

## **Determining input loss in SIS receivers**

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### **Abstract**

*We present measurements on two high frequency S(uperconductor) I(nsulator) S(uperconductor) heterodyne receivers at 660 GHz and 810 GHz. Their noise temperatures are 163 K and 860 K, respectively. The contribution of the high frequency (RF) input loss to this number is measured with the commonly used method of intersecting lines. The result is compared to a more rigorous analysis using the full quantum theory of mixing. We will show that in this frequency range the method of intersecting lines in general leads to an overestimation of the RF input loss. Even at 660 GHz, with a well matched SIS junction, the input noise contribution is overestimated by about 25 percent. This is due to the summation of the contributions due to the vacuum fluctuations, which is the highest contribution, the mismatch at the intermediate frequency (IF), and the fact that the mixer output noise is not completely independent of local oscillator (LO) power. At 810 GHz, due to a much stronger loss-induced mismatch of the SIS junction, the mixer output noise varies considerably with LO-power. The large loss of approximately 8 dB directly in front of the mixer enhances this contribution to the measured RF input loss. The contribution of the IF-mismatch is likewise enhanced. Together with the contribution of the vacuum fluctuations, this can lead to an overestimation of the RF input noise by more than 100 percent, depending on the actual mismatch of the mixers. We therefore conclude that, for the severe mismatch that occurs in Nb based SIS mixers above 700 GHz, the method of intersecting lines gives incorrect results and the full analysis should be applied.*

### **1 Introduction**

The noise temperature of niobium SIS mixers is now, by better junctions and by better design, reduced to only a few times the quantum limit, especially at frequencies below 700 GHz. Frequently receiver noise temperatures at these frequencies are dominated by the other noise sources than the mixer [1], [2]. The reduction in signal to noise ratio by losses in the focusing optics or in the unavoidable cryostat windows often constitutes at least 25% of the total receiver noise.

Accurate analysis of the distribution of the total receiver noise over its various contributions is important. Not only to locate the areas where a receiver might be improved, but also to compare several designs, often from different laboratories.

In recent years a particularly attractive and quick method, the so called intersecting line method (ISLM), developed in Ref. [3] and [4], has come in general use to determine the noise contribution of the RF-input loss, in front of the mixer. The main advantage of the method is that no additional parameters of the mixer have to be determined. From the receiver noise temperature and the conversion loss at a number of local oscillator settings, the input noise is determined by a simple linear fit. This measurement can be done at any telescope receiver without extra facilities.

The accuracy of the method was discussed in Ref [4] and it was concluded that the method should be as accurate as the measurement of the receiver noise temperature and that the several correction terms should all be smaller than the quantum temperature  $hf/k$ , except for mixers that are severely mismatched to the local oscillator (LO). The influence of the noise originating from the vacuum fluctuations, the quantum noise, is only briefly discussed for mixers that are perfectly matched to the LO. It is concluded that for that case the unavoidable quantum noise is fully included in  $T_{rf}$ .

We noticed, from our own experience and from other measurement data [1], [4] - [6], that the estimate of the noise contribution of the input loss by this method is always somewhat high and that the losses responsible for the measured input noise temperature of the system can only be partly explained from an inspection of the optics. Especially our measurements just below and just above the gap frequency of niobium at 660 GHz and 810 GHz, with comparable optics, raised questions about the accuracy of the ISLM. The optics of the 810 GHz test set up always contributed at least three times as much noise to the receiver as the 660 GHz test set-up, according to the input noise temperature measured by the ISLM.

In this paper we will determine the contribution of the RF losses to the noise temperature of the receivers at both frequencies in two ways. The results of the ISLM and of a more elaborate approach, where the noise contribution of all the components of the receiver is determined separately with help of the quantum theory of mixing [7] will be compared. We will argue that the ISLM will only give a correct result in the case of a mixer with an almost ideal embedding impedance and a low RF loss in the warm as well as in the cold optics or electronic circuitry in front of the mixer.

## 2 Method

We will in short describe the two methods that we use to determine the high frequency input loss of SIS mixers. They both are based on the measurement of the output power of the total receiver in a certain bandwidth as a function of the input power of the receiver. As input source we use a blackbody of 295 K (hot) and 77 K (cold) physical temperature.

All temperatures used in the equations, unless explicitly stated as physical temperatures, are noise temperatures that represent equivalent noise input powers given by  $kBT$ , with  $k$  the Boltzmann constant,  $B$  the detection bandwidth and  $T$  the noise temperature.

