Bias Dependence of the Thermal Time Constant in Nb Superconducting Diffusion-Cooled HEB Mixers

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In this paper, we present an experimental study of the intermediate frequency bandwidth of a Nb diffusion-cooled hot-electron bolometer mixer for different bias voltages. The measurements show that the bandwidth increases with increasing voltage. Analysis of the data reveals that this effect is mainly caused by a decrease of the intrinsic thermal time of the mixer and that the effect of electro-thermal feedback through the intermediate frequency circuit is small. The results are explained using a qualitative model, in which the time constant is calculated using a weighted average of the diffusion constant as a function of the relative length of the hotspot, and accounting for the lower diffusion constant in the superconducting domain. Thus, we show that for a diffusion-cooled hot-electron bolometer, the intermediate frequency bandwidth is determined not only by the length of the microbridge, as is generally believed, but also by the length of the electronic hotspot.

I. Introduction

The strong need for sensitive heterodyne mixers at terahertz frequencies is an important stimulus for the ongoing development of hot-electron bolometer (HEB) mixers. Besides the rapid progress on the experimental side, new light has been shed on the physical mechanisms that govern the HEB in heterodyne operation. Classically, the HEB is described as a lumped element, making use of the steep rise of the resistance as a function of temperature *R(T)* close to the critical temperature T_c [1-3]. Under the usual operating conditions of the mixer the presence of a temperature profile in the microbridge has to be taken into account [4], and the experimental dc resistive transition is no longer relevant [5]. A new concept has been introduced in which heterodyne mixing in HEBs is described in terms of an electronic hotspot (EHS) of which the length L_h oscillates at the intermediate frequency (IF) [6,7]. Recently, a similar physical model was introduced in a somewhat different mathematical approach, although there, the authors continue to relate the mixing properties to the experimentally observed *R(T)* [8].

II. The time constant of a diffusion-cooled HEB

An important figure of merit of a HEB mixer is the IF gain bandwidth, defined as the frequency where the gain has dropped 3 dB below its zero frequency value (roll-off frequency f_{-3dB}), and being physically determined by the speed with which the thermal energy can be removed from the microbridge. Based on the ratio between the thermal length $\lambda_{th} = (D\tau_{e\text{-}ph})^{1/2}$ (D is the electronic diffusion constant and and $\tau_{e\text{-}ph}$ is the electron-phonon coupling time) and the length of the microbridge L_b, HEBs can be divided into two classes.

If $\lambda_{th}/L_b < 1$, the hot electrons will primarily lose their energy via inelastic electron-phonon scattering within the microbridge. In this case one speaks of a phonon-cooled HEB [1]. Based on an electron-phonon coupling time of about 12 ps for a thin NbN film, the IF bandwidth can be 10 GHz and thus large enough for practical applications [9]. However, due to relatively long phonon escape time of approximately 40 ps, the IF bandwidth is in practice limited to 3-4 GHz. If, on the other hand, $\lambda_{th}/L_b > 1$, the heat transfer in the microbridge will be dominated by outdiffusion of hot electrons to the contacts. The idea of using diffusion-cooling rather than phonon-cooling in order to achieve a large bandwidth was proposed by D. Prober in 1993 [2]. Experimentally, the crossover from phonon-cooling to diffusion-cooling has been demonstrated by varying the length of the microbridge [10,11]. Obviously, if the mixer is made very short, the IF roll-off frequency can be very high. Recently, an IF bandwidth of 9.2 GHz was reported for a 100 nm long Nb HEB, whereas a 80 nm long HEB showed no clear roll-off up to 15 GHz [12].

