Effect of voltage modulation on the shape of the depletion layer of a submillimeter wave Schottky varactor

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Abstract

Schottky varactor frequency multipliers are used to generate local oscillator power at millimeter and submillimeter wavelengths. Multiplication efficiency depends on varactor parameters such as capacitance–voltage characteristic and possible current saturation. The aim of this work is to develop the equivalent circuit of the submillimeter wave Schottky varactor in order to model the effect of fast voltage modulation on the three-dimensional shape of the depletion layer. The effect of voltage modulation can be studied by solving the potential and electron conduction currents in the epitaxial layer of the Schottky varactor. In this work the potential and the electron currents have been calculated from device physics by using numerical methods. According to our results the shape of the depletion layer is strongly affected, when the voltage modulation over the pumped varactor is rapid. The dynamic shape of the depletion layer during the voltage sweep affects the junction capacitance as well as the electron velocity saturation. These effects should be included to the equivalent circuit of the Schottky varactor. This can be done by modifying the model obtained by using static equations.

1 Introduction

Frequency multipliers are used to generate the all-solid-state local oscillator power of heterodyne receivers at millimeter and submillimeter wavelengths [1]. These local oscillators are needed in many future scientific satellites (e.g. SWAS, Odin, FIRST and SMIM). At millimeter and submillimeter wavelengths a Schottky varactor is the most commonly used multiplier device, although several novel varactors (SBV, QWD, BNN, bbBNN, HEMV) have been proposed [2].

The aim of this work is to develop the equivalent circuit of the Schottky varactor in order to find a model, which is physically valid at millimeter and submillimeter wavelengths. The most important task is to analyse the shape of the depletion layer during the voltage sweep, when the submillimeter wave frequency multipliers are pumped rapidly. This voltage modulation affects the three-dimensional shape of the transition front so that during a voltage sweep the shape of the front differs from the shape obtained using static solution. This has an effect on the junction capacitance of the Schottky varactor, which means that the correction factor due to the edge effects must be modified compared to the value obtained by using a static solution.

2 Formulation of the problem

A circular metallic anode is assumed to be at the top of the epitaxial semiconductor (GaAs), as shown in Figure 1. The radius of the anode is R_0 and the thickness of the epitaxial layer is t_e. The approximate shape of the depletion layer is shown in Figure 1, when the anode is charged to a static potential $\phi_0 = V - \phi_{bi}$.

At the static situation the electric equilibrium is reached and no currents are flowing in the epitaxial layer. When the potential of the anode is increased (or decreased), the potential of the epitaxial layer has to change, which means that currents begin to flow

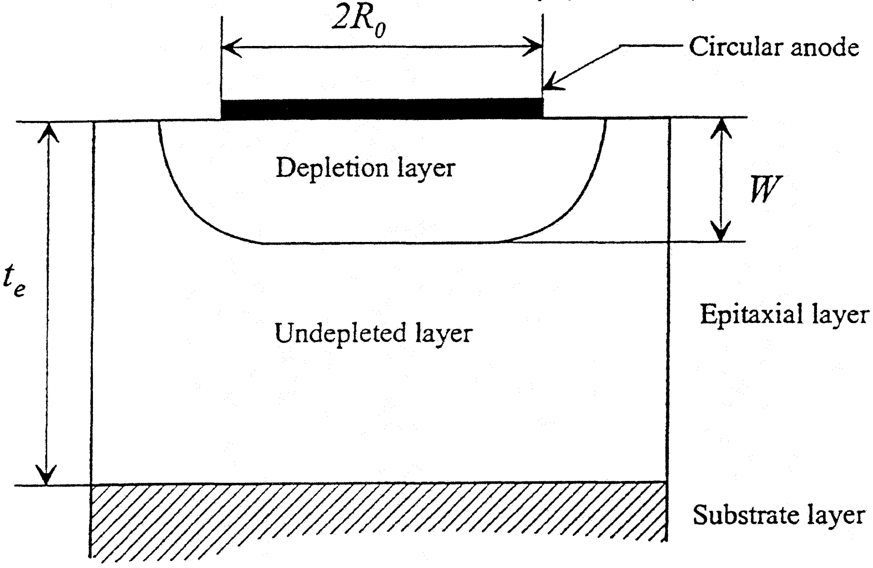

Figure 1: Schematic of the Schottky varactor.