In both methods we use the output power of the receiver  $P_{out}$  written as:

$$P_{out} = \left\{ (P_{in} + kT_{rf}) G_{rf} G_{mix} G_{4.2K} G_{if} + P_{mix}^{out} G_{if} + kT_{if} G_{if} \right\} B \quad (1)$$

with  $P_{in}$  the input power of the receiver,  $kT_{rf}$  the noise power added by the RF input losses and  $G_{rf}$  the transmission of the RF input section.  $G_{4.2K}$  is the transmission at 4.2K physical temperature, inside the dewar,  $G_{if}$  is the total IF gain,  $T_{if}$  is the noise temperature of the IF amplifier(s), and  $B$  is the detection bandwidth.  $P_{mix}^{out}$  is the output noise of the mixer per unit bandwidth, and  $G_{mix}$  is the total mixer gain

For the ISLM its convenient to write the total receiver noise temperature  $T_{rec}$ , using the notation of Eq. (1), as

$$kT_{rec} = kT_{rf} + \frac{P_{mix}^{out} + kT_{if}}{G_{rf}G_{4.2K}} \frac{1}{G_{mix}} \quad (2)$$

$T_{rec}$  is measured several times for different sub-optimum local oscillators powers. The ISLM uses the fact that for LO powers below the optimum level  $P_{mix}^{out}$  is approximately constant, as had been shown in Ref. [8]. With this assumption  $T_{rf}$  is found from the intercept of  $T_{rec}$  at different LO powers against the corresponding value of  $1/G_{mix}$ . A more complete description of the intersecting line method (ISLM) can be found in Ref.'s [3] and [4].

In the second method for determining  $T_{rf}$  we use again Eq. (1) for the output power of the mixer, but now the terms other than that with  $T_{rf}$  are determined separately. For that purpose we apply the quantum theory of mixing (QTM) in the three port approximation [7]. To do so the receiver output power has to be measured as a function of bias voltage at the optimum LO power level, and without LO-power. In addition the junction IV-curve has to be measured at the same local oscillator settings.

The bandwidth is set by an accurate bandfilter in the IF chain. The IF gain and noise temperature are determined by the well established shotnoise method [9], that uses the output noise of the junction without LO power as calibrated input noise source for the IF amplifiers.

$G_{mix}$  and  $P_{mix}^{out}$  are calculated from the QTM.  $G_{mix}$  is given by the sum of the conversion from the upper side band,  $G_{m_{01}}$ , and from the lower side band,  $G_{m_{0-1}}$ , to the IF

$$G_{mix} = G_{m_{01}} + G_{m_{0-1}} = 4G_{load} \left( G_{usb} |Z_{01}|^2 + G_{lsb} |Z_{0-1}|^2 \right) \quad (3)$$

$G_{load}$  is the real part of the terminating admittance at the IF port of the mixer,  $G_{usb}$  that at the upper side band port and  $G_{lsb}$  that at the lower side band port.  $Z_{ij}$  is the ij-element of the conversion matrix  $Z$  obtained by inverting the matrix  $YT$ ,

$$YT_{ij} = Y_{ij} + \begin{pmatrix} Y_{usb} & 0 & 0 \\ 0 & Y_{load} & 0 \\ 0 & 0 & Y_{lsb} \end{pmatrix} \quad i, j = -1, 0, 1 \quad (4)$$

with  $Y_{ij}$  the small signal admittance matrix that connects the small signal voltages and currents in the junction. The elements of  $Y_{ij}$  are given by equations (4.49-4.51) in Ref [7]. The diagonal matrix contains the full embedding admittance of the mixer, given by the terminating admittance at the upper side band,  $Y_{usb}$ , at the lower side band,  $Y_{lsb}$ , and at the IF port,  $Y_{load}$ . The embedding impedances for different frequencies are determined from the measured DC IV-curves with and without local oscillator power.

To determine the output noise of the mixer per unit bandwidth we use the expression derived in Ref. [11], which has the advantage that the contribution of the quantum noise to the output noise of the mixer is explicitly stated.

$$P_{mix}^{out} = G_{load} \sum_{i,j=-1,0,1} Z_{oi} \overline{Z_{oj}} H_{ij} + \frac{1}{2} h \sum_{i=-1,0,1} G m_{oi} |i f_{lo} + f_{if}| \quad (5)$$

of which the first term is the contribution of the shotnoise and the second term is identified as the mixer noise output due to the unavoidable half quantum of noise input per side band due to the vacuum fluctuations [12]. As is clearly explained in this reference the sum of both terms gives the correct output noise of the mixer for every embedding impedance.

