

# PERFORMANCE OF SUB-HARMONIC MIXING WITH AN SIS JUNCTION

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## Abstract

The performance of superconductor-insulator-superconductor (SIS) sub-harmonic mixers is thoroughly investigated in this paper. On account of the importance of the harmonic effect, a full five-port model combined with an enhanced Newton solution is applied to calculate the local oscillator (LO) waveform. The mixer performance is studied as functions of principal parameters including the embedding admittances, LO power and bias voltage. The results are compared with those from the quasi five-port model.

## I. Introduction

It is well known that the local oscillator (LO) power for SIS mixers is proportional to the square of the LO frequency<sup>1</sup>, and that it is still difficult to develop broadband LO sources in the terahertz regime. Hence sub-harmonic SIS mixers, which are pumped by a LO signal of a frequency about one half the RF frequency, are of particular interest in the terahertz regime as far as the LO frequency and power are concerned. Conventional sub-harmonic mixers (e.g., Schottky ones) usually utilize anti-parallel diodes to suppress the fundamental mixing<sup>2</sup>, thereby achieving good sub-harmonic mixing performance. Sub-harmonic mixers with SIS junctions, however, cannot adopt such a scheme because of their symmetric I-V characteristic. The main goal of this research is to investigate the sub-harmonic mixing behavior of SIS mixers with a single junction device. Shen et al<sup>3</sup> and Edward Tong et al<sup>4</sup> have previously studied this theme merely focusing on the investigation of the mixer gain.

The three-port model, the simplest model for SIS junction, cannot be adopted to simulate the mixing behavior of sub-harmonic SIS mixers since all frequencies beyond the fundamental frequency are assumed to be short-circuited by the junction capacitance. The quasi five-port model can be applied for such simulations<sup>5</sup>, by assuming a sinusoidal LO voltage across the junction. For more precise analyses, here we use the full five-port model with a spectral domain approach calculating the waveform of the LO voltage<sup>4</sup>. By simulating a sub-harmonic mixer pumped at 250 GHz, we have thoroughly studied the performance of sub-harmonic SIS mixers, including the mixer's conversion efficiency, noise temperature, input coupling and output coupling.

## II. Theory of Simulation

## II.a Large-Signal Analysis

In the full five-port approximation, the junction's LO wave form is expressed as a finite Fourier series:

$$V_J(t) = V_0 + \text{Re}(V_1 e^{j\omega_p t}) + \text{Re}(V_2 e^{j2\omega_p t}) \quad (1)$$

by neglecting those harmonic terms beyond  $n=2$ . Here  $\omega_p$  is the LO angular frequency. For a given value of the LO source current ( $I_{LO}$ ), Voltages  $V_1$  and  $V_2$  should satisfy the constraints imposed by the embedding network at frequencies  $\omega_p$  and  $2\omega_p$ <sup>4</sup>. A globally convergent multi-dimensional Newton's method<sup>6</sup> is applied to solve  $V_1$  and  $V_2$ . This algorithm optimizes the revising vector of each iteration so as to avoid the oscillating effect that often occurs in conventional Newton's iterations. This method allows any reasonable initial trial solution and the convergence is always achieved in our calculation.

In our simulation, we calculate the mixer's performance as the parameter of interest varies step by step. At each step, the solving process of the nonlinear equation can be accelerated by choosing the previous  $V_1$  and  $V_2$  as the trial solution of the next one, thereby reducing the iteration to only 2~3 loops.

## II.b Small-Signal Analysis

The small-signal analysis has been well discussed in many papers<sup>1,7</sup>. Here we only list those results and performance parameters discussed in the following parts. By using the Fourier series of the phase factor, we easily get the mixer's impedance matrices [ $Z_{mn}$ ] and shot-noise correlation matrix [ $H_{mn}$ ]. The subscript of the matrices  $m=-2,-1,0,1,2$  denotes the sideband whose frequency is  $m f_m + f_n$ . The associated conversion efficiency from the sub-harmonic sideband ( $n = 2,-2$ ) is

$$G_n = 4 \text{Re}(Y_{emb}(n\omega_p + \omega_0)) \text{Re}(Y_{emb}(\omega_0)) |Z_{on}|^2, n = 2,-2. \quad (2)$$

The mixer noise includes the thermal noise and the shot noise. The Josephson noise is not discussed here as we can apply a DC magnetic field to suppress the Josephson effect, and choose the bias point away from the region where the Josephson noise is considerable. The noise temperature of SIS mixers is

$$T_m = \frac{P_0^n}{4k\Delta f \text{Re}(Y_{emb}(2\omega_p)) |Z_{o2}|^2}, \quad (3)$$

where  $\Delta f$  is the bandwidth of the intermediate frequency (IF) and  $P_0^n$  is the IF noise power, which can be expressed by the noise correlation matrix and the impedance matrix.

