A distributed device model for phonon-cooled HEB mixers predicting IV characteristics, gain, noise and IF bandwidth

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Abstract

A distributed model for phonon-cooled superconductor hot electron bolometer (HEB) mixers is given, which is based on solving the one-dimensional heat balance equation for the electron temperature profile along the superconductor strip. In this model it is assumed that the LO power is absorbed uniformly along the bridge but the DC power absorption depends on the local resistivity and is thus not uniform. The electron temperature dependence of the resistivity is assumed to be continuous and has a Fermi form. These assumptions are used in setting up the non-linear heat balance equation, which is solved numerically for the electron temperature profile along the bolometer strip. Based on this profile the resistance of the device and the IV curves are calculated. The IV curves are in excellent agreement with measurement results. Using a small signal model the conversion gain of the mixer is obtained. The expressions for Johnson noise

and thermal fluctuation noise are derived. The calculated results are in close agreement with measurements, provided that one of the parameters used is adjusted.

I. Introduction

Previously presented HEB models (the "point bolometer " or "standard" model) [1,2] assume a uniform electron and phonon temperature along the superconductor strip. Although these models are quite successful to explain many experimental results, some discrepancies have been reported:

It has been shown that the models are not capable of estimating the absorbed LO power correctly when operating at frequencies above the quasiparticle bandgap [3]. Also accurate measurements and calculations have shown such the "point bolometer" model cannot explain the dependence of the measured conversion loss and the output noise temperature on the bias voltage [4]. Nevertheless in order to optimize the device performance for space applications, an accurate model is needed.

One-dimensional models have been developed assuming that the electron temperature varies along the superconductor strip [5,6]. This assumption leads to different heating efficiencies for the absorbed LO and DC power implying that the resistance change due to a small change in absorbed LO power is not the same as that for absorbed DC power. This is due to the fact that the DC power is only absorbed in the part of the strip where the electron temperature is above the critical temperature whereas LO power absorption is uniform.

In previous one-dimensional hot spot models [5,7] the temperature dependence of resistivity is assumed to be a step function, which goes from zero to normal resistivity at the critical temperature. This and other assumptions were made in order to make it possible to obtain an analytical solution for the temperature profile. The main disadvantage of this model is that while the temperature profile does not exceed the critical temperature no hot spot is formed, and the model predicts zero resistance. Consequently it is not possible to calculate reasonable IV curves at low bias voltages and low LO powers. In practice the resistivity transition is smooth and measurements show that the transition width is about 1.2 K [8]. In order to obtain complete IV curves it is important to model the transition in a more realistic way.

The model presented here assumes that the temperature dependence of the resistivity, although step like, is continuous and has a Fermi form, and the transition width can be chosen as a variable (similar to a model for Nb diffusion cooled bolometers c.f. [9].). This results in a non-linear heat balance equation, which is solved for the electron temperature profile numerically. In section II the one-dimensional heat balance equation and our large signal model are discussed. Section III describes the small signal model. The noise is discussed in section IV. Section V explains a possible method for calculating the IF impedance and the bandwidth of the bolometer. Section VI summarizes the results.

II. Large signal model

Figure 1 shows the schematic heat flow in a small segment of a bolometer. The electrons are heated by the absorbed LO power (P_{LO}) and DC bias power. P_{LO} is absorbed uniformly along the bolometer with length $2L$, but the absorption of DC power is not uniform and depends on the bias current I_0 and the local resistivity $\rho(T_e)$. In steady state this heat is partly transferred to the phonons ($P_{e\to p}$) and partly to the electrons in the neighboring segments due to the electron thermal conductivity. Here it is assumed that power that is carried by the phonons ($P_{e\to p}$) is directly transferred to the substrate ($P_{p\to substrate}$). There the thermal conductivity of the phonons in the direction of the bolometer strip is neglected. The electron thermal conductivity, λ_e, is a function of the electron temperature. Below the critical temperature, T_c, it is proportional to T_e^3 and above T_c it is proportional to T_e^1 [10]. A denotes the cross section area of the bolometer strip.