in the undepleted layer. The currents move the transition front between the depleted and undepleted layers. If the potential of the anode is increased very slowly, the shape of the transition front during the transient is equal to the front obtained using the static solution. However, if the potential is increased fast enough, the shape of the transition front is affected by the electron velocity saturation as well as by the edge effects due to the circular anode. The shape of the transition front during the voltage modulation can be determined by using the device equations (Poisson's equation, current continuity equation, etc.) and boundary conditions (potential of anode, potential of substrate, radius of anode, etc.). This two-dimensional problem (actually the problem is three-dimensional but cylindrically symmetric) is non-trivial and the effect of the modulation on the shape of the transition front must be studied by using numerical methods.

Because the problem is very complicated, the following assumptions have been made.

1. Because the conductivity of the substrate is much greater than the conductivity of the epitaxial layer, the substrate layer is assumed to be a perfect conductor and, therefore, the potential of the substrate layer is zero.
2. Because the permittivity of GaAs is greater than the permittivity of air, the normal component of the electric field is assumed to be zero in the air-GaAs interface.

3. Because the Debye length L_D in the epitaxial layer is small, the transition front between the depleted and undepleted layer can be assumed to be abrupt.
4. The radius of the epitaxial layer is assumed to be four times the radius of the anode. This decreases the time required in numerical analyses compared to the case, when the real radius of the epitaxial layer is employed. This assumption can be made because the potential strongly decreases, when the distance from the axis of the cylindrical epitaxial layer increses. The potential is negligible at the distance of 2-3 times R_0 away from the axis.

These four assumptions must be traded against the decreased complexity of the problem as well as the the decreased time required in the numerical simulations.

3 Device equations

3.1 Poisson's equation

The potential ϕ in the epitaxial layer can be found by using the Poisson's equation [3]

$$\nabla^2\phi = -\frac{\rho}{\epsilon}, \tag{1}$$

where ρ is the net volume charge and ϵ is the permittivity. The electric field is

$$\vec{\mathcal{E}} = -\nabla\phi \tag{2}$$

and the net volume charge is

$$\rho = q(N_D - n), \tag{3}$$

where the q is the charge of the electron and N_D is the doping density. The density of free electrons is

$$n = n_i e^{(E_F - E_i)/kT}, \tag{4}$$

where n_i is the intrinsic electron density, E_F is the Fermi energy level, E_i is the intrinsic Fermi level, k is Boltzmann's constant and T is the temperature. In the undepleted layer the density of free electrons is equal to N_D and therefore the net volume charge is zero. In the totally depleted layer the density of free electrons is zero and the net charge per unit volume is equal to qN_D. In a static situation (no currents flow in the undepleted layer) the net volume charge depends on the potential as

$$\rho = qN_D(1 - e^{q\phi/kT}). \tag{5}$$

3.2 Current density

The electron current density is the sum of the drift and diffusion components [4, 5]

$$\vec{J_n} = q\mu(\mathcal{E})n\vec{\mathcal{E}} + qD_n\nabla n, \tag{6}$$

where $\mu(\mathcal{E})$ is the electron mobility and the diffusivity D_n is given by the Einstein relation

$$D_n = \frac{kT}{q}\mu(\mathcal{E}). \tag{7}$$

In the depleted layer the density of free electrons is zero and therefore the electron current is also zero. In the undepleded layer the density of free electrons is constant N_D and, therefore, the electron current is

$$\vec{J_n} = q\mu(\mathcal{E})N_D\vec{\mathcal{E}}. \tag{8}$$

Because in the simulation program the transition front is assumed to be abrupt, the diffusion current is always zero. In reality, the diffusion current affects the density of free electrons near the transition front. However, the diffusion current does not affect the conduction current in the epitaxial layer and therefore the diffusion current can be negleted.