It is also necessary to concern the mixer's input and output coupling efficiencies in practical receiver systems. In our simulation, the input and output coupling efficiencies are written as:

$$C_{in} = \frac{4 \operatorname{Re}(Y_{emb}(2\omega_p)) \operatorname{Re}(Y_{in})}{|Y_{emb}(2\omega_p) + Y_{in}|^2}, \quad (4)$$

$$C_{out} = \frac{4 \operatorname{Re}(Y_{emb}(\omega_0)) \operatorname{Re}(Y_{out})}{|Y_{emb}(\omega_0) + Y_{out}|^2}. \quad (5)$$

The input admittance at the signal sideband  $Y_{in}$  and the output impedance of the IF port  $Y_{out}$  can be expressed as :

$$Y_{in} = \frac{1}{Z_{22}} - Y_{emb}(2\omega_p), \quad (6)$$

$$Y_{out} = \frac{1}{Z_{00}} - Y_{emb}(\omega_0). \quad (7)$$

It should be mentioned that as we assume  $\omega_0 \ll \omega_p$ , the embedding impedances at the upper and lower sideband are approximately the same. In the following parts, the embedding impedance at the IF port and each sideband are normalized by the junction's normal state resistance ( $R_N$ ) and separated into resistance and reactance as:

$$\tilde{Y}_{emb}(\omega_0) = \frac{1}{R_{if}} + jB_{if}, \quad (8)$$

$$\tilde{Y}_{emb}(\omega_p \pm \omega_0) = \tilde{Y}_{emb}(\omega_p) = \frac{1}{R_{rf}} + jB_{rf}, \quad (9)$$

$$\tilde{Y}_{emb}(2\omega_p \pm \omega_0) = \tilde{Y}_{emb}(2\omega_p) = \frac{1}{R_{hm}} + jB_{hm}. \quad (10)$$

### III. Results and Discuss

The performance of sub-harmonic SIS mixers is affected by a variety of parameters including the embedding admittances at the five frequency ports, the bias voltage and the LO current. To illustrate how these parameters affect the mixer performance, we examined the performance variation under the condition that only one or two parameters changed while others were fixed.

In our simulation, the  $I$ - $V$  curve of a physical Nb/AlO<sub>x</sub>/Nb SIS junction was used. Its gap voltage  $V_{gap}$  is  $2.76mV$  and normal-state resistance  $R_N$  is  $17.6\Omega$ . Because the performance is directly influenced by the effective embedding admittance  $\tilde{Y}'_{emb}$ :

$$\tilde{Y}'_{emb}(\omega) = \tilde{Y}_{emb}(\omega) + j\omega R_N C_J, \quad (11)$$

the  $\omega R_N C_J$  product was not regarded as an independent variable here.

#### III.a Optimum Bias Voltage and LO Source Current

Firstly, a preliminary optimization was carried out to obtain a set of parameters enabling reasonably good mixer performance on the first photon step. Based on this optimization, we obtained the embedding admittance at each sideband and the IF port such as:  $\tilde{Y}'_{emb}(\omega_p) = 1/2.3 + j0.2$ ,  $\tilde{Y}'_{emb}(2\omega_p) = 1.0$  and  $\tilde{Y}(\omega_0) = 1/2.0$ .

With different LO pumping currents  $\gamma=(I_{LO}R_N/V_{gap})$  from 0.4~1.2, a series of responding conversion efficiencies and noise temperatures were calculated (see Fig. 1(a) and (b)). Obviously,  $G_2$  increases with  $\gamma$  while  $\gamma<1.0$  and tends to fall while  $\gamma>1.0$ . If a proper LO current is applied, positive conversion efficiency is observed just below the gap voltage. Accompanying with the positive conversion efficiency, the differential resistance of the pumped  $I$ - $V$  curve is nearly zero or merely negative, which is actually not desirable for practical SIS receivers. In Fig. 1(b), the noise temperature on the first photon step is 23K, which is about 1~2 times as large as the quantum limit ( $\hbar\omega_s/2k_b$ ). The noise temperature on the second photon step is not optimum because the embedding admittances were optimized for the first photon step.