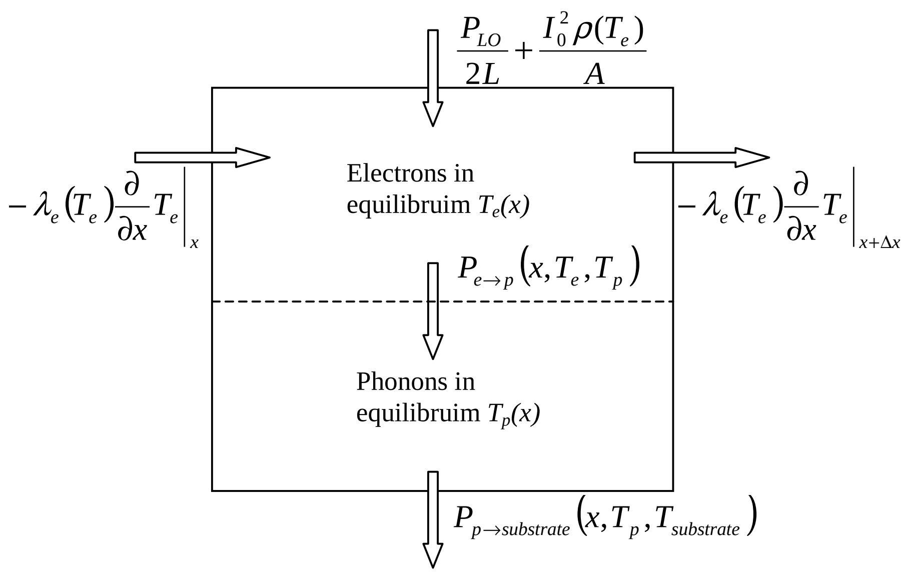

Figure 1. The heat flow in a small segment of the bolometer.

The electron-phonon and phonon-substrate thermal coupling can be written as:

$$P_{e\to p} = \sigma_e\left(T_e^n - T_p^n\right) \tag{1}$$

$$P_{p\to substrate} = \sigma_p\left(T_p^4 - T_{substrate}^4\right) \tag{2}$$

where σ_e, σ_p are the electron-phonon and phonon-substrate cooling efficiency respectively [8]. For NbN n is found to be equal to 3.6 in (1) [8].

The heat balance equations for a small segment of the bolometer can be written as:

$$\frac{\partial}{\partial x}\left[\lambda_e(T_e)\frac{\partial}{\partial x}T_e\right] - \sigma_e\left(T_e^{3.6} - T_p^{3.6}\right) + \frac{P_{LO}}{2L} + \frac{I_0^2\rho(T_e)}{A} = 0 \tag{3.I}$$

$$\sigma_e\left(T_e^{3.6} - T_p^{3.6}\right) = \sigma_p\left(T_p^4 - T_{substrate}^4\right) \tag{3.II}$$

Solving (3.II) approximately for T_p and substituting in (3.I) yields:

$$\frac{\partial}{\partial x}\left[\lambda_e(T_e)\frac{\partial}{\partial x}T_e\right]-\sigma_{eff}\left(T_e^{3.6}-T_{substrate}^{3.6}\right)+\frac{P_{LO}}{2L}+\frac{I_0^2\rho(T_e)}{A}=0 \qquad (4)$$

where $\sigma_{eff}=\frac{8\sigma_e\sigma_p\sqrt{T_{substrate}}}{8\sigma_p\sqrt{T_{substrate}}+7\sigma_e}$.

Note that the dimension of all the terms in (4) is W/m. and σ_{eff} and λ_e depend on the cross section area of the bolometer [7]. The corresponding dimensions of σ_{eff} and λ_e are given in table 1.

The electron temperature dependence of the resistivity is assumed to be:

$$\rho(T_e)=\frac{\rho_N}{1+e^{-\frac{T_e-T_c}{\delta T}}} \qquad (5)$$

where the ρ_N is the normal resistivity, T_c is the critical temperature and δT is a measure for the transition width. Figure 2 shows the resistivity as a function of temperature. If we define ΔT as the width of the transition form 10% to 90% of the normal resistivity, it is possible to show from (5) that:

$\Delta T = 2\ln 9\cdot\delta T \approx 4.4\cdot\delta T$.

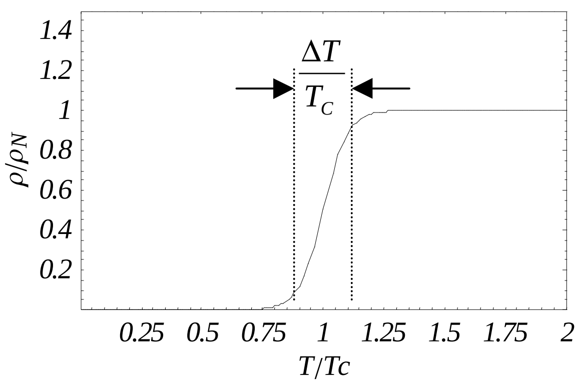

Figure 2. Resistivity as a function of temperature.