As the continuity equation for the electron current is

$$\nabla \cdot \vec{J_n} - q\frac{\partial n}{\partial t} = 0, \tag{9}$$

the velocity of the transition front can be derived to be

$$\vec{v} = -\mu(\mathcal{E})\vec{\mathcal{E}}. \tag{10}$$

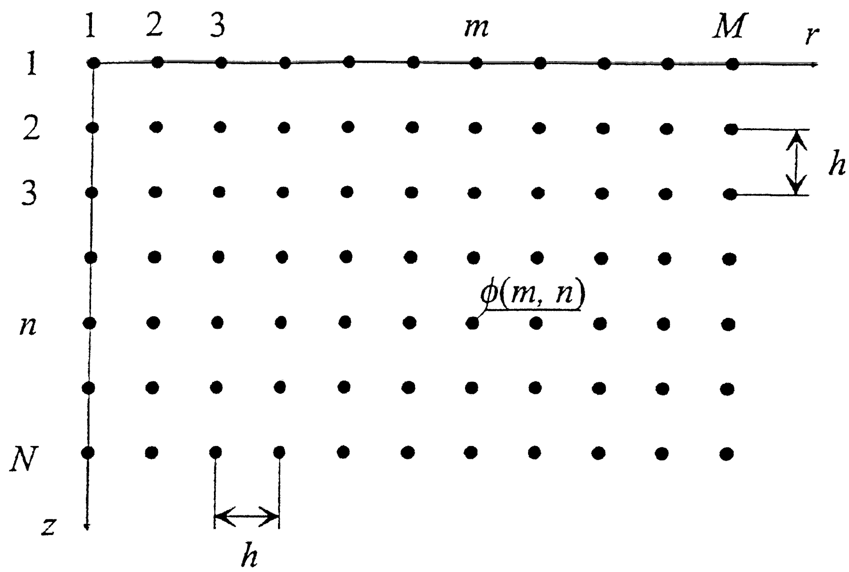

Figure 2: Mesh used in simulations.

4 Numerical method

4.1 Algorithm

Because the problem can not be solved by using analytical methods, the effect of the voltage modulation on the shape of the transition front has been studied by using a numerical method. The Poisson's equation has been solved numerically by using the finite difference method, where the epitaxial layer is divided into a dense mesh of size $M \times N$ as shown in Figure 2. The potential of each mesh point $\phi(m, n)$ is determined by using discrete Poisson's equation and iterative over-relaxation method [6, 7].

The transition front between the depleted and undepleted layers has been modelled by using discrete corner points as shown in Figure 4. The corner points are moved according to the electric field, which is calculated from the potential by using numerical version of equation (2). The net volume charge of each mesh point is determined accoring to the information of the corner points. If the mesh point is inside the transiton front (depleted layer) the net volume charge is N_D and otherwise (undepleted layer) the

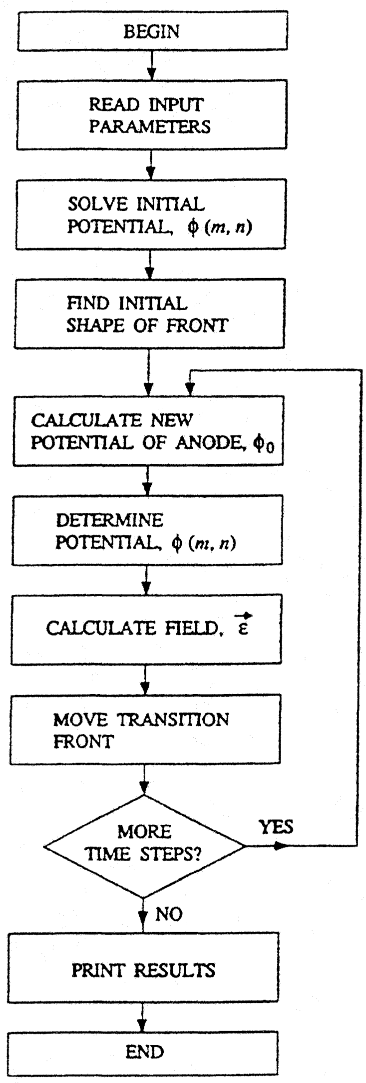

Figure 3: The computer algorithm.

charge is zero at the mesh point.