In order to study the effect of the LO current, we plot  $\alpha_1, \alpha_2, G_2$  and  $T_m$  as a function of  $\gamma$  at a fixed bias point of  $V_{bias}=2.2mV$ . As shown in Fig. 2(a),  $\alpha_1$  varies almost linearly with  $\gamma$  and  $\alpha_2$  increases till  $\gamma=1.8$ , but begins to decrease while  $\gamma>1.8$  possibly due to the junction non-linearity fading out under a large LO power. In Fig. 2(b),  $G_2$  and  $T_m$  have their best values when  $\gamma$  is about 1.0~1.2, and start to deteriorate beyond this range.

The full five-port model gives different result from that by the quasi five-port model does when the harmonic effect is not negligible. The harmonic effect can be evaluated by the ratio of  $\alpha_2/\alpha_1$ . We plot the ratio as a function of  $V_{bias}$  in Fig. 3(a). It is interesting that the harmonic effect is most obvious in the intersectional region of the first photon step and the second one. In this region the noise temperature calculated by the quasi five-port model is lower than that by the full five-port model due to omitting the noise contribution of the harmonic part (shown in Fig. 3(b)). The conversion efficiency calculated by the quasi five-port model is larger on the first photon step but smaller in the second photon step. In the region where  $\alpha_2/\alpha_1<0.1$ , there is almost no difference between the two models.

### III.b Effect of the RF Termination

Just like the fundamental mixing, the sub-harmonic mixer has a good performance when the junction capacitance is nearly tuned out (by an external inductance, for example.) The contours of the performance as a function of  $\tilde{Y}'_{emb}(\omega_p)$  are demonstrated in Fig. 4 with the supposition of  $V_{bias}=2.2mV$ ,  $\gamma=1.2$ ,  $\tilde{Y}'_{emb}(2\omega_p)=1.0$ ,  $\tilde{Y}_{emb}(\omega_0)=0.5$ . Positive conversion efficiency is observed in the region of  $R_{rf}>2$  and  $(B_{rf}-\omega_p R_N C_J)=-1\sim-2$ . Beyond this region, either a smaller  $R_{rf}$  or a larger  $|B_{rf}-\omega_p R_N C_J|$  will cause a poor coupling between the junction and the LO source, and bring about low conversion efficiency and large noise temperature.

Being sensitive with the value of  $(B_{rf}-\omega R_N C_J)$ , the lowest noise temperature is achieved when the junction capacitance is exactly tuned out and  $R_{rf}$  lies in the region of 2~3. The noise temperature oscillates while  $R_{rf}>3$  and the value of  $(B_{rf}-\omega R_N C_J)=0$ .

Directly influenced by  $\tilde{Y}_{emb}(2\omega_p)$  instead of  $\tilde{Y}_{emb}(\omega_p)$  as expressed in Eq. (4) and (6), the input coupling efficiency  $C_{in}$  appears insensitive to  $\tilde{Y}_{emb}(\omega_p)$ . Only in the region where positive conversion efficiency occurs,  $C_{in}$  have small values. The  $C_{out}$  behaves more irregularly. As indicated in Fig. 4(d), while the effective embedding admittance is inductive, there appears a large area that corresponds to a negative  $C_{out}$ . It should be noted that this area covers the region of positive conversion efficiency and also its peripheral area with small negative conversion efficiency.

### III.c Effect of the Harmonic Termination

Being different from the performance of the fundamental mixing, which is insensitive to the embedding admittance at the harmonic sidebands,  $\tilde{Y}_{emb}(2\omega_p)$  plays a more important role in the sub-harmonic mixing. As shown in Fig. 5, the highest conversion efficiency resides in the region of  $R_{hm}=0.4\sim 4$  and  $(B_{hm} - 2\omega_p R_N C_J)=0\sim 1$ . It is indeed indicated that besides the necessity of tuning out the capacitance at the fundamental sidebands, good performance requires the capacitance tuned out at the harmonic sidebands as well.

The mixer noise temperature and input coupling efficiency behave very similarly as the conversion efficiency. Their behaviors are nearly symmetrical along the line of  $(B_{hm} - 2\omega_p R_N C_J)=0$ . The region relating to the highest  $G_2$  is also that with the lowest  $T_m$  and the best  $C_{in}$ , however  $C_{out}$  has a large gradient there and varies very sensitively with  $(B_{hm} - 2\omega_p R_N C_J)$ .

