ELECTROMAGNETIC ANALYSIS OF FINLINE MIXERS

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ABSTRACT

We describe the electromagnetic design of antipodal finline SIS mixers. An important part of the mixer is the section of antipodal finline which transforms the high impedance of the waveguide ($\approx 300\Omega$) to the low impedance of the microstrip ($\approx 15\ \Omega$) in which the tunnel junction is located. We review the numerical methods that were used to calculate the electrical properties of the finlines, taking into account the finite thickness of the metallisation. We also explain how the transmission line taper was synthesized using an optimum taper method. The design procedure is illustrated by outlining the experimental behaviour of a 230GHz mixer, and scale model results are presented for a mixer which is currently being developed for 345 GHz.

INTRODUCTION

On previous occasions we have reported the design and performance of an antipodal finline mixer at 230 GHz. The arrangement comprises a diagonal waveguide horn, an antipodal finline transition and a miniature superconducting microstrip line which contains the tunnel junction. This mixer combines the directivity and low sidelobe levels of metallic horns with the simplicity, ease of manufacture and repeatability of planar-circuit technology. The noise performance and bandwidth of the mixer are comparable to the best waveguide mixers, and the mixer is easy to operate and does not employ mechanical tuning (Yassin *it al, 1997)*.

A key feature of the design is the use of a low impedance antipodal finline to transform the waveguide into a microstrip line. The two superconducting films which form the finline are deposited on one side of a quartz substrate and are separated by a thin oxide layer. Since the thickness of the oxide layer cannot easily be made thicker than $\approx$ 400nm and the width of the microstrip line cannot be made narrower than $\approx 3\mu m$ the embedding impedance of the device is low. More precisely, taking a microstrip width of $w = 3\ \mu m$, a dielectric thickness of $h = 400\,nm$ (dielectric constant of SiO_2=5.8), and a metallization thickness of $t = 400\,nm$ we obtain a source impedance of $15.5\,\Omega$ (Yassin and Withington, 1995).

One disadvantage of the finline mixer is that the waveguide-to-microstrip transition is rather difficult to design. In this paper we shall describe the electromagnetic methods that were used to design the finline tapers. As is well known, a difficulty when calculating the properties of miniature superconducting microstrip lines is that the metallization thickness cannot be neglected when it is comparable to the transmission line dimensions. In the case of microstrip lines the effect can be taken into account by using methods such as conformal mapping. In the case of finlines the problem is particularly difficult because of the complicated geometry and the dispersive nature of the multilayer structure. To date, we have calculated the electrical properties of finlines by using a combination of the ridged waveguide model and the spectral domain method, corrected for thickness by Wheeler's approximation. We have verified the conclusions of our theoretical work by performing measurements on scale models and investigating in some detail the performance of a 230 GHz mixer. In this paper we shall also describe a method based on matching the electromagnetic fields at the discontinuities in the cross section of the waveguide. An important advantage of this method is that it is capable of dealing with metallisation thickness rigorously and can be applied easily to a whole variety of finline configurations. Although the calculations can only yield cutoff frequencies, rather than propagation constants and characteristic impedances, knowledge of the former is sufficient to synthesize finline tapers, as we shall see later.

TAPER SYNTHESIS

The conventional way to taper a quasi-TEM transmission line is to taper the geometry according to the impedance profile. This method has two disadvantages. The first is due to the fact that the characteristic impedance of a non-pure TEM mode is not uniquely defined and the second is that the calculation of the characteristic impedance of finlines with thick metallisation and all gap dimensions is not easy. We design our transmission lines according to an "Optimum Taper Method" which only requires knowledge of the propagation constant (and the cutoff frequency) as a function of lateral dimensions. This method is based on minimizing the coupling coefficient χ^{-+} between the incident and reflected waves along a quasi-TEM transmission line. The end product is a minimum-length high-pass section which gives a return loss lower than a specified design value R_{max} at frequencies above a predetermined frequency f_0.