The computer algorithm of the simulation program is shown in Figure 3. At the beginning of the simulation the potential of the epitaxial layer is determined by using the numerical version of equations (1) and (5) and by using the input parameters ($N_D, \epsilon_r, R_0, t_e$, etc.) [6, 7]. Then the initial shape of the transition front is determined from this numerically solved potential. The discrete corner points of the transition front are placed smoothly over the transition front so that the vertical (or horizontal) spacing between corner points is equal to the spacing of the initial mesh as shown in Figure 4.

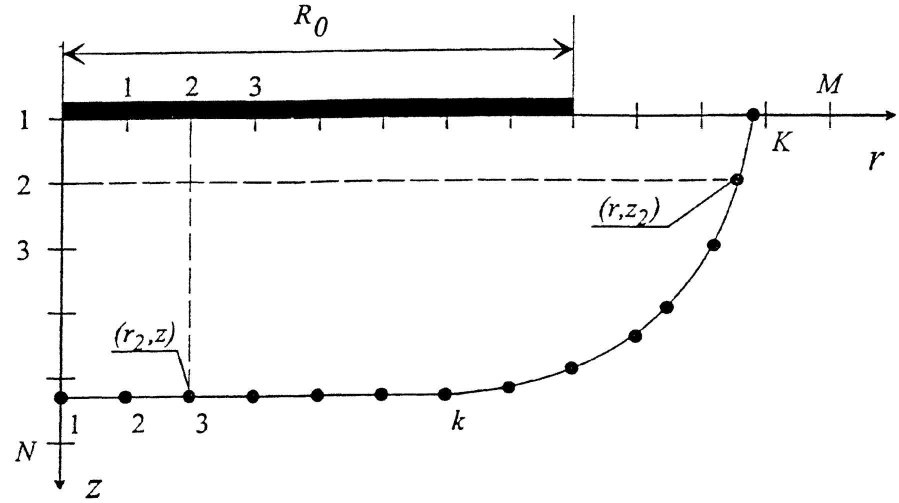

Figure 4: Corner points of the transition front.

At the beginning of each time step, the potential of the anode is determined according to the input parameters of the program. Then the net volume charge of each mesh points is determined from the shape of the transition front (charge is zero in the undepleted layer and equal to qN_D in the depleted layer). After that the potential of each mesh point is calculated numerically by using the iterative over-relaxation method. When the potential is known the electric field is found by using a numerical version of equation (2). Finally the corner points of the transition front are moved according to the electric field. The movement of the corner point is given by

$$\Delta s = -\mu(\mathcal{E}_n)\mathcal{E}_n \Delta t, \tag{11}$$

where Δt is the time step and $\mathcal{E}_n$ is the normal component of the electric field at the corner point.

4.2 Junction capacitance

By using static equations, the junction capacitance of millimeter wave Schottky varactor is given by [7]

$$C_j = \frac{\epsilon A}{W}\gamma_C, \tag{12}$$

where A is the area of anode, W is width of depletion layer and the net correction factor compared to the simple plate capacitance is given by

$$\gamma_C = \left(1 + 1.5\frac{W}{R_0} + 0.3\frac{W^2}{R_0^2}\right), \tag{13}$$

where R_0 is the radius of anode.