### III.d Effect of the IF Termination

According to our result, the IF load impedance affects the output coupling efficiency and the conversion efficiency, but does not influence the noise temperature  $T_m$  and the input coupling efficiency. The dependences of  $G_2$  and  $C_{out}$  on  $\tilde{Y}_{emb}(\omega_0)$  are plotted in Fig. 6 with the parameters listed in the figure caption. Either capacitive or inductive  $\tilde{Y}_{emb}(\omega_0)$  will deteriorate the performance while a wider range of resistance ( $R_{if}=2\sim 5$ ) is suitable. It is expectable that a reasonably large  $R_{if}$  is advantageous to the output matching, because the output resistance (approximately equal to the differential resistance of the pumped  $I-V$ ) is quite large at the practical bias point. The result also indicates that larger  $R_{if}>5$  is harmful to good output efficiency.

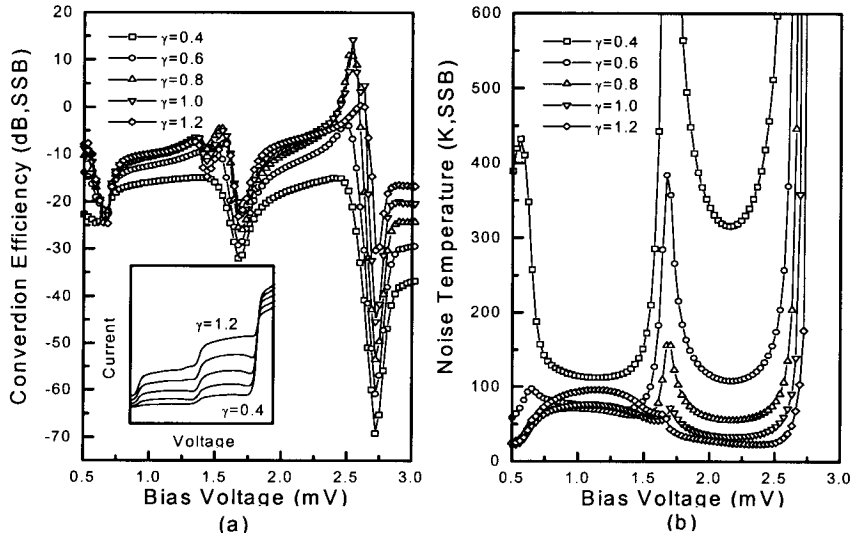
## IV Summary

We have used a full five-port model to simulate a sub-harmonic SIS mixer pumped at 250GHz. It is indicated that to obtain good sub-harmonic mixing performance, the junction capacitance should be tuned out at both the fundamental and harmonic frequency. The best termination resistant at  $\omega_p$  is 2~3 and a wider

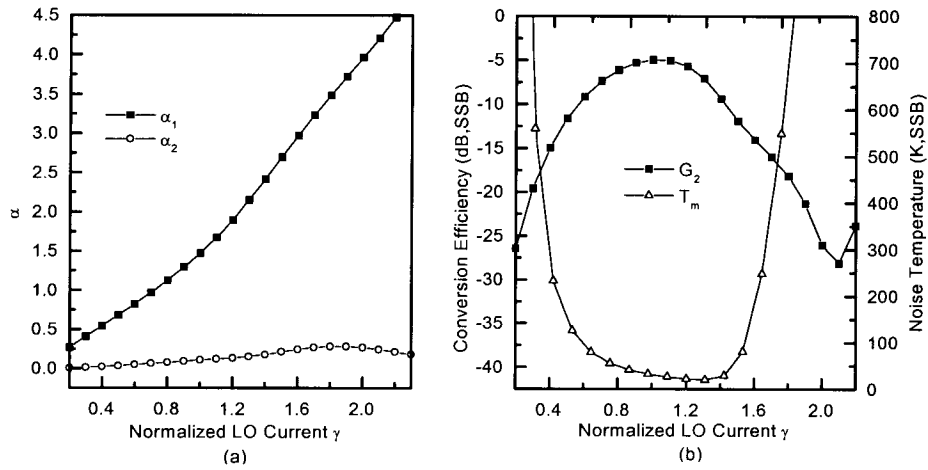
range from 0.4 to 4 at  $2\omega_p$ . The IF termination only influences the conversion efficiency and the IF output coupling efficiency. A pure resistive IF load in the range of 2~5 will gain good conversion efficiency and the best output coupling. The optimum  $\gamma$  in our simulation is 1.2, and the corresponding  $\alpha_1$  is 1.7. It is necessary to emphasize that the lowest noise temperature of the sub-harmonic mixer is close to the quantum noise limit. It provides strong evidence that sub-harmonic mixers should be useful at sub-millimeter wavelengths.