The shotnoise term in Eq. (5) is derived in Ref. [7], where the elements of the current correlation matrix  $H$  are given by Eq. (4.69). The input power per unit bandwidth from the blackbody load of temperature  $T$  at frequency  $f$  is calculated by the Planck formula [11],

$$P(T, f) = \frac{hf}{\exp\left(\frac{hf}{kT}\right) - 1} \quad (6)$$

$T_{rf}$  and  $G_{rf}$  are related via

$$T_{rf} = \frac{(1 - G_{rf})}{G_{rf}} T_{295K} + \frac{(1 - G_N)}{G_N G_{rf}} T_{77K} \quad (7)$$

with  $G_N$  the known transmission of the infrared filter of the dewar at 77 K. An equivalent noise temperature for  $G_{4.2K}$  is neglected because the noise power  $kBT_{4.2K}$  is very small.

$G_{rf}$  (and so  $T_{rf}$ ) and  $G_{4.2K}$  are determined by fitting the calculated receiver output power according to Eq. (1) to the measured output power, for the hot and the cold input. By adapting  $G_{4.2K}$  the overall gain is changed but the noise input stays equal, and so this parameter is mainly adapted to fit the output power at a hot input. Next, by  $G_{rf}$  which hardly changes the calculated output for a hot input, the output for a cold load input is fitted.

This gives a second way to determine  $T_{rf}$  that is much more elaborate and complicated. In the next section the result of the two methods will be compared.

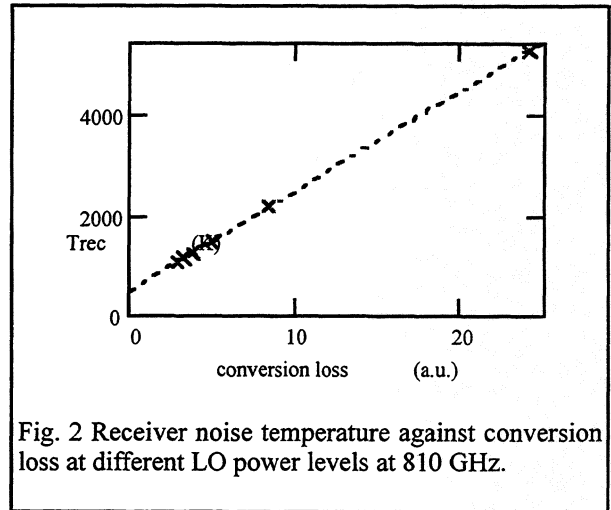
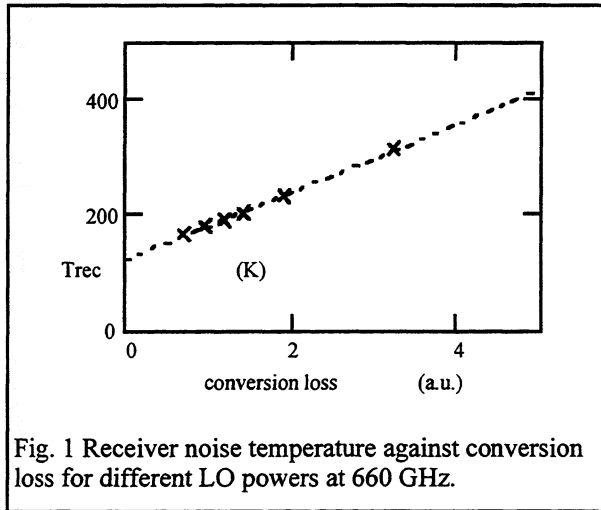
### 3 Results

All measurements are done in a liquid helium dewar at a temperature of 4.2 K. The vacuum window of the dewar and the 77 K-infrared filter are sealed with Teflon or Mylar foils. The thickness of the foils is optimized to achieve a transmission of about 98 % in the current frequency band.

For 660 GHz the local oscillator and the signal are combined with a beamsplitter. The calculated reflection of the beamsplitter is 5%. At 800 GHz we use a Martin-Puplett diplexer with a transmission loss of about 7 %.