Equation (4) is solved numerically for $T_e(x)$ with the boundary conditions:

$T_e(-L)=T_e(L)=T_{bath}$

where $2L$ is the bolometer length. Note that the x-axis is in the direction of the bolometer strip and the origin is at the center of the bolometer.

Once the temperature profile is calculated the resistance of the bolometer takes the following form:

$$R=\frac{1}{A}\int_{-L}^{L}\rho(T_e(x))\,dx=\frac{1}{A}\int_{-L}^{L}\frac{\rho_N}{1+e^{-\frac{T_e(x)-T_c}{\delta T}}}dx \qquad (6)$$

III. Small signal model and conversion gain

The bolometer circuit is shown in figure 3. The mixing term at IF causes resistance fluctuations in the bolometer. The corresponding small signal voltage drives a current through the amplifier at the IF frequency. Note that this current goes through the bolometer as well and causes additional heating and consequently additional change in the resistance, which must be taken in to account. This effect is referred to as *electrothermal feedback.*

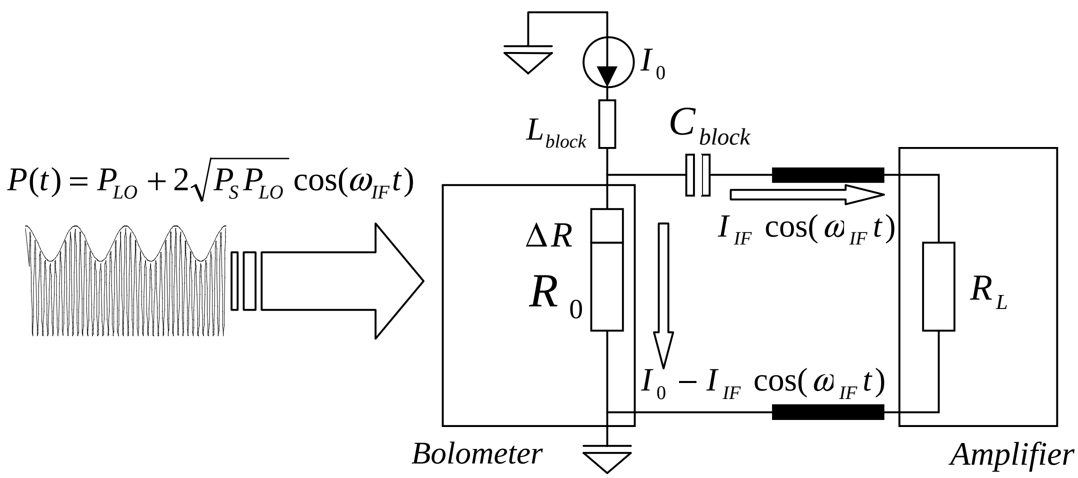

Figure 3. The bolometer circuit

From the above circuit the conversion gain of the mixer is derived as [7]:

$$G = \frac{P_L}{P_S} = \frac{2 \cdot I_0^2 \cdot R_L \cdot C_{RF}^2 \cdot P_{LO}}{(R_0 + R_L)^2 \cdot \left(1 - C_{DC} \cdot I_0^2 \frac{R_L - R_0}{R_L + R_0}\right)^2} \tag{7}$$

where P_L is the power in the load resistance and P_S is the absorbed signal power. C_{DC} and C_{RF} are the heating efficiencies of the absorbed DC and RF power respectively and defined as:

$$C_{DC} = \left.\frac{\partial R_0}{\partial P_{DC}}\right|_{\Delta P_{LO}=0} \text{ and } C_{RF} = \left.\frac{\partial R_0}{\partial P_{LO}}\right|_{\Delta P_{DC}=0} . \tag{8}$$

In order to calculate C_{DC}, the LO power is kept constant and the resistance change due to the small change in the bias current is calculated at each operating point. Special care has to be taken when calculating C_{RF} because keeping the current constant and applying a small change in LO power will change the absorbed DC power as well, which in turn changes the resistance. This contribution has to be deducted from the total resistance change in order to calculate the resistance change due to the change in the absorbed RF power only.

Assuming equal values for C_{DC} and C_{RF}, (7) is reduced to the corresponding relation known from the point bolometer models [1,4].

IV. Noise

The calculation of the Johnson noise contribution from the HEB noise model in [4] neglected the fact that the dissipated power in the noise source is actually dissipated within the HEB bridge, resulting in additional heating. Ignoring this term leads to a discrepancy between the results when the voltage noise source is replaced by an equivalent current noise source. A complete derivation for Johnson noise is given in the appendix, taking this term in to account.