### References

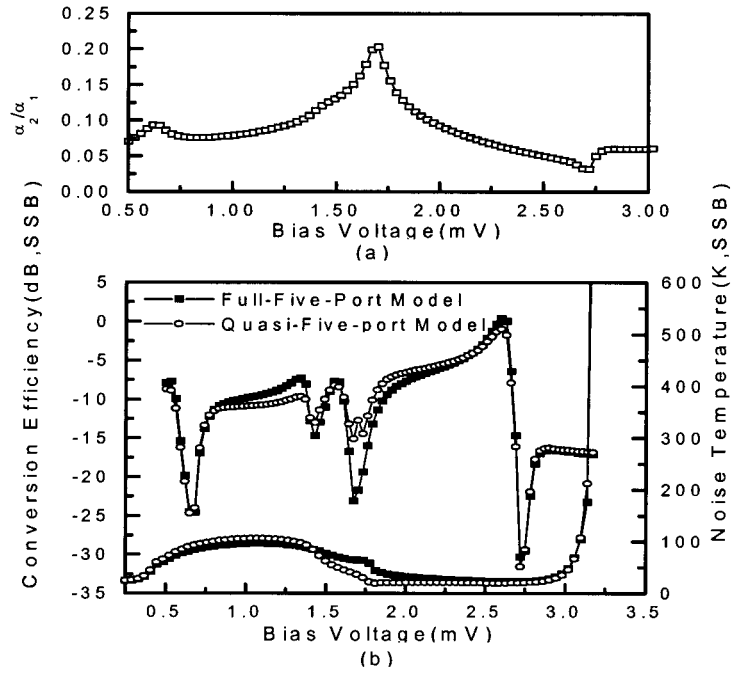
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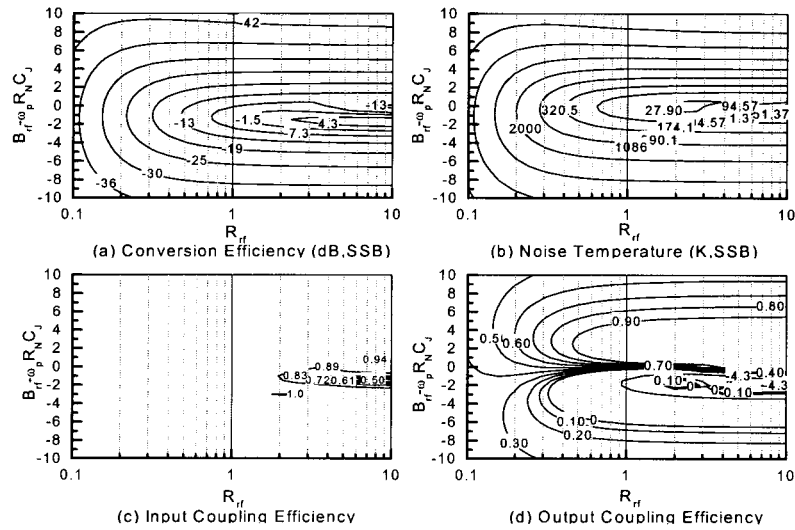
**Fig. 1** The simulated performance shown as functions of bias voltage for normalized LO Source Current  $\gamma = 0.4 \sim 1.2$ . Embedding admittances included are:  $Y'_{emb}(\omega_p) = 1/2.3 + j1.2$ ,  $Y'_{emb}(2\omega_p) = 1.0$  and  $\tilde{Y}(\omega_0) = 1/2.0$ . The inset depicts the pumped  $i-v$  curves.



**Fig. 2** Performance of the same mixer of Fig. 1 with fixed bias voltage 2.2mV as function of normalized LO source  $\gamma$ . (a)  $\alpha_1$  and  $\alpha_2$ , (b) mixer gain and noise temperature.