The reflection coefficient of a TEM taper of length L can be approximated by

$$R(f_\circ, L) = -\int_0^L \chi^{+-} \exp[-2j \int_0^z \beta(f_\circ, z')dz']dz \ . \tag{1}$$

To solve this equation we need to know the propagation constant, a problem we shall consider in the next section. In addition it has been shown that for any finline structure, the coupling coefficient can be written as (Hinken, 1983)

$$\chi^{+-} = \frac{1-(f_c/f_\circ)^2/2}{1-(f_c/f_0)^2} . \frac{1}{f_c} . \frac{df_c(z)}{dz} \tag{2}$$

where z is the coordinate along the taper and f_c is the cutoff frequency of the finline. The above equation can be written in the form

$$R(f_\circ, L) = C \int_0^{2\theta} K(\xi) \exp[-j\xi] d\xi \tag{3}$$

where $\xi(z) = \int_0^z 2\beta(f_0, z')dz'$, $2\theta = \xi(L)$ and C is a normalization constant so that

$$\int_0^{2\theta} K(\xi)d\xi = 1 \tag{4}$$

which yields

$$C = ln\left[\frac{f_{c1}}{f_{c2}}\left(\frac{1 - f_{c2}^2/f_o^2}{1 - f_{c1}^2/f_o^2}\right)^{\frac{1}{4}}\right] \ . \tag{5}$$

Here, f_{c1} and f_{c2} are the cutoff frequencies at the two ends of the taper. For a given distribution $K(\xi)$ we can then calculate the required taper length (or the value of θ) which yields a return loss which is less than a specified design value R_{max}. Moreover the distribution which gives the minimum value of R for a given length is the Dolph-Chebyshev polynomial (Spolder, 1979). Synthesis of the finline taper then proceeds as follows:

1. Determine the initial and final gaps of the finline section s_1, s_2 and calculate the corresponding cutoff frequencies f_{c1}, f_{c2}.

2. Choose a function $K(\xi)$ which gives the coupling coefficient distribution along the taper in the ξ-space, where the variable ξ is related to the variable z along the taper by $\xi(z) = \int_0^z 2\beta(f_0, z')dz'$. Calculate the normalization constant C and use eqn. (3) to determine the value of θ for the specified value of R_{max}.

3. Use the relation between the coupling coefficient and the cutoff frequency for both the unilateral and the antipodal configurations to obtain the cutoff frequency distribution $f_c(\xi)$. This relation can be written as

$$\begin{array}{l} f_c(\xi) = f_{c1}\left[F/2 + \sqrt{F^2/4 + (1-F).\exp[4CI(\xi)]}\right]^{-\frac{1}{2}} \\ F = (f_{c1}/f_o)^2 \\ I(\xi) = \int_0^\xi K(\xi')d\xi' \end{array} \ . \tag{6}$$

4. Calculate the corresponding finline gap from the known cutoff frequency using the transverse resonance approach or any other method (eg. spectral domain analysis)and then synthesize the taper using the relation

$$\Delta z = \frac{\Delta\xi}{2\beta(f_0, \xi}) \ . \tag{7}$$

The design of the microstrip taper was also based on the above method, but in this case an analytical expression could be derived to compute the characteristic impedance Z_0 as a function of the longitudinal coordinate z which can be written as (McGinnis and Beyer, 1988)

$$Z_0(z) = Z_{01}\exp\left\{\frac{1}{2}\ln(\frac{Z_{02}}{Z_{01}})\left[\sin[\pi(\frac{z}{L} - \frac{1}{2})] + 1\right]\right\} \ . \tag{8}$$

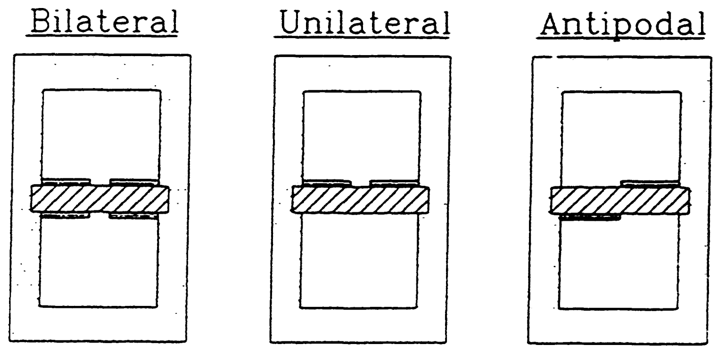

FIGURE I   The main finline configurations

This approach is convenient since the characteristic impedance of a microstrip line is unambiguously defined and can be calculated accurately using the conformal mapping method (Yassin and Withington, 1995).