The contribution of the Johnson noise to the total output noise is:

$$T_{Jn}^{out} = \frac{4R_L R_0 T_0}{(R_0 + R_L)^2 \left(1 - C_{DC} I_0^2 \frac{R_L - R_0}{R_L + R_0}\right)^2} \qquad (9)$$

The expression for the DSB Johnson receiver noise temperature is simply obtained by dividing the output noise (9) by two times the gain (7):

$$T_{Jn,DSB}^{in} = \frac{R_0 T_0}{I_0^2 \cdot C_{RF}^2 \cdot P_{LO}} \qquad (10)$$

In the one-dimensional hot spot model the electron temperature and the resistivity vary spatially, which requires us to modify (10) as:

$$T_{Jn,DSB}^{in} = \frac{\int_{-L}^{L} \rho\left(T_e(x)\right) T_e(x) dx}{I_0^2 \cdot C_{RF}^2 \cdot P_{LO} \cdot A} \qquad (11)$$

From what has been derived originally in the point bolometer model [1,2] the output thermal fluctuation (TF) noise is written as:

$$T_{TFn}^{out} = \frac{I_0^2 \cdot R_L}{\left(R_{bo} + R_L\right)^2 \cdot \left(1 - C_{DC} I_0^2 \cdot \frac{R_L - R_0}{RL + R_0}\right)^2} \left(\frac{\partial R}{\partial T_e}\right)^2 \cdot \frac{4 T_e^{\,2}}{c_e V} \tau_e \qquad (12)$$

where V is the bolometer volume, c_e is the electron thermal capacity, and τ_e is the electron relaxation time.

The double side band receiver noise temperature due to the TF noise is obtained by dividing the TF output noise (12) by two times the gain (7):

$$T_{TFn,DSB}^{in} = \frac{1}{C_{RF}^2 \cdot P_{LO}} \cdot \left(\frac{\partial R}{\partial T_e}\right)^2 \cdot \frac{T_e^{\,2}}{c_e V} \tau_e \qquad (13)$$

This expression also has to be modified for the one-dimensional distributed model. The model presented here assumes that the electron temperature fluctuations in the bolometer segments are uncorrelated. The derivation is given in the appendix B in detail and here we only recall the result:

$$T_{TFn,DSB}^{in} = \frac{1}{C_{RF}^2 \cdot P_{LO}} \int_{-L}^{L} \frac{\rho_n^2 e^{-2\cdot\left(\frac{T_e - T_c}{\delta T}\right)}}{\delta T^2 \cdot \left(1 + e^{-\left(\frac{T_e - T_c}{\delta T}\right)}\right)^4} \cdot \frac{T_e^2 \cdot \tau_e}{c_e \cdot A^3} \cdot dx \qquad (14)$$

Note that c_e and τ_e depend on the electron temperature, which in turn depends on x.
The discussion in appendix B indicates that the output fluctuation noise is essentially independent of the correlation length assumed.

The measured noise at the output of the mixer has three contributions and can be written as:

$$T_{noise}^{out} = T_{TFn}^{out} + T_{Jn}^{out} + 2 \times 295 \times G \qquad (15)$$

The third term is the ambient temperature (22 °C =295 K) at the input of the mixer, which contributes to the output noise during this measurement.

The total receiver noise temperature is calculated using the usual expression:

$$T_{RX,DSB} = T_{TFn,DSB}^{in} + T_{Jn,DSB}^{in} + \frac{T_{IF}}{2G} \qquad (16)$$

where T_{IF} is the noise contribution from the low noise IF amplifier (≅7 K) and G is the conversion gain (7).

V. IF Impedance and Bandwidth

We are currently studying a possible method to calculate the IF impedance and Bandwidth of the bolometer. The method is based on solving the time varying small signal one-dimensional heat balance equation at the operating point. Adding the time varying terms to (4) yields:

$$-c_e.(T_e).\frac{\partial}{\partial t}T_e + \frac{\partial}{\partial x}\left[\lambda_e(T_e)\frac{\partial}{\partial x}T_e\right] - \sigma_{eff}\left(T_e^{3.6} - T_{substrate}^{3.6}\right) + \frac{P_{LO}}{2L} + \frac{I_0^2 \rho(T_e)}{A} + \left(\frac{2\sqrt{P_S P_{LO}}}{2L} + p\right) \cdot e^{j\omega t} = 0 \qquad (17)$$

where p denotes the small signal power due to the electrothermal feedback per unit length. The electron temperature can be written as:

$$T_e(x) = T_0(x) + \tilde{T}(x) \cdot e^{jwt} \qquad (18)$$

Inserting (18) in (17) and separating the time varying part, an equation is obtained which can be solved for $\tilde{T}(x)$. Preliminary studies show a capacitive behavior of the bolometer, as is well-known from the point bolometer model [1].