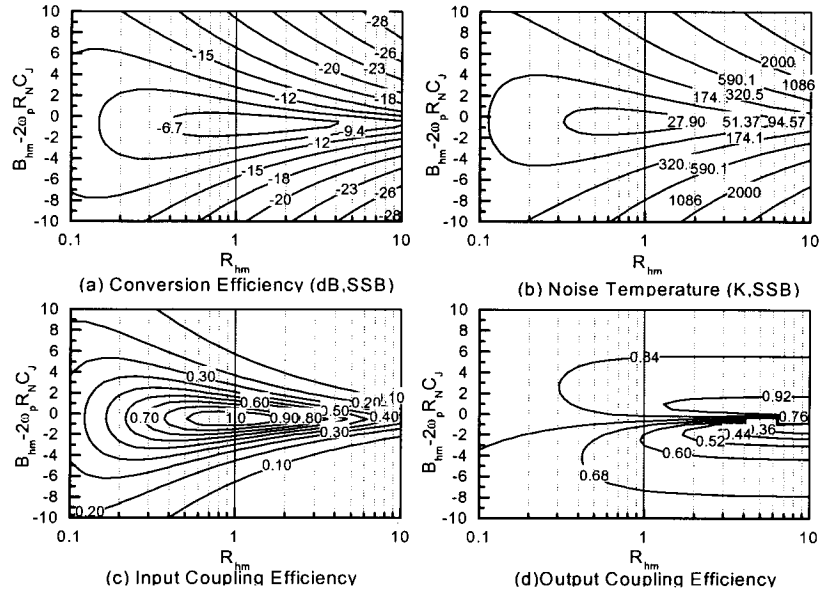


**Fig. 3** Comparison between the results from the full five-port Model and that of the quasi five-port Model.(a)  $\alpha_2/\alpha_1$ , the in indicator of harmonic effect, as function of bias voltage, (b)conversion efficiency and noise temperature calculated from the two models.

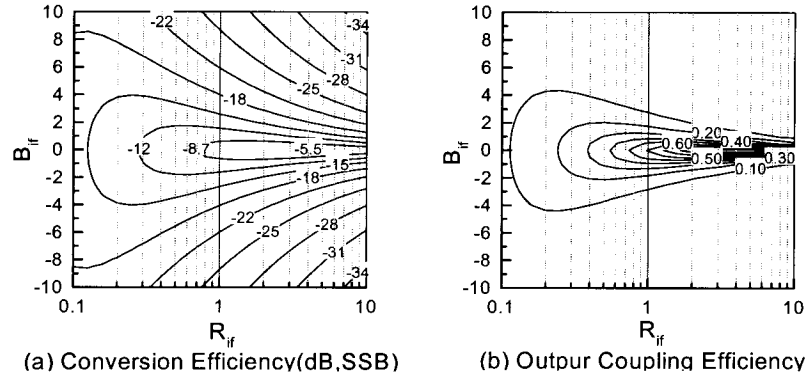


**Fig. 4** Contours of all performances of SIS harmonic mixer as a function of  $\tilde{Y}'_{emb}(\omega_p)$ . The parameters involved are:  $V_{bias} = 2.2mV$ ,  $\gamma = 1.2$ ,  $\tilde{Y}'_{emb}(2\omega_p) = 1.0$ ,  $\tilde{Y}(\omega_0) = 1/2.0$ . (a) Conversion efficiency of the upper harmonic sideband, (b) noise temperature of the mixer, (c) input efficiency and (d) output efficiency.





**Fig. 5** Contours of all performances of SIS harmonic mixer as a function of  $\tilde{Y}'_{emb}(2\omega_p)$ . The parameters involved are:  $V_{bias}=2.2mV$ ,  $\gamma=1.2$ ,  $\tilde{Y}'_{emb}(\omega_p)=1/2.3+j0.2$ ,  $\tilde{Y}_{emb}(\omega_0)=1/2.0$ . (a) Conversion efficiency, (b) noise temperature of the mixer, (c) input efficiency and (d) output efficiency.



**Fig. 6** Contours of (a) conversion efficiency and (b) output efficiency plotted as a function of the  $\tilde{Y}_{emb}(\omega_0)$ . The parameters involved are:  $V_{bias}=2.2mV$ ,  $\gamma=1.2$ ,  $\tilde{Y}'_{emb}(\omega_p)=1/2.3+j0.2$ ,  $\tilde{Y}_{emb}(\omega_0)=0.5$ ,  $\tilde{Y}'_{emb}(2\omega_p)=1.0$ .