We employed the above approach to design the waveguide to microstrip transition and found that a taper length of two wavelengths is sufficient to match the 300Ω impedance of the loaded waveguide to the 15.5Ω impedance seen by the junction.

CALCULATION OF THE ELECTRICAL PARAMETERS OF FINLINES

A schematic view of the well known finline configurations is shown in Fig. 1.

This structure has been investigated thoroughly in the literature, assuming infinitely-thin metallisation. It is, however, extremely difficult to identify a single full-wave analysis that can be applied to all types of finlines and yet take into account the finite thickness of the fins. One must, therefore, use several analysis techniques and approximations that can, jointly, be used to cover the whole of the waveguide to microstrip transition.

To put the analysis into context we first consider a magnified view of a chip geometry which was designed for a 230 GHz mixer as shown in Fig. 2.

The RF transmission line comprises a transition from waveguide, to modified antipodal finline, to microstrip. The fins (which constitute the base and the wiring layers) are made out of Nb and are separated by a 300 nm of SiO_2 and deposited on a 170 μm thick quartz substrate. In the region before the fins overlap the thickness of the SiO_2 layer is much less than that of the quartz substrate, and the transmission line behaves like a unilateral finline. The impedance in this section is brought down from several hundred ohms to 80 Ω as the finline gap is reduced from the height of the waveguide to about one micron. As the fins overlap the structure needs to be treated as an antipodal finline whose properties are mainly determined by the thin oxide layer rather than the thick quartz sheet, which from this stage onwards merely supports the structure. When significant overlap is achieved, the section behaves like a parallel-plate waveguide with an effective width equal to that of the overlap region. When the width becomes

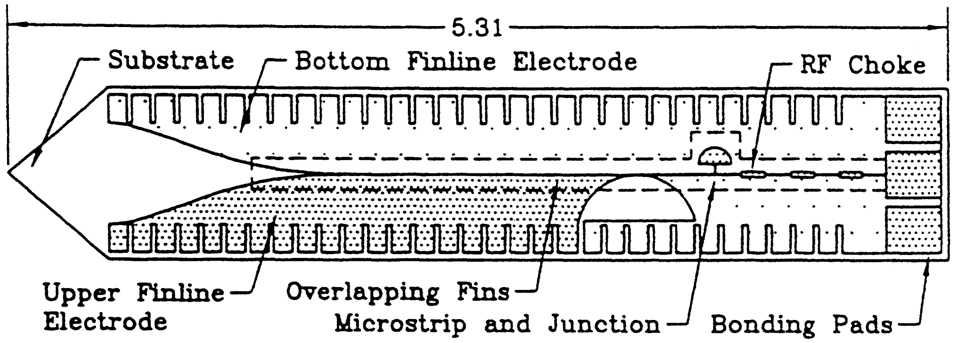

FIGURE II A schematic view of the mixer chip geometry.

large enough such that fringing effects can be ignored, transition to microstrip is performed which in turn is tapered to the required width of about 4 μm. We therefore conclude that the type of sections that we need to analyse, in order to synthesize the transition, are as follows:

1. A unilateral finline on a quartz substrate.
2. An antipodal finline with small overlap.
3. A parallel plate waveguide and microstrip with a large width-to-height ratio.

We shall now describe the various methods that may be employed to analyse the individual sections.

Spectral Domain Analysis (SDA)

Spectral Domain Analysis is a well established full-wave method for analysing multi-layer planar circuits with infinitely thin metallisation. Extensive theoretical work, which was carried on unilateral finline, revealed that there is only a mild dependence of the characteristic impedance on the film thickness and that the dependence of the propagation constant on the thickness is extremely small over much of the waveguide band (Kitazawa and Mittra, 1984). We therefore conclude that the Spectral Domain Method is recommended for calculating the electrical properties of the first section of the transition since it accurately accounts for dispersion which is significant in unilateral finlines with relatively large gaps. The method that we used, which is regarded as being computationally efficient, is well documented in the literature and therefore only a brief description is required here.