We now consider the time constant of a diffusion-cooled superconducting HEB mixer in a bit more detail. For a diffusion-cooled *metallic* microbridge, the thermal relaxation time is given by [4,13]:

$$\tau_{th} = \frac{L_b^2}{\pi^2 D} \qquad [1]$$

This expression does take into account the presence of a temperature profile, but assumes no spatial variation of the thermal diffusivity, being determined by the (local) ratio of the thermal conductivity k and the electronic heat capacity c_e. In case of a diffusion-cooled metallic microbridge, this ratio is the same as the electronic diffusion constant, since heating and heat transport both take place via the electrons. In practice, however, the rf pumped and dc biased HEB is cooled well below T_c, and operates in an electronic hotspot state. In this case, the center of the bridge is normal (hotspot region), whereas the ends of the microbridge are superconducting. In the normal region, the diffusion constant is independent of the local temperature, because both k and c_e have a linear temperature dependence. In the superconducting parts, k decreases sublinearly with decreasing temperature, whereas the c_e will be larger than in the normal part of the microbridge [14]. Thus, the thermal diffusivity will be

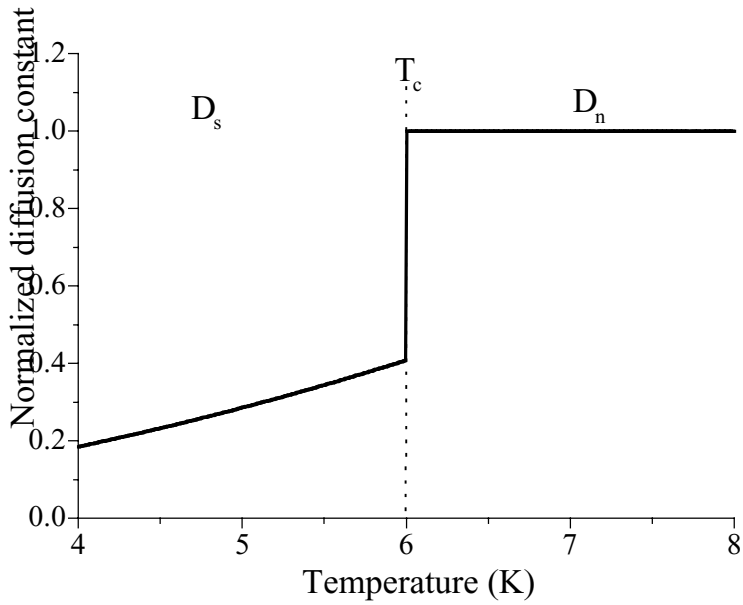

Fig. 1: Thermal diffusivity of Nb as a function of temperature, normalized to the normal state value D_n (1.6 cm^2/s). Calculated is the ratio between the thermal conductance k and the electronic heat capacity c_e of a thin Nb film with a critical temperature of 6 K.

considerably lower in the superconducting regions. This can be seen in Fig. 1, where we plot the normalized ratio k/c_e below and above T_c for a typical 10 nm thin Nb film on the basis of expressions for c_e and k given in Refs. 14 and 15 (D_n and D_s refer to the diffusion constant in the normal and superconducting state, respectively). An abrupt decrease in k/c_e occurs at T_c because of the discontinuity of c_e. In our experiments, the device is usually operated at a bath temperature around 4.2 K and it can be seen in the figure that in the temperature range between 4.2 K and 6 K, $D_n \approx 3\text{-}5\ D_s$. Therefore, we hypothesize that if the length of the hotspot decreases, e.g. due to a decrease of the bias voltage, the effective (or average) diffusion constant along the microbridge will also decrease, resulting in a slower response time of the device.

In this work, we verify this hypothesis. We present the experimental results of the measurement of the IF bandwidth of a Nb diffusion-cooled HEB for different bias voltages i.e. for different lengths of the electronic hotspot. The measured time constants are corrected for electro-thermal feedback through the IF circuit using the EHS model, in order to infer the intrinsic time constant of the mixer. Finally, the time constants are compared with the calculated time constant on the basis of a qualitative model.

