

# **EFFICIENT ANALYSIS OF QUASI-OPTICAL FILTERS BY THE BI-RME METHOD**

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*ABSTRACT – In this paper, we present an efficient algorithm for the analysis of Frequency Selective Surfaces, which consists of a periodic array of arbitrarily shaped apertures in a conductive screen. The effectiveness of our algorithm strongly depends on the application of the Boundary Integral-Resonant Mode Expansion (BI-RME) method. The computational efficiency and the flexibility of the method are discussed through some examples.*

## 1. INTRODUCTION

Frequency selective surfaces (FSSs) typically consist of a metal screen, perforated by periodic apertures of arbitrary shape, possibly backed by a dielectric layer (Fig. 1). FSSs find several applications as antenna sub-reflectors, quasi-optical filters and polarizer grids in the microwave and in the millimeter-wave range [1].

In this paper, we present a novel approach for the analysis of FSSs. In our analysis, the shape of the apertures can be given arbitrarily. Moreover, we consider both thin and thick metal screens.

Our approach is based on the equivalence theorem and on the representation of the fields as a modal expansion. The fields in free space are given as a summation of Floquet modes, whereas the fields on the aperture  $S$  are expanded on the modal fields of the waveguide of cross-section  $S$ .

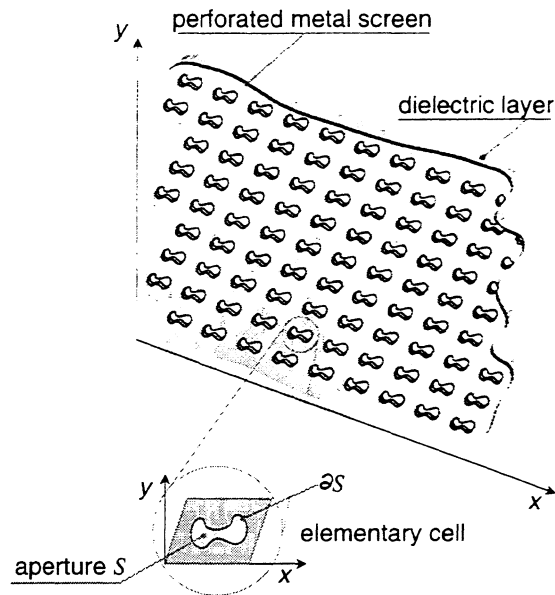


Fig. 1 – Structure of a frequency selective surface.

The analysis requires the determination of the mode spectrum of the waveguide and the calculation of the coupling integrals between the waveguide modes and the Floquet modes on the aperture  $S$ .

The waveguide modes are determined by the Boundary Integral–Resonant Mode Expansion (BI–RME) method [2] in a fast and reliable way. Moreover, the calculation of the coupling integrals can be obtained as a by–product of the BI–RME analysis.

This algorithm has been implemented in a computer code. An example proves the effectiveness and the accuracy of the method.

## 2. ANALYSIS OF THIN FSSs

An algorithm for the analysis of periodic arrays of apertures in a thin–screen metal layer is presented in [3]. Using the equivalence theorem, the problem is split in two parts (Fig. 2). The aperture  $S$  is closed by a metal screen and two sheets of (unknown) magnetic current densities  $\mathbf{M}$  and  $-\mathbf{M}$  are defined on the opposite sides of the screen.

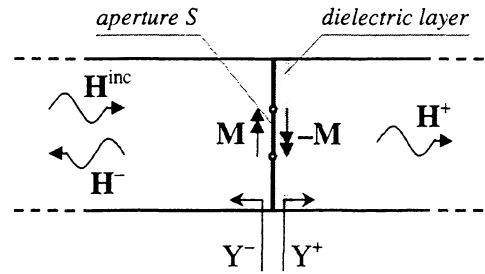


Fig. 2 – Equivalent problem for the analysis of thin frequency selective surfaces.

The continuity of the tangential component of the electric field is automatically guaranteed by the proper choice of the magnetic current densities, equal in amplitude and opposite in phase.

The value of the unknown  $\mathbf{M}$  is determined by imposing the continuity of the tangential component of the magnetic field across the aperture  $S$

$$\mathbf{H}^{inc}(\mathbf{r}) + \mathbf{H}^-(\mathbf{r}) = \mathbf{H}^+(\mathbf{r}) \quad \text{on } S \quad (1)$$

where  $\mathbf{H}^{inc}$  is the (given) incident magnetic field, i.e., a plane wave incident at an arbitrary angle. Moreover,  $\mathbf{H}^+$  and  $\mathbf{H}^-$  are the magnetic fields which propagate in direction  $z$  and  $-z$ , respectively, due to the magnetic current densities defined on  $S$ .

