r = 3.8$. Dimensions are in μm.

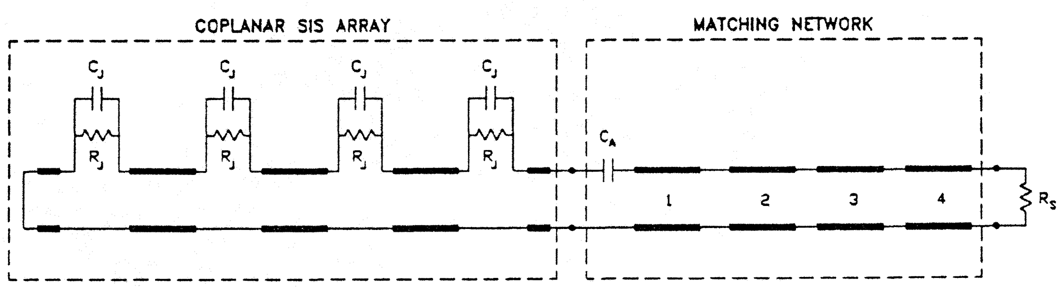

Fig. 6. Circuit of the 250 GHz SIS mixer used in the example. The matching network contains a capacitor C_A in series with four transmission line matching sections. The source resistance $R_S = 50\ \Omega$.

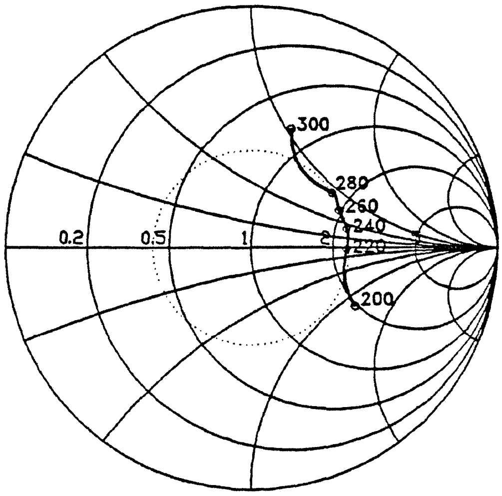

Fig. 7. Embedding admittance of the SIS mixer in Fig. 6 after optimizing the matching network to give the widest possible bandwidth with $|\rho| \leq 0.4$ (dotted circle). Admittances are normalized to the optimum source conductance of the array $1/R_{S,opt}$.

TABLE II

Circuit	b
Bode limit for parallel RC	85%
Fano limit with series L	55%
Circuit of Fig. 6	36%
RC with parallel L tuning	24%

Acknowledgements

The author would like to thank M. J. Feldman, J. Granlund, S.-K. Pan, and M. W. Pospieszalski for their helpful discussions and comments during the course of this work.

References

[1] H. W. Bode, "Network analysis and feedback amplifier design," New York: Van Nostrand, 1945.

[2] R. M. Fano, "Theoretical limitations on the broadband matching of arbitrary impedances," *J. Franklin Inst.*, vol. 249, pp. 57-83 and 139-155, Jan. and Feb. 1950.

[3] A. R. Kerr and S.-K. Pan, "Some recent developments in the design of SIS mixers," *Int. J. Infrared Millimeter Waves*, vol. 11, no. 10, pp. 1169-1187, Oct. 1990.

[4] A. R. Kerr, S.-K. Pan, A. W. Lichtenberger and D. M. Lea, "Progress on tunerless SIS mixers for the 200-300 GHz band," *IEEE Microwave and Guided Wave Letters*, vol. 2, no. 11, pp. 454-456, Nov. 1992.

[5] Q. Ke and M. J. Feldman, "Optimum source conductance for high frequency superconducting quasi-particle receivers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-41, no. 4, pp. 600-604, April 1993.

[6] A. R. Kerr, S.-K. Pan, and M. J. Feldman, "Integrated tuning elements for SIS mixers," *Int. J. Infrared Millimeter Waves*, vol. 9, no. 2, pp. 203-212, Feb. 1988. This paper was presented at the International Superconductivity Electronics Conference, Tokyo, Japan, Aug. 1987.

[7] A. R. Kerr, S.-K. Pan, S. Whiteley, M. Radparvar, and S. Faris, "A fully integrated SIS mixer for 75-110 GHz," *IEEE Int. Microwave Symp. Digest*, pp. 851-854, May 1990.

[8] F. Schafer, E. Kreysa, T. Lehnert, and K. H. Gundlach, "A planar SIS receiver with log-periodic antenna for submillimeter waves," *Proceedings of the Fourth International Symposium on Space Terahertz Technology*, pp. 661-665, March 1993.

[9] A. Karpov, M. Carter, B. Lazareff, M. Voss, D. Billon-Peron, K. H. Gundlach, "Wideband fixed tuned and tuneable SIS mixers for 230 GHz and 345 GHz receivers," *Proceedings of the Fourth International Symposium on Space Terahertz Technology*, p. 11, March 1993.

[10] W. R. McGrath, P. Febvre, P. Batelaan, H. G. LeDuc, B. Bumble, M. A. Frerking, and J. Hernichel, "A submillimeter wave SIS receiver for 547 GHz," *Proceedings of the Fourth International Symposium on Space Terahertz Technology*, pp. 50-58, March 1993.

[11] J. Mees, A. Skalare, M. M. T. M. Dierichs, H. van der Stadt, R. A. Panhuyzen, Th. de Graauw, and T. M. Klapwijk, "Double dipole antenna SIS receivers at frequencies above 500 GHz," *Proceedings of the Fourth International Symposium on Space Terahertz Technology*, pp. 59-71, March 1993.

[12] G. Pance and M. J. Wengler, "Integrated tuning elements for millimeter and submillimeter SIS mixers," *IEEE International Microwave Symposium Digest*, pp. 337-340, June 1992.

[13] MMICAD is a microwave integrated circuit analysis and optimization program, and is a product of Optotek, Ltd., Ontario, Canada K2K-2A9.