III. Experiment

In this section, we present the measurement of the IF bandwidth of a Nb diffusion-cooled HEB around 700 GHz using two local oscillators. The device used in the

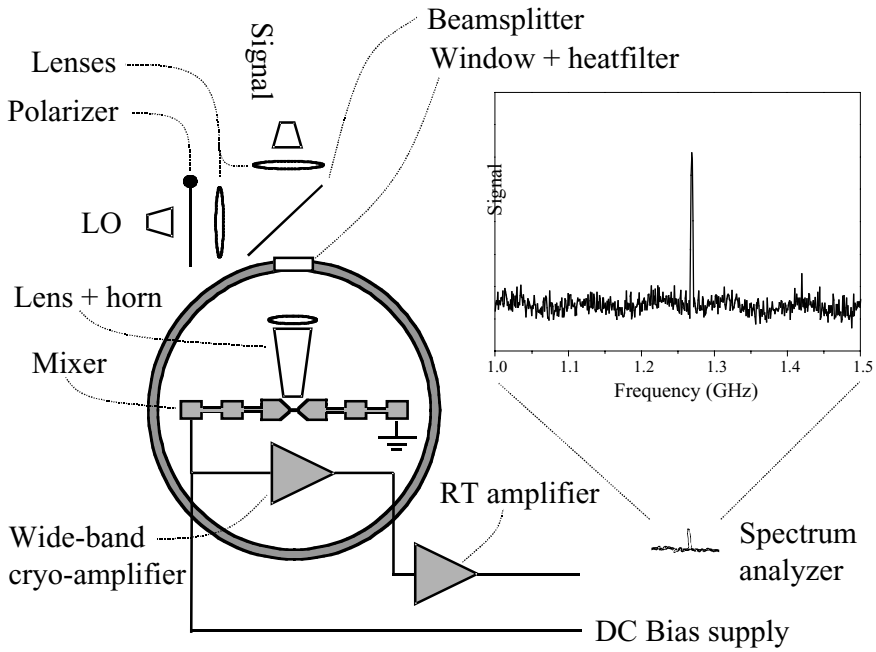

Fig. 2: Schematic representation of the experimental setup used for the gain vs. intemediate frequency measurement. The inset shows an actual measured spectrum.

experiment has a length of 300 nm, a width of 200 nm, and a normal state resistance R_n of 53 Ω. The critical temperature T_c of the microbridge is 5.9 K. The (normal state) diffusion constant was independently determined to be 1.6 cm^2/s. The fabrication procedure of the device has been given in detail in Ref. 16. The device is mounted in a fixed-tuned waveguide, designed for frequencies around 700 GHz. A schematic representation of the experimental set-up used for the measurements is given in Fig. 2. The rf power to pump the device is provided by a carcinotron with a doubler, whereas a Gunn oscillator with a quadrupler acts as a (weak) signal source. The IF output signal is amplified by a 0.1-8 GHz Miteq cryo-amplifier with an effective input impedance R_L of 32 Ω [17] and a 6 dB attenuator at the input to isolate the device from reflections. A 0.1-14 GHz Miteq amplifier is used to amplify the signal at room temperature. The amplified signal is measured with a spectrum analyzer. Special attention is paid to the gain versus frequency calibration of the IF chain and saturation of the second amplifier. During the experiment, the signal frequency is kept constant and the LO frequency is varied. The LO power is kept constant by adjusting the polarizer in the optical path and maintaining a constant current through the device with an accuracy better than 1 μA. All measurements reported here are performed in a single run, because of small changes in the electrical behavior of the device upon thermal cycling. The bath temperature T_b during the experiment is 4.5 K.

The measured relative conversion gain as a function of the intermediate frequency at a bias voltage V_b = 0.30 mV is shown in Fig. 3a. The calculated