Let E_x and E_z represent the tangential electric fields across and along the gap of the finline and let J_x and J_z represent the transverse and longitudinal currents in the fins. In ordinary spatial analysis, the fields and currents are related by an integral equation with the dyadic Green's function as the kernel. In SDA however we relate the Fourier transform of these quantities by a matrix as follows

$$\begin{pmatrix} \tilde{J}_x(\alpha_n) \\ \tilde{J}_z(\alpha_n) \end{pmatrix} = \begin{bmatrix} G_{xx}(\alpha_n,\beta) & G_{xz}(\alpha_n,\beta) \\ G_{zx}(\alpha_n,\beta) & G_{zz}(\alpha_n,\beta) \end{bmatrix} \begin{pmatrix} \tilde{E}_x(\alpha_n) \\ \tilde{E}_z(\alpha_n) \end{pmatrix} \tag{9}$$

where $\alpha_n = \frac{n\pi}{b}$is the Fourier parameter of the x-coordinate and $G(\alpha, \beta)$ is the dyadic Green's function in the Fourier domain. Expressions for $G(\alpha, \beta)$ corresponding to a three layer waveguide (unilateral finline) and two layer waveguide (bilateral finline) may be found in the literature. The dyadic Green's function for the antipodal finline is a combination of the two layer and three layer systems. It is worthwhile noting that the simplest expressions for $G(\alpha, \beta)$ are those found using a transverse-resonance-like method called the immittance approach (Zhang and Itoh, 1987). In fact the poles of $G(\alpha, \beta)$ give the roots of the transverse resonance equations of a loaded waveguide.

The above matrix equation is solved using Galerkin's method. The electric fields E_x, E_y are expanded in terms of known basis functions ξ and η

$$\tilde{E}_x(\alpha_n) = \sum_{i=1}^{M} a_i \tilde{\xi}_i(\alpha_n) \tag{10}$$

$$\tilde{E}_z(\alpha_n) = \sum_{i=1}^{M} b_i \tilde{\eta}_i(\alpha_n) \tag{11}$$

$$\tag{12}$$

After taking the inner products of the above equations with $\tilde{\xi}_i$ and $\tilde{\eta}_i$, using Parseval's theorem and taking advantage of the fact that at any point across the finline either the current or the tangential electric field vanishes, we obtain a 2M $\times$ 2M set of linear equations. The coefficients of the resulting determinant have the form

$$A_{ij}^{uv} = \sum_{n=0}^{\infty} \tilde{\xi}_i(\alpha_n) G_{uv}(\alpha_n, \beta) \tilde{\eta}_j(\alpha_n)\ . \tag{13}$$

A wide range of basis functions have been chosen to solve the resulting set of homogeneous equations. We recommend the Legendre polynomials for the transverse field and sinusoidal functions for the longitudinal field. For a unilateral finline we may choose

$$\xi_m(x) = P_{2(m-1)}(2x/w) \tag{14}$$

$$\eta_m(x) = \sin(\frac{2m\pi x}{w})\ . \tag{15}$$

The reason for this choice is that Legendre polynomials yield accurate results for both wide and narrow gaps provided that at least the first five terms are used. Moreover since those polynomials form a complete set and satisfy the electric fields edge condition for large values of m, they may be used for more computationally demanding applications such as higher order modes or loss calculations.

In our designs we employed the Spectral Domain Technique to calculate the cutoff frequency and propagation constant at the start and the end of the taper