Because the shape of the depletion layer during a voltage modulation differs from that obtained by using the static equations, the junction capacitance during a transient must be determined by using the net charge of the depletion layer. The net charge can be calculated by using information of the corner points and it is given by the numerical equation

$$Q_{tot} = qN_D \sum_{k=2}^{K} 2\pi \frac{r_k + r_{k-1}}{2} \frac{z_k + z_{k-1}}{2}(r_k - r_{k-1}), \tag{14}$$

where r_k and z_k are the coordinates of the corner point k. The junction capacitance at the time step t is given by

$$C_j = \frac{\Delta Q}{\Delta V_j} = \frac{Q_{t+1} - Q_{t-1}}{V_{t+1} - V_{t-1}}, \tag{15}$$

where Q_t is the net charge of the depletion layer at the time step t and V_t is the voltage over the depletion layer.

5 Results

The simulation program can be used to analyse any millimeter or submillimeter wave Schottky varactor. Because the time required to analyse the varactor during a single pump cycle is very long (about 10 h), the program can not be integrated to the harmonic balance method. For that reason we have analysed the varactor during a linear voltage sweep, which is a good approximation for the modulation of the anode voltage, when the charge of the depletion layer is loaded or unloaded during the pump cycle [8]. The results have been used to develop the equivalent circuit, which can be employed in the harmonic balance method.

5.1 Shape of the transition front

Because the effect of the voltage modulation depends on the parameters of the varactor as well as on the frequencies and power levels of the multiplier, the actual shape of the depletion front must be analysed separately for each case. As an example we have analysed a very small area Schottky varactor, whose parameters are given in Table 1. The thickness of the epitaxial layer has been optimized for 1 THz operation so that the epitaxial layer is just depleted during the pump cycle [9]. At the beginning of the simulated voltage sweep the applied voltage V is slightly smaller than the contact potential ϕ_{bi}, so that the epitaxial layer is almost totally undepleted. During the simulation the potential of the anode has been decreased by 5 mV/fs. The total time of the voltage sweep is 0.5 ps, which is half of the cycle at the output frequency. This kind of a voltage sweep simulates the situation, where the varactor is pumped with large input power.

Table 1: Parameters of the 1 THz range Schottky varactor.

Radius of the anode	R_0	0.7	μm
Thickness of the epitaxial layer	t_e	0.2	μm
Doping density	N_D	$1.0 \cdot 10^{17}$	cm^{-3}
Series resistance	R_s	12	Ω
Junction capacitance at zero bias	C_0	1.8	fF

The transition front between the depleted and undepleted layer is shown in Figure 5, where time difference between two fronts is 50 fs. At the beginning of the voltage sweep the epitaxial layer is almost totally undepleted and the shape of the transition front is equal to the shape which can be obtained by static equations. When the currents begin to flow in the epitaxial layer, the volume of the depleted layer increases. Because the potential of the anode is decreased very fast, the electric field in the epitaxial layer is high and so the transition front moves with the maximum velocity of electrons.

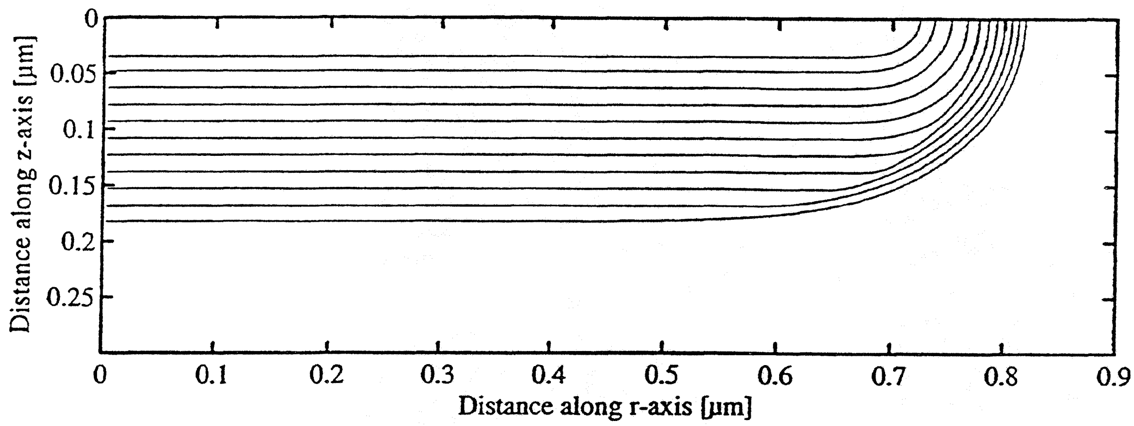

Figure 5: Example of the shape of the transtion front during a voltage ramp.