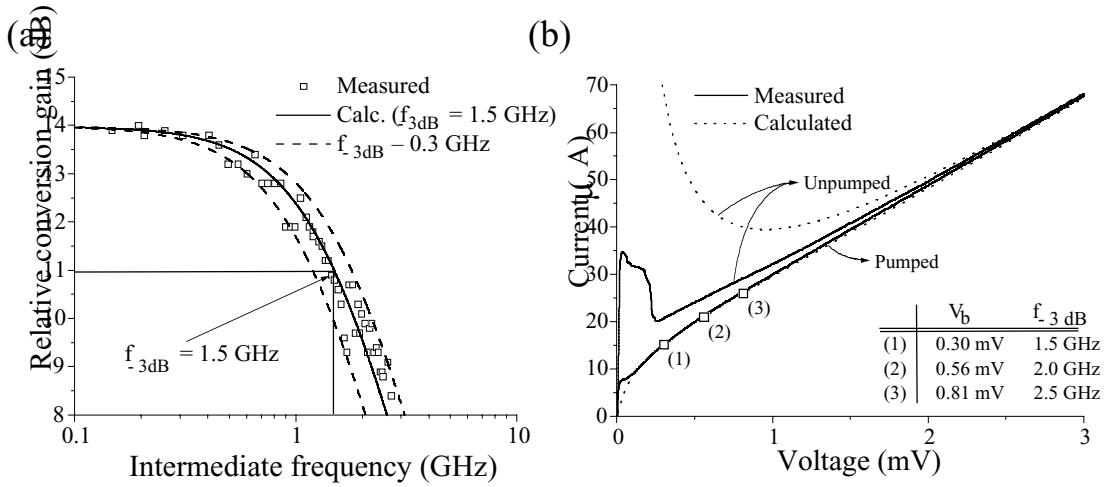

Fig. 3: (a) Relative conversion gain as a function of the intermediate frequency. The solid line is a one-pole calculation with a roll-off frequency f_{-3dB} of 1.5 GHz. The dashed lines represent the error margins (1.5 ± 0.3 GHz). (b) Unpumped and pumped I(V) curves. The bias points where the IF bandwidth has been measured are indicated by the open squares. Also shown are the calculated I(V) curves based on the EHS model. The coupled LO power in the calculation is 37 nW.

bandwidth from a one-pole fit to the data is 1.5 GHz, corresponding to an effective thermal time of $\tau_{eff} = (2\pi f_{-3dB})^{-1}$ = 106 ps. The maximum error in the measurement is 0.3 GHz and the corresponding curves with a roll-off frequency f_{-3dB} of 1.2 GHz and 1.8 GHz are also plotted in Fig. 3a. Fig. 3b shows the unpumped and pumped I(V) curves of the device. The three bias points at which the IF bandwidth has been measured are indicated in the figure, and the corresponding values for the bandwidth are given in the table in the inset. Clearly, the IF bandwidth increases with increasing bias voltage, up to 2.5 GHz at V_b = 0.81 mV. The same effect was observed by Skalare *et.al.* on a comparable device, but the physical origin of the effect was not addressed [17].

IV. Data analysis

In this section we calculate the intrinsic time constant from the measured time constant, which is affected by selfheating of the bolometer. We first discuss how to take into account selfheating in the framework of an electronic hotspot. Then the electro-thermal behavior of the mixer is modeled using the appropriate heat balance equations. From the calculated pumped I(V) curve we calculate all relevant parameters to correct the measured time constant for electrothermal feedback.

IV.a Selfheating through the IF load in the hotspot framework

In order to understand our experimental results and infer the intrinsic time constant of the mixer, the data have to be corrected for electro-thermal feedback through the IF load [3,19]. The measured effective time constant τ_{eff} of a HEB mixer is related to the intrinsic time constant τ_{th} via

$$\tau_{eff}(V) = \frac{\tau_{th}}{1+\alpha(V)\beta(V)} \qquad [2]$$

with $\alpha(V)$ and $\beta(V)$ given by

$$\alpha(V) = \frac{I_{dc}^2\left(\frac{dR}{dT}\right)}{G} \quad \text{and} \quad \beta(V) = \frac{R_{dc}-R_L}{R_{dc}-R_L} \qquad [3]$$

Here, I_{dc} is the dc current flowing through the device and R_{dc} the dc resistance (V_{dc}/I_{dc}) in the operating point. The factor $\alpha(V)$ reflects the selfheating of the device, whereas $\beta(V)$ accounts for the finite impedance of the IF load, basically suppressing the effect of electro-thermal feedback. In the lumped element approach, $\alpha(V)$ can be calculated directly from the pumped $I(V)$ curve [19]. In the hotspot framework, however, a temperature profile is present, and the same change in rf and dc heating has a different effect on the resistance of the HEB [6,7]. Therefore, $(dR/dT)G^{-1}$ should be rewritten as

$$\left(\frac{dR}{dT}\right)G^{-1} = \frac{C_{rf}G_{rf}+C_{dc}G_{dc}}{G_{rf}+G_{dc}} \qquad [4]$$

with

$$C_{rf,dc} = \left(\frac{dR}{dP_{rf,dc}}\right) \quad \text{and} \quad G_{rf,dc} = \left(\frac{dT_{avg}}{dP_{rf,dc}}\right)^{-1} \qquad [5]$$