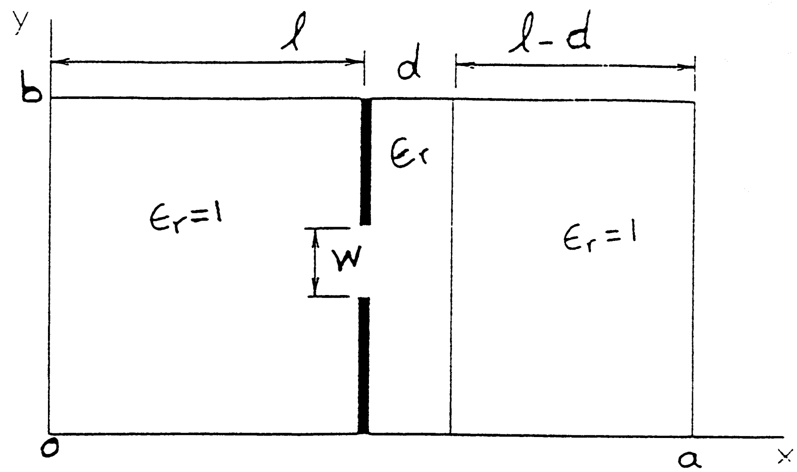

FIGURE III Transverse Resonance Calculation for Unilateral finline

and to validate the results obtained using the transverse resonance method, which we shall describe later. We also employed this method to provide a list of numbers to taper the second section. For this section we corrected for the finite thickness of the fins by applying Wheeler's approximation to the overlap region. The recommended basis functions for the antipodal finline are also a combination of Legendre polynomials and sinusoidal functions (Mirshekar-Syahkal and Davis, 1982)

The Transverse Resonance Approach

The Spectral Domain Method yields accurate results for unilateral finlines. When employed design tapers in conjunction with the optimum taper method described previously, however, a doubly iterative procedure results, which makes it computationally complicated. An alternative method which is simpler and yet gives accurate results is a transverse resonance method which takes into account the presence of the dielectric (Schieblich, Piotrowski and Hinken, 1984). We shall describe how this method can be applied to thin metallisation although it could be extended to handle thick films with the disadvantage of extra complexity. Let a and b be the width and height of the waveguide respectively and k_c the cutoff wavenumber as shown in Fig. 3.

The transverse resonance equation for a loaded waveguide is

$$-\cot(k_c l) - \cot[k_c(l-d)] + \frac{B}{Y} = 0 . \qquad (16)$$

Expressions for the normalized susceptance were then derived using an equivalent circuit which for a centred gap gave the following result

$$\frac{B}{Y} = \frac{b}{\pi} k_c [2P_1 + \epsilon_r (P_2 + P_3)]$$

$$\begin{aligned} P_1 &= \ln[csc(\tfrac{\pi w}{2b})] \\ P_2 &= r_d . \arctan(\tfrac{1}{r_d} + \ln\sqrt{1 + r_d^2}) \\ P_3 &= r_b . \arctan(\tfrac{1}{r_b} + \ln\sqrt{1 + r_b^2}) \\ r_d &= w/d \quad r_b = w/b \end{aligned} \qquad (17)$$

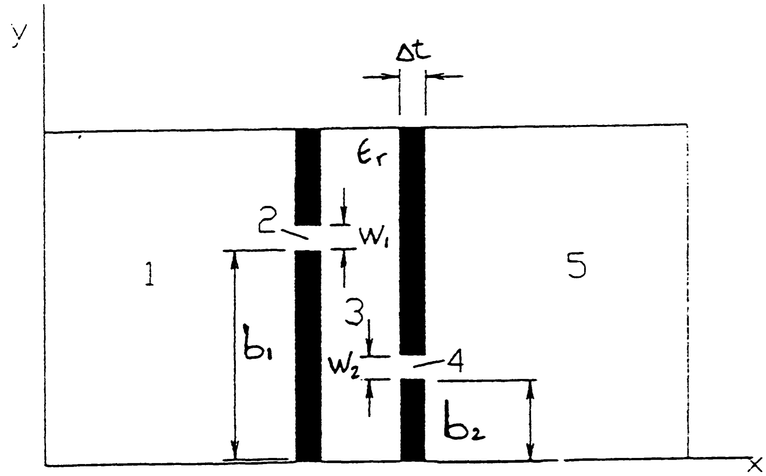

FIGURE IV A cross section for the three finline structures

The propagation constant β is found from the calculated cutoff frequency using the equivalent dielectric constant ϵ_{eq}

$$\begin{aligned} \beta/k_o &= \sqrt{\epsilon_{eq}[1-(\tfrac{f_c}{f})^2} \\ \epsilon_{eq} &= (f_{co}/f_c)^2 \end{aligned} \tag{18}$$

where f_{co} is the cutoff frequency for $\epsilon_r = 1$. We have designed and tested scale model unilateral finline tapers of length 1λ and measured excellent return losses across the full waveguide band.