The mixers are fixed tuned waveguide mixers and are described in detail in Ref. [2] and [13] In the present measurement set up the lowest receiver noise temperature at 660 GHz is 163 K ,

and at 810 GHz 860 K. The center frequency of the cooled HEMT amplifier is 1.4 GHz and its noise temperature is 4 K. The IF output power is measured in a 100 MHz bandwidth around the



center frequency.

The result of the hot/cold load measurements at different local oscillator powers below the optimum are given in Fig. 1 for 660 GHz and in Fig. 2 for 810 GHz. The intersection point with the vertical axis is 125K at 660 GHz and 470 K at 810 GHz. These temperatures represent again equivalent noise powers.

The measured output power as a function of bias voltage for a hot and a cold input at optimum LO-power are given in Fig. 3 and 4 for 660 GHz and 810 GHz resp. by the solid lines. The calculated output powers are given by the (+) signs for the hot load input and the (o) signs for the cold load input. The wiggles on the calculated curves stem from noise in the measured IV-curves of the junctions.

To obtain the agreement as shown in the Fig. 3 and 4 ( $G_{rf}$ ,  $G_{4.2K}$ ) is chosen (0.78, 0.75) for 660 GHz and (0.59, 0.36) at 810 GHz. Using Eq. 7 these values for  $G_{rf}$  give  $T_{rf}$  equal to 82K for 660 GHz and 183K for 810 GHz.

The embedding impedance normalized to the junction normal state impedance is  $0.5 + 0.1i$  for 660 GHz and  $0.15 - 0.04i$  at 810 GHz. The embedding impedances are not obtained from a real fit to the pumped IV-curve in the sense of Ref. [10]. They are determined from calculations of the embedding circuitry and then slightly adapted to obtain best agreement between the measured and calculated pumped IV-curve.

## 4 Discussion

At both frequencies the two values of  $T_{rf}$ , determined by the two methods, do not correspond. In addition the difference in  $T_{rf}$  of 125K at 660 GHz and 470K at 800 GHz, as obtained by the ISLM, is much larger than expected from an inspection of the optics.

To investigate the difference between the two methods we focus on the assumption that  $P_{mix}^{out}$  is constant. Smaller effects due to mismatch at the IF port of the mixer are not discussed. It can be seen immediately from Eq. (2), that any component of  $P_{mix}^{out}$  depending linearly on  $G_{mix}$  results in

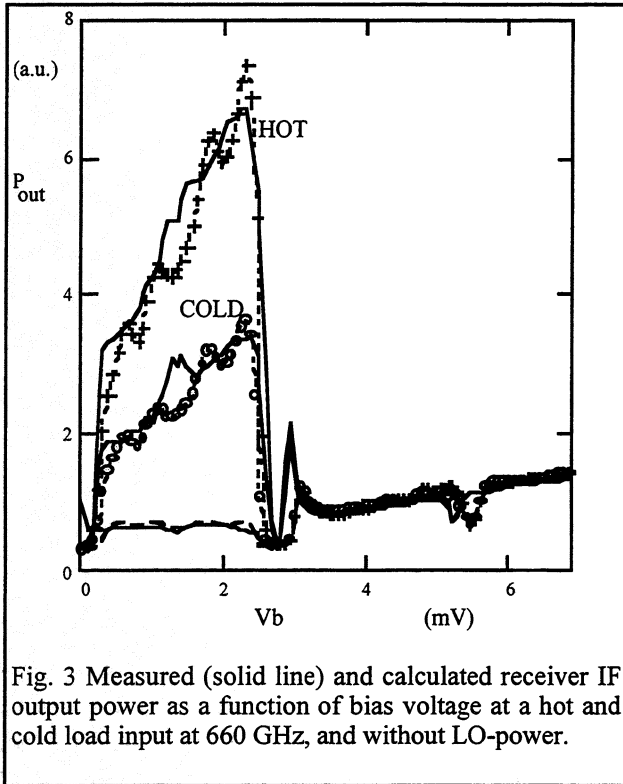


Fig. 3 Measured (solid line) and calculated receiver IF output power as a function of bias voltage at a hot and cold load input at 660 GHz, and without LO-power.