The electric field is extremely high near the edge of the anode, which means that the transition front moves there with the maximum, saturated velocity of electrons. So, the shape of the transition front is almost equal to the shape of a quarter of a circle as shown in Figure 5. This means that the volume of the depletion layer increases faster than can be assumed by using static equations. At the end of the voltage sweep the velocity of the transition front near the center of the anode is still equal to the maximum velocity of electrons. However, near the edge of the anode the velocity of the transition front decreases, because the electric field decreases, as the distance from the edge of the anode increases. This means that at the end of the the sweep the increase of the net charge is slower than can be assumed by using static equations.

5.2 Equivalent circuit

The calculated capacitance of the R_0 =0.7 μm varactor during a 1 THz voltage sweep is shown in Figure 6. At the beginning of the voltage sweep (applied voltage is almost equal to ϕ_{bi}) the numerically obtained junction capacitance is larger than the static capacitance, because the net volume of the depletion layer increases faster than can be assumed by using static equations. At the end of the voltage sweep (epitaxial layer is almost totally depleted and $V \approx -1.5$) the net volume charge increases slowly and

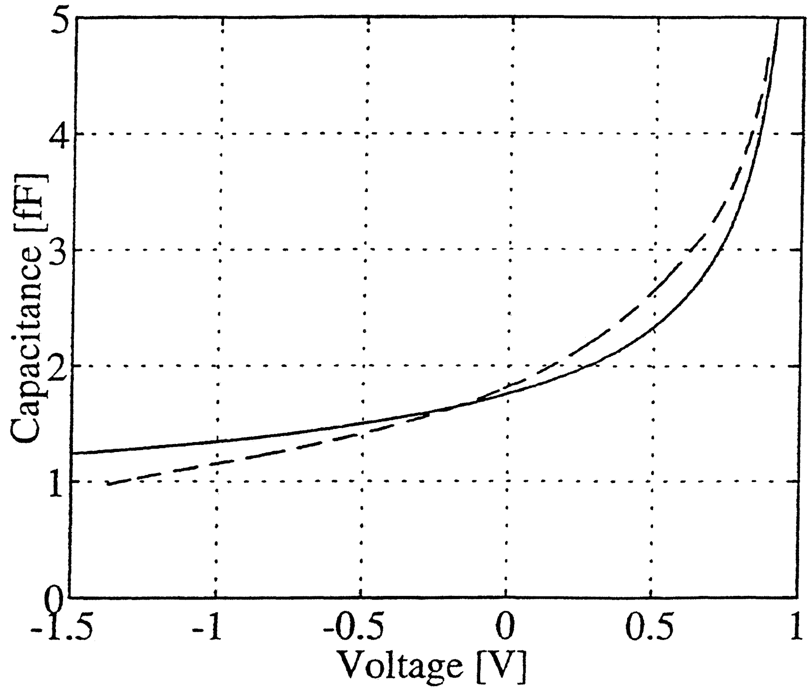

Figure 6: The capacitance of the varactor. Solid line is obtained with static equations and dashed line with a numerical solution using a 1 THz sweep starting from $V = \phi_{bi}$.