The Field Matching Method

The arrangement for calculating the properties of finline using field matching is shown in Fig. 4 (Saad and Schunemann, 1982). This arrangement allows us to find a solution which is applicable to the three main finline configurations. For example, the gap centred unilateral finline is obtained by taking $b_2 = 0, d_2 = b$ and an antipodal finline is obtained for $b_1 = 0, b_2 = b - d_1$. Assuming that the fields are at cutoff, the arrangement may be considered to be a parallel plate waveguide with conductors located at $x = 0$ and $x = a$. The fins and dielectric are then considered as discontinuities and the fields are matched at the four interfaces. Since media 1 and 5 are identical, an eigenvalue problem is obtained which leads to a linear set of algebraic equations.

The solution can elegantly be represented by the equation

$$\mathbf{Ax} = \mathbf{b} \tag{19}$$

where $\mathbf{b}$ is an N-element constant vector, $\mathbf{x}$ is the unknown N-element vector whose components give the expansion coefficients of the fields in the five regions (once those coefficients are known the fields are known) and $\mathbf{A}$ is an N $\times$ N matrix whose components are functions of the unknown cutoff wavenumber k_c. Explicit expressions for the elements A_{mn} and the components of b_n are given in (Saad and Schunemann, 1982). It is interesting to notice that this method is can be used of calculating the parameters of the higher order modes in addition to those of the dominant mode which is required for designing the taper.

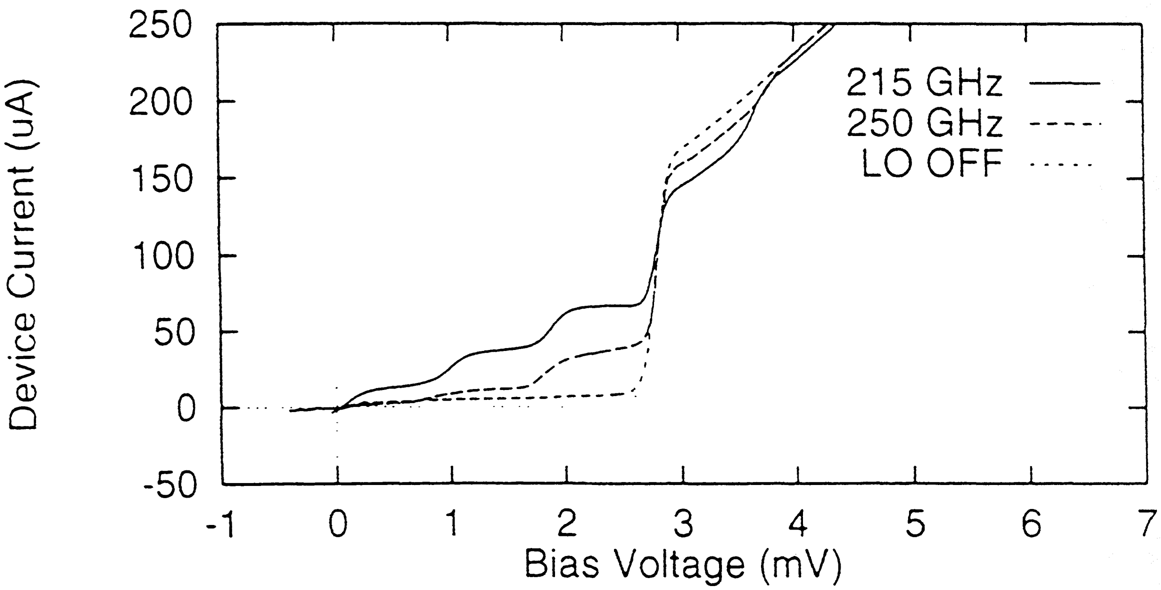

FIGURE V Pumped I-V Curves showing the effect of the tuning stub

Experimental Results

At present we are in the process of designing a broad band finline mixer for 345GHz. The performance of this mixer will be compared with that of a broadband probe-type mixer, and the one with the best performance will be used in a receiver that is being developed explicitly for observing highly-redshifted extragalactic spectral lines (WEASEL). We have therefore designed and tested several scale model finline taper sections using the method described above. In Fig. 5 we show the return loss of a back-to-back unilateral finline with a taper length of $\approx 1\lambda$. The waveguide dimensions were a = 47.5 mm and b = 22.2 mm and the thickness of the quartz substrate was 6.0 mm, which scales the real mixer by a factor of about 75.