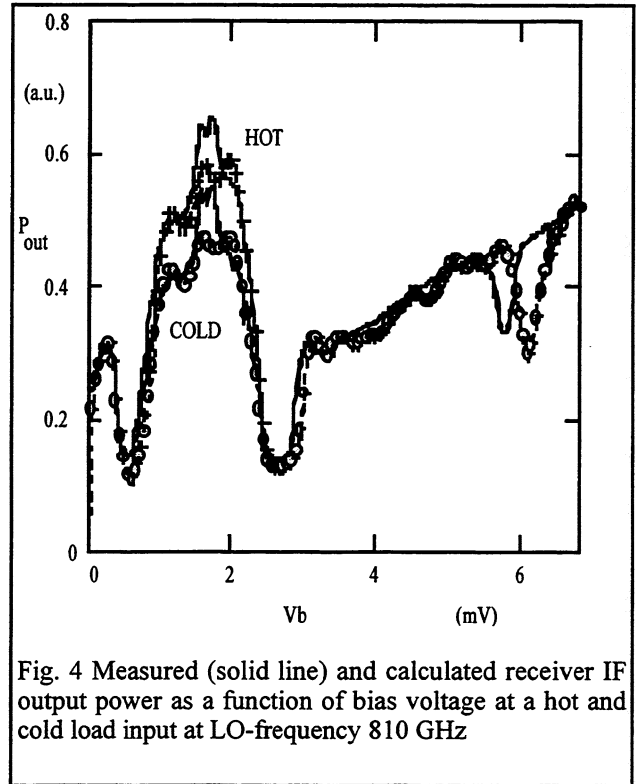


Fig. 4 Measured (solid line) and calculated receiver IF output power as a function of bias voltage at a hot and cold load input at LO-frequency 810 GHz

a constant contribution to  $T_{rf}$  which would have no effect on the quality of the linear fit to the results.

If we insert Eq. (3) and (5) in Eq. (2), given that the term with  $G_{m_{00}}$  in Eq. (5) is approximately constant, it is clear that the quantum noise term of Eq. (5) will always add to  $T_{rf}$ . Unfortunately, for a mixer with a lossy input coupling, the contribution of the quantum noise to  $T_{rf}$  is larger by the unknown factor  $1/(G_{rf}G_{4.2K})$ .

Furthermore, if we investigate the dependence of the shotnoise term on  $G_{mix}$ , we get the result as is given in Fig. 5 and 6. The mixer output power is calculated for the embedding impedances given above at several LO-power levels. In each figure two curves are given. The lower curve shows the dependence of only the shotnoise term on  $G_{mix}$ . The upper curve is a linear fit to the total output noise, calculated for eight different values of the LO power. We verified that for an embedding impedance equal to one (normalized to the junction normal state resistance) the calculated shotnoise output is indeed almost constant with  $G_{mix}$ .

Already at 660 GHz, where the receiver noise temperature is quite low, the imperfect embedding impedance causes a positive slope of 11 K. At 810 GHz, due to the imperfect compensation of the junction capacitance by the lossy integrated tuning structures [13], the embedding impedance is worse than at 660 GHz. Consequently the slope of the lower line is higher, 38 K, which is well above  $0.5 hf/k$ . The total output noise, given in both Fig.'s by the upper line has a slope which is approximately  $0.5 hf/k$  higher than the slope of the lower curve. For 660 GHz the total slope is 27K, and at 810 GHz it is 58K.

We can use the values for  $G_{rf}$  and  $G_{4.2K}$  to calculate a correction to the  $T_{rf}$  that has been measured by the ISLM. The small value of  $G_{4.2K}$  (0.36) at 810 GHz resulting from the second method to determine  $T_{rf}$  is not unreasonable if compared to the calculated transmission of the integrated tuning structure [13]. We subtract the value of the slope of the upper curves in Fig. 5

and 6 divided by  $G_{rf}G_{4.2K}$ . This corrected value of  $T_{rf}$  is in good agreement with the value determined directly from the second method. Phrased differently: A simulation of the ISLM using the QTM and the values for  $G_{rf}$  and  $G_{4.2K}$  as determined by the second method, yields a value for  $T_{rf}$  that is in good agreement with the value measured by the ISLM for both mixers.

We conclude that the assumption that the mixer output noise is independent of the mixer gain for low local oscillator power does not hold for both our mixers and that thus the ISLM gives too high values for  $T_{rf}$ .

Even if the embedding impedance would be perfect and the input loss both at ambient temperature *and* at 4.2 K would be negligible, a half quantum of unavoidable noise would be included in  $T_{rf}$ . As is shown in Ref.[14] this is due to the fact that we calculate the receiver noise temperature from the hot/cold load measurement using the Planck formula (Eq. 6) for the input noise from the loads. This is consistent since the quantum noise is included in the output noise of the mixer (Eq.5).