Here, $C_{rf,dc}$ reflects the change in resistance due to a change in dissipated rf or dc power, respectively, $G_{rf,dc}$ is the effective thermal conductance for rf or dc heating, and T_{avg} is defined as the average temperature along the microbridge. Both $C_{rf,dc}$ and $G_{rf,dc}$ can be calculated using the appropriate heat balance equations for a HEB. Note that in the case that $C_{rf} = C_{dc}$ and $G_{rf} = G_{dc}$, Eq. 4 reduces to the expression given by Refs. 3 and 19.

IV.b Electro-thermal analysis of the mixer

We now proceed by modeling the electro-thermal behavior of the HEB mixer using the physically justified electronic hotspot model [6]. Here, we introduce two

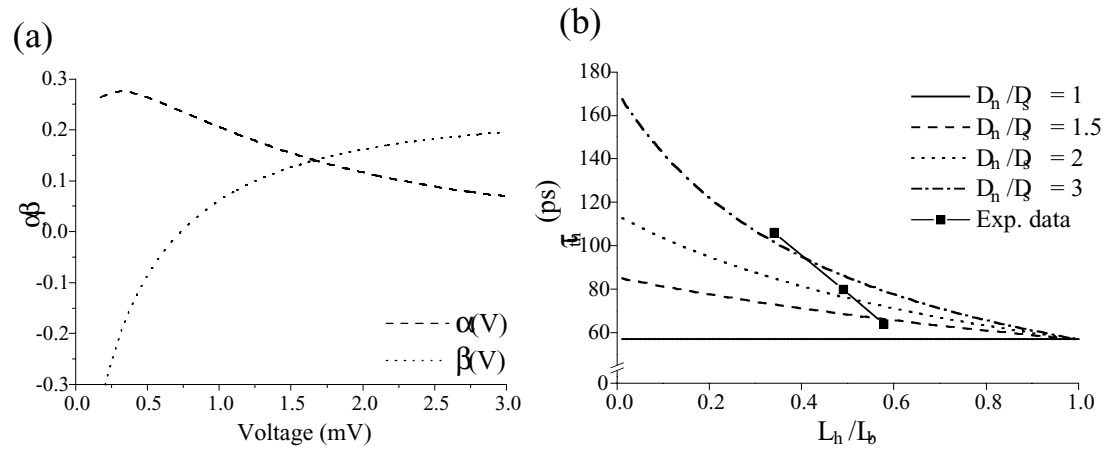

Fig. 4: (a) Calculation of α(V) and β(V) from the theoretical pumped I(V) characteristic and Eqs. 3-5. (b) Intrinsic time constant as a function of the relative length of the hotspot. The squares indicate the experimental results. The curves are calculated using different values of the ratio D_n/D_s (D_n = 1.6 cm^2/s, L_b = 300 nm).

modifications with respect to the model presented in Ref. 6, which physically describe a diffusion-cooled HEB more accurately, although they are found to have little influence on the actual outcome of the calculations. First, we do not take the thermal conductivity k to be constant but assume that it is linearly increasing with temperature in both normal and superconducting parts according to the Wiedemann-Franz law [20]. Second, we neglect the heat transfer from electrons to phonons, justifiable for a diffusion-cooled HEB, and allowing an analytical solution to the problem. As a result, the heat balance equation for a biased and pumped diffusion-cooled HEB takes the following form: Inside the hotspot, we have