Using the Floquet theorem [4], the magnetic fields in free space are expressed as a summation of Floquet modes  $\mathcal{H}_m$ . In particular, the fields  $\mathbf{H}^+$  and  $\mathbf{H}^-$  are defined as follows

$$\mathbf{H}^-(\mathbf{r}) = -\sum_m Y_m^- \mathcal{H}_m \int_S \mathbf{M}(\mathbf{r}) \cdot \mathcal{H}_m^* dS \quad (2)$$

$$\mathbf{H}^+(\mathbf{r}) = \sum_m Y_m^+ \mathcal{H}_m \int_S \mathbf{M}(\mathbf{r}) \cdot \mathcal{H}_m^* dS \quad (3)$$

where  $Y_m^-$  is the modal admittance of the  $m$ -th Floquet mode, and  $Y_m^+$  is the modal admittance seen through the dielectric layer (Fig. 2).

Using (2)–(3) in (1), we obtain an integral problem, which is solved by the Method of the Moments (MoM). In the application of the MoM, the magnetic current density  $\mathbf{M}$  is

expanded on a set of basis functions, corresponding to the magnetic modal fields  $\mathbf{h}_q$  of the waveguide of cross-section  $S$

$$\mathbf{M} = \sum_q x_q \mathbf{h}_q \quad (4)$$

As discussed in [3], the choice of entire domain basis functions, which satisfy the right boundary conditions, is very convenient, since it leads to a matrix problem of small dimension.

After expanding  $\mathbf{M}$  using (4) and testing the continuity condition (1) using the same functions  $\mathbf{h}_q$ , the following matrix problem results

$$[\mathbf{A}][\mathbf{x}] = [\mathbf{B}] \quad (5)$$

where

$$A_{qi} = \sum_m (Y_m^+ + Y_m^-) \int_S \mathbf{h}_q \cdot \mathcal{H}_m^* dS \int_S \mathbf{h}_i \cdot \mathcal{H}_m dS \quad (6)$$

$$B_i = 2 \int_S \mathbf{h}_i \cdot \mathbf{H}^{\text{inc}} dS \quad (7)$$

If  $\mathbf{H}^{\text{inc}}$  is a plane wave, it can be expressed as a combination of Floquet modes. Thus, the integral appearing in (7) can be written as a combination of the integrals appearing in (6), i.e., the coupling integrals between the waveguide modes and the Floquet modes.

Therefore, this analysis requires *i*) the determination of the mode spectrum of a waveguide of cross-section  $S$ , and *ii*) the calculation of the coupling integrals between the waveguide modes and the Floquet modes.

If the aperture  $S$  has a conventional shape (rectangular or circular), both the mode spectrum and the coupling integrals are known analytically [5]. On the contrary, if the shape of  $S$  is arbitrary, a fast and reliable algorithm is required, to numerically evaluate the waveguide modes as well as the coupling integrals.

In this work, we apply the Boundary Integral-Resonant Mode Expansion (BI-RME) method [2] to perform the evaluation of the mode spectrum of the waveguide. The BI-RME method provides the cutoff frequencies and the normalized modal fields in a prescribed frequency band. This method is accurate and reliable, with no risk of either missing some modes or yielding spurious solutions.

With regards to the calculation of the coupling integrals, it can be proved that the surface integrals on  $S$  can be reduced to contour integrals on the aperture boundary  $\partial S$

$$\int_S \mathbf{h}'_i \cdot \mathcal{H}^*_{m'} dS = \frac{k_m}{\kappa_i (k_m^2 - \kappa_i^2)} \int_{\partial S} \frac{\partial \psi_i}{\partial n} \xi_m^* d\ell \quad (8)$$

$$\int_S \mathbf{h}'_i \cdot \mathcal{H}^*_{m''} dS = 0 \quad (9)$$

$$\int_S \mathbf{h}'_i \cdot \mathcal{H}^0_{m'} dS = 0 \quad (10)$$

$$\int_S \mathbf{h}''_i \cdot \mathcal{H}^*_{m'} dS = -\frac{1}{\kappa_i} \int_{\partial S} \varphi_i (\mathbf{n} \cdot \mathcal{H}^*_{m'}) d\ell \quad (11)$$

$$\int_S \mathbf{h}''_i \cdot \mathcal{H}^*_{m''} dS = -\frac{\kappa_i}{(\kappa_i^2 - k_m^2)} \int_{\partial S} \varphi_i (\mathbf{n} \cdot \mathcal{H}^*_{m''}) d\ell \quad (12)$$

$$\int_S \mathbf{h}''_i \cdot \mathcal{H}^0_{m'} dS = -\frac{1}{\kappa_i} \int_{\partial S} \varphi_i (\mathbf{n} \cdot \mathcal{H}^0_{m'}) d\ell \quad (13)$$

In expressions (8)–(13), the superscripts prime, double prime and zero refer to TM, TE and TEM modes, respectively. Moreover,  $\psi_i$  and  $\varphi_i$  are the potentials of the TM and TE waveguide modes, and  $\xi_m$  is the potential of the Floquet modes;  $\kappa_i$  and  $k_m$  are the resonance wave-numbers of the waveguide and Floquet modes, respectively. Finally,  $\mathbf{n}$  is the outward unit vector normal to  $\partial S$ .