As is shown in Ref. 14, the receiver noise temperature determined from the hot/cold load measurement is lower by exactly  $0.5 hf/k$  if the quantum noise is included in the input signal. The input noise power per unit bandwidth (P) from the calibration loads is than calculated using the Callen & Welton formula  $P = 0.5 hf \coth(0.5 hf/kT)$  instead of the Planck formula. This emphasizes that to appreciate published values of  $T_{rf}$  measured by the ISLM for almost perfect mixers, one needs specific information about the calculation of the receiver noise temperature [1] [4].

For this analysis we prefer to use the Planck formula to calculate the input signal of the mixer because the contribution of the quantum noise to the output of the mixer can then be written explicitly in Eq. 5. If the quantum noise is included in the input signal one has to be very careful in introducing the input losses and the added quantum noise of these losses to end up with the correct output noise power of the mixer.

From a practical point of view one could attempt to correct a value of  $T_{rf}$  measured by the ISLM. Subtraction of  $0.5hf/k(1/G_{rf})$  or  $0.5hf/k(1/G_{rf} - 1)$ , depending on how  $T_{rec}$  is calculated, gives a better estimate for the quality of the input coupling.  $G_{rf}$  can be determined from the measured  $T_{rf}$  assuming that all of the loss occurs at 295K. This works reasonably well as long as  $G_{4.2K}$  is close to one. For niobium SIS mixers above the gap frequency of niobium, which generally have a low value for  $G_{4.2K}$  due to the unavoidable loss in the integrated tuning structure, values of  $T_{rf}$  measured by the ISLM and adapted in this way will still be much too high. In that case an estimate for  $G_{4.2K}$  is necessary and the full analysis should be applied.

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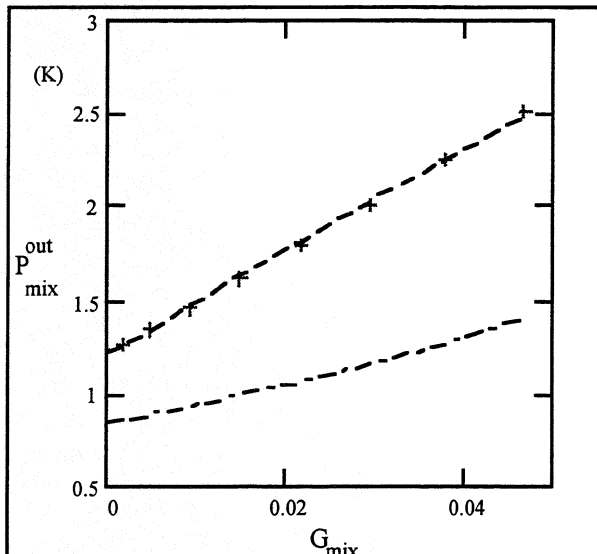


Fig. 5 Calculated mixer output noise at 660 GHz against mixer gain, for different LO power levels. The dash-dotted curve, the dependence of the shotnoise on  $G_{mix}$ , has a slope of 11.4K. The slope of the curve including the quantum noise is 27K, given by the upper line fitted to the calculated points.

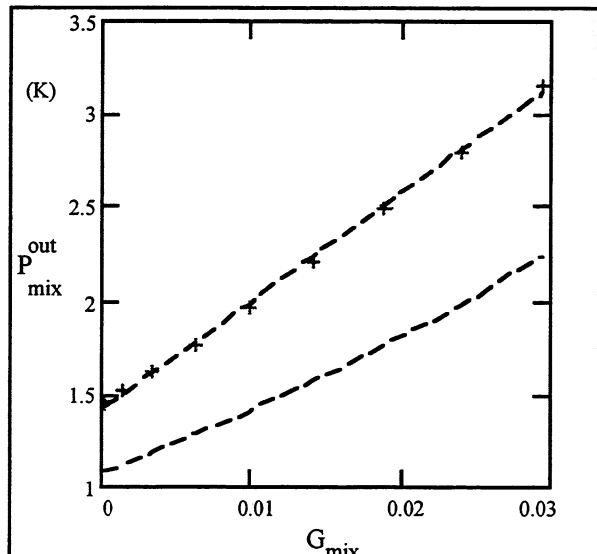


Fig. 6 Calculated mixer output noise at 810 GHz against mixer gain, for different LO power levels. The slope of the upper curve, given by the straight line fitted to the calculated points, is 58K. The lower curve gives only the contribution of the shotnoise to the mixer output noise and has a slope of 38K.

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