$$-\frac{d}{dx}\left(k(T)\frac{dT}{dx}\right) = j^2\rho + p_{rf} \qquad [6]$$

with j the dc current density in the microbridge and p_{rf} the rf power density (per unit volume). Outside the hotspot we use the same equation, with the exception that there the dc dissipation is zero ($j^2\rho \rightarrow 0$). In the analysis, it is assumed that rf power is absorbed homogeneously in the microbridge, which is true if the frequency of the radiation is above the gap frequency of the superconductor. In our case, the radiation frequency is 700 GHz, whereas the gap frequency is ≈ 450 GHz. The equations are solved by requiring that $T = T_b$ at the end of the bridge and $T = T_c$ at the hotspot boundary. The current required to sustain a hotspot of length L_h when a radiation power density p_{rf} is absorbed, follows from matching dT/dx at the hotspot boundary.
In Fig. 3b, we plot the calculated pumped and unpumped *I(V)* characteristics together with the experimental data. A good agreement is found between measurement and calculation in the case of the pumped *I(V)* curve. In the case of the unpumped

characteristic, deviations between model and measurement are observed at low bias voltages, which show that the measured minimum dc current needed to sustain a hotspot is smaller than predicted by the model. This observation has been discussed in more detail in Ref. 21. Using the simulated pumped *I(V)* characteristic, we have calculated $(dR/dT)G^{-1}$ as a function of bias voltage according to Eqs. 4-5. The result is shown in Fig. 4a.

In Table 1, the results of the measurement and calculation for the three different bias points are summarized, from which it becomes clear that the intrinsic time constant of the microbridge increases with increasing length of the hotspot [22]. Note that in our particular case the effect of electro-thermal feedback is largely suppressed by the IF circuit, because the impedances of the IF load and mixer are close to each other. As a result, the effective and intrinsic thermal times do not differ much.

Table 1: Results of experiment and analysis

V_b (mV)	L_h (nm)	τ_{eff} (ps)	α(V)	β(V)	τ_{th} (ps)
0.30	113	106	0.28	-0.21	100
0.56	148	80	0.26	-0.058	79
0.81	173	64	0.23	0.021	64

V. Discussion

In order to verify whether the outcome of the experiment is physically sensible, we compare the results with a calculation of the time constant on the basis of Eq. 1, taking into account the difference in the diffusion constants of the normal and superconducting parts of the microbridge. We assume that we can use a linearly weighted average value of the diffusion constant according to

$$D_{avg} = \frac{D_n L_h + D_s (L_b - L_h)}{L_b} \qquad [7]$$

with D_n and D_s the diffusion constant in the normal and superconducting regions, respectively. In Fig. 4b, we plot the calculated time constant for different of values of D_n/D_s as a function of L_h/L_b. The ratio s D_n/D_s have been chosen such that they are comparable to the values expected on the basis of Fig. 1. In the case $D_n/D_s = 1$, the bandwidth does not depend on the length of the normal domain and thus the time constant is the same as given by Eq. 1, with $D_n = D_s = 1.6$ cm^2/s. It is important to note that the experimentally obtained values for the thermal time constant are, as expected, always larger than the value predicted by Eq. 1. Clearly, the predicted trend and the magnitude of the observed effect from Eq. 7 correspond to the experimental observations. However, we do not obtain nor expect a one-to-one correspondence between measurement and calculation, because the model used is oversimplified. A

more realistic calculation would have to include the full time-dependent heat-balance equations, taking into account the bias-dependent modification of the time constant due to electro-thermal feedback.

The results of the measurement and analysis presented here are in contrast to measurements of the IF bandwidth in a Nb *phonon-cooled* HEB mixer, in which case the bias dependence could very well be explained by electro-thermal feedback through the IF load [19]. This observation can be understood by realizing that in a phonon-cooled device the thermal path to the contacts is not relevant since the energy from the hot electrons is removed via inelastic scattering with phonons.

VI. Conclusions

To conclude, the essential outcome of this experiment is that it demonstrates that the intrinsic time constant of a diffusion-cooled HEB is not only depending on the length of the microbridge, but also on length of the hotspot in the bridge, being determined by the bias conditions of the system. The observations can be understood to a first order by using a qualitative model in which the reduced diffusion constant in the superconducting regions is taken into account.

Acknowledgements

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