Integrals (8)–(13) involve the potential  $\varphi_i$  of the TE waveguide modes and the normal derivative of the potential  $\psi_i$  of the TM modes, on the aperture boundary  $\partial S$ . It is worthy noting that both these quantities are provided by the BI–RME method as a by-product of the mode calculation. Moreover, the Floquet modes are known analytically. Therefore, the calculation of the coupling integrals can be efficiently performed by a numerical integration.

Occasionally, if  $\kappa_i^2 = k_m^2$ , relations (8) and (12) are not applicable: in these cases, the numerical integration on the surface  $S$  is required.

### 3. ANALYSIS OF THICK FSSs

The theory presented in the previous section can be extended to the analysis of thick-screen FSSs. The approach is very similar: using the equivalence theorem, the problem is split in three parts (Fig. 3).

The electric and magnetic fields are expressed as a summation of Floquet modes  $\mathcal{H}_m$  in regions I and III, and as a summation of waveguide modal fields  $\mathbf{h}_q$  in region II. The magnetic current densities  $\mathbf{M}_1$  and  $\mathbf{M}_2$  ensure the continuity of the fields across the apertures  $S_1$  and  $S_2$ .

By imposing the continuity condition at the apertures  $S_1$  and  $S_2$  and applying the MoM, an integral problem similar to (5) is obtained.

It results that, also in this case, the only significant computational efforts are the determination of the mode spectrum of the waveguide of cross-section  $S$  and the calculation of the coupling integrals (8)–(13) between the waveguide modes and the Floquet modes.

Therefore, all the advantages deriving from the use of the BI-RME method well apply also in the case of thick-screen FSSs.

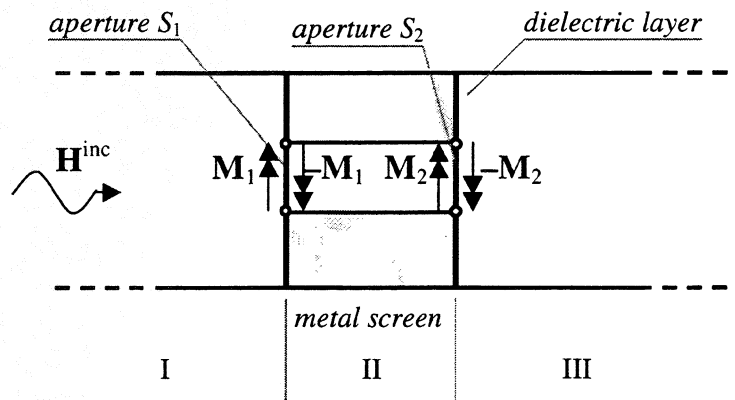


Fig. 3 – Equivalent problem for the analysis of thick-screen frequency selective surfaces.

#### 4. VALIDATION OF THE METHOD

In order to validate the method described in this work, we report the analysis of a FSS firstly presented by Mittra [1]. The filter (Fig. 4) consists of a free-standing array of apertures in a thin metal screen. Each aperture has a Jerusalem-cross shape.

The analysis with our code on a workstation Sun Ultra 10 required only 30" for calculating 30 waveguide modes and the corresponding samples on the boundary  $\partial S$ , and 2'30" for calculating the reflection and transmission coefficients in 15 frequency points. Fig. 5 shows that our results are in good agreement with the ones reported in [1].

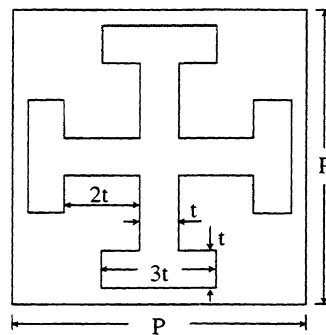


Fig. 4 – Unit cell of the FSS proposed in [1]. Dimensions:  $t=0.19$  cm,  $P=1.52$  cm.

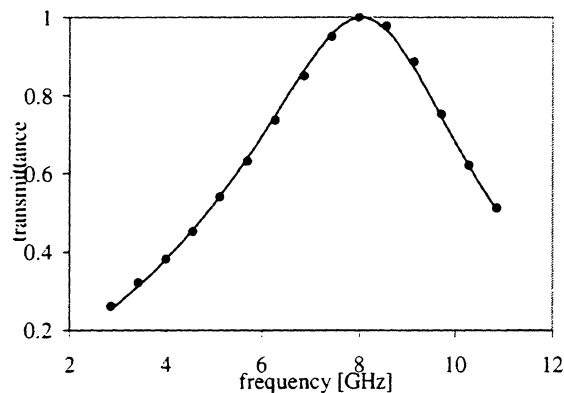


Fig. 5 – Transmittance of the Jerusalem cross FSS (solid line = our simulation, markers = from literature [1])

## 5. CONCLUSIONS

This paper presents an efficient algorithm for the analysis of thin and thick frequency selective surfaces.

The analysis requires the determination of the mode spectrum of an arbitrarily shaped waveguide and the calculation of the coupling integrals between the waveguide modes and the Floquet modes. We apply the BI-RME method for the determination of the mode of the waveguide, and obtain the calculation of the coupling integrals as a by-product of the BI-RME analysis.

An example demonstrates the accuracy and the rapidity of the method.

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