It can be seen that the return loss is -15 db over most of the waveguide band (4-6 GHz). We matched the empty to the loaded waveguide using one dimensional linear dielectric tapers of length $1/2\lambda$, at both ends of the substrate. We have chosen this arrangement for its simplicity although we were able to verify that the periodic variations which limited the return loss to its present values are in fact resonances which result from the mismatches at the dielectric interfaces and they are not intrinsic to the transition itself.

Finally we present the latest results which were obtained with the 230GHz finline mixer which was developed for the MARS project (Padman and Blundell, 1995). We tested the mixer over a frequency range of 213-265GHz and obtained an average noise temperature of $\approx$ 60K over this range. In fact the frequency band of operation was limited by the availability of local oscillator power rather than a rapid deterioration of the mixer performance. Our measurements clearly show that the bandwidth of the mixer is determined by the tuning stub rather than the finline transition itself. Examples of the performance of the 230GHz mixer are shown in Figs. 5 and 6.

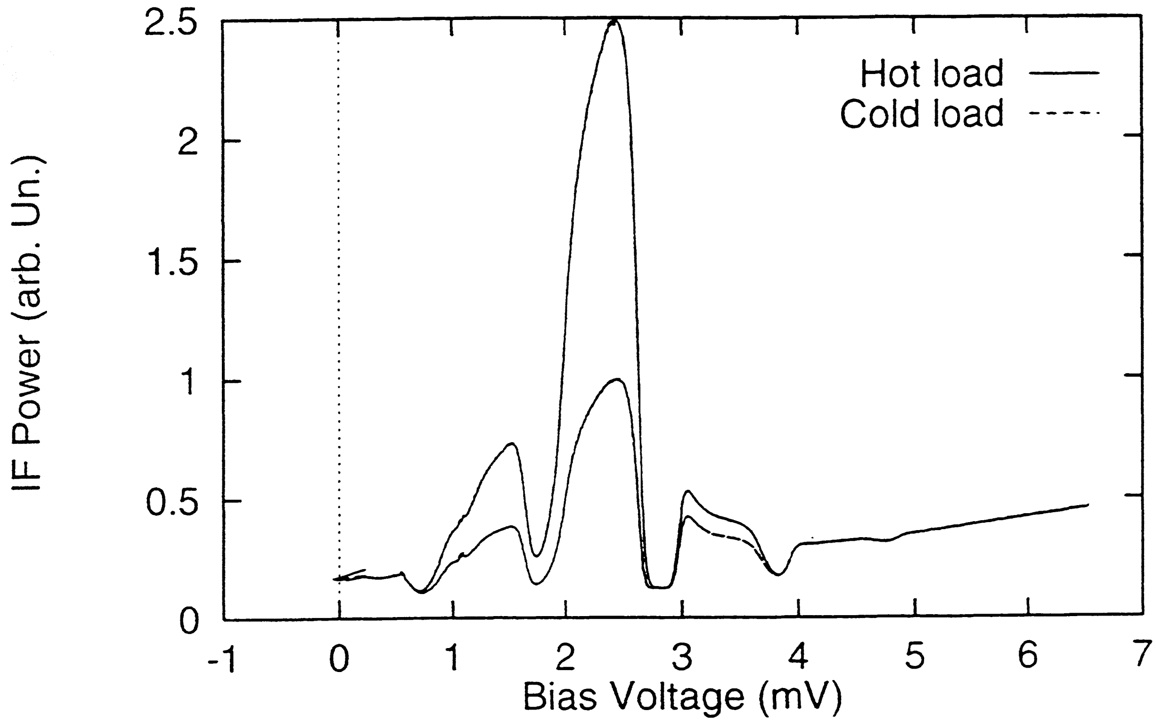

FIGURE VI Hot and cold load measurements at 250 GHz

ACKNOWLEDGEMENTS:

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