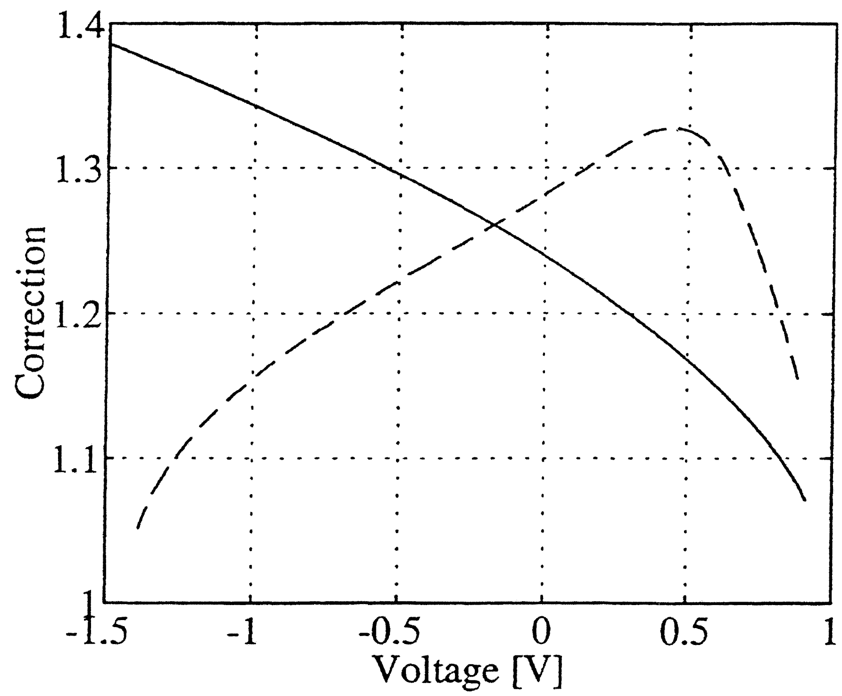

Figure 7: The correction factor γ_C of the varactor. Solid line is obtained with static equations and dashed line with a numerical solution.

thus the numerically obtained capacitance is smaller than the static capacitance. This means that during a fast voltage modulation the C_{max}/C_{min}-ratio is larger than can be assumed by static equation.

At submillimeter wavelengths the maximum output power of the Schottky varactor frequency multiplier is strongly affected by the electron velocity saturation [10]. The saturation effect can be included in the model of the varactor by limiting the electron conduction current to the value of the maximum current [9]

$$i_{max} = AN_D q v_m \gamma_C, \tag{16}$$

where v_m is the maximum velocity of electrons. The net correction factor γ_C can be calculated from the numerically obtained capacitance by using equation (12). As shown in Figure 7, at the beginning of the voltage sweep the numerically obtained correction factor is larger than the static value. However, at the end of the voltage sweep the numerically obtained correction factor is small. This means that at the beginning of the voltage sweep the electron conduction current may be much larger than can be assumed by using static equations. This helps to pump the varactor more effectively.

5.3 Doubler for 1 THz

As an example we have simulated a doubler for 1 THz. At small input power levels the effect of voltage modulation is small and thus the capacitance-voltage characteristic derived using static equations can be used. However, when the input power increases, the effect of voltage modulation increases and the numerically obtained capacitance characteristic should be used. According to our results the theoretical output power without circuit losses at 1 THz is about 200 μW, when a doubler is pumped with 2.0 mW input power at 500 GHz. This is 1.5 dB higher than the simulated output power obtained by using the static equations. However, the output power is still much smaller than the power obtained, if the electron velocity saturation is omitted.

Above we have used a very small area Schottky varactor ($R_0 = 0.7\mu$m), in which case the edge effects are shown clearly. If the area of the varator is larger than assumed above, the edge effects are smaller. This means that the effect of voltage modulation is not as large as above. Therefore, in the case of large area Schottky varactor the static equations can be employed.

6 Conclusions

The shape of the transition front between the depleted and undepleted layer in a small area Schottky varactor is strongly affected by the fast voltage modulation during the pump cycle of a submillimeter wave frequency multiplier. The effect can be studied by numerically solving the device equations. According to our results the C_{max}/C_{min}-ratio of the junction capacitance is larger than can be assumed by static equations. In addition, the electron conduction current is larger than derived from static equations, which helps to pump the Schottky varactor more effectively at THz range. These two effects together result in simulations in higher output powers than those obtained with static equations including the electron velocity saturation.

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