A simple way to estimate the IF bandwidth is to assign a time constant for the phonon-cooling process and another time constant for the diffusion-cooling process. The diffusion-cooled power can be calculated by evaluating the gradient of electron temperature at the pads. The power cooled by phonons is then the difference between the total absorbed power and the diffusion-cooled power. The effective mixer time constant can be obtained by weighting the time constants with their contributions to the total cooling power.

VI. Results

The results of our calculations and measurements presented here apply to a 0.4 μm long, 4 μm wide and 5 nm thick NbN HEB device on MgO (Device M2-1). The parameter values used in (4) and (5) are summarized in table 1.

Parameter	$\lambda_e(T_e)$	σ_{eff}	δT	T_c	$T_{substrate}$	R_N
Value	$6\times10^{-18}\times T_e^3$	2×10^{-4}	0.3	8.5	4.2	75
Dimension	Wm/K	W/(m.K$^{3.6}$)	K	K	K	Ω

Table 1. The parameter values used in (4) and (5).

Equation (4) is solved numerically. Figure 4 shows the temperature profile obtained for 40 μA current and different absorbed LO powers.

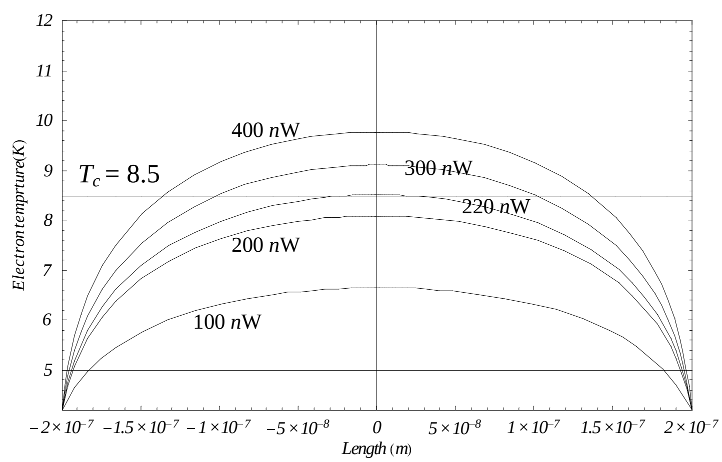

Figure 4. The electron temperature profile across the bolometer at 40 μA bias current and mentioned absorbed LO power.

Figure 5 shows the calculated and measured IV curves for device M2-1. Except at bias voltages below .4 *m*V, the calculated and measured IV curves are in excellent agreement. In general the calculated IV curves tend to bend more at lower voltages than the measured ones. On the other hand below .4 *m*V the noise is very high and the conversion gain is very low so this region is hardly of any interest.

The optimum operating point was observed around 40 μA and 0.8 *m*V. Therefore we focus on the middle curve which corresponds to 250 *n*W absorbed LO power and calculate the conversion gain and the noise.

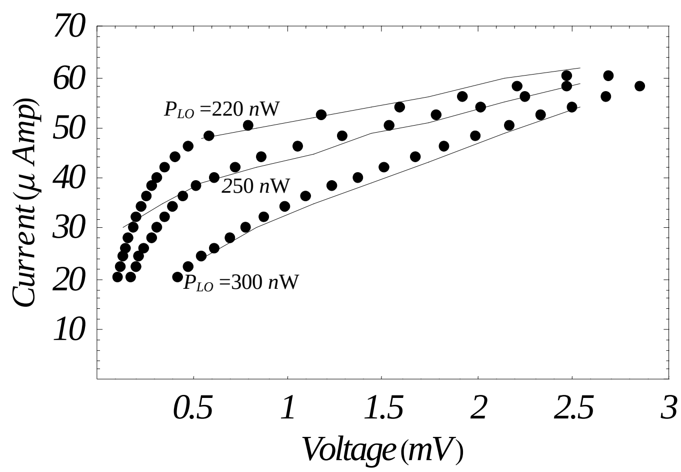

Figure 5. The measured (solid lines) and calculated (dots) IV cures for device M2-1.

The calculated C_{DC} and C_{RF} (8) for the 250 *n*W absorbed LO power curve is shown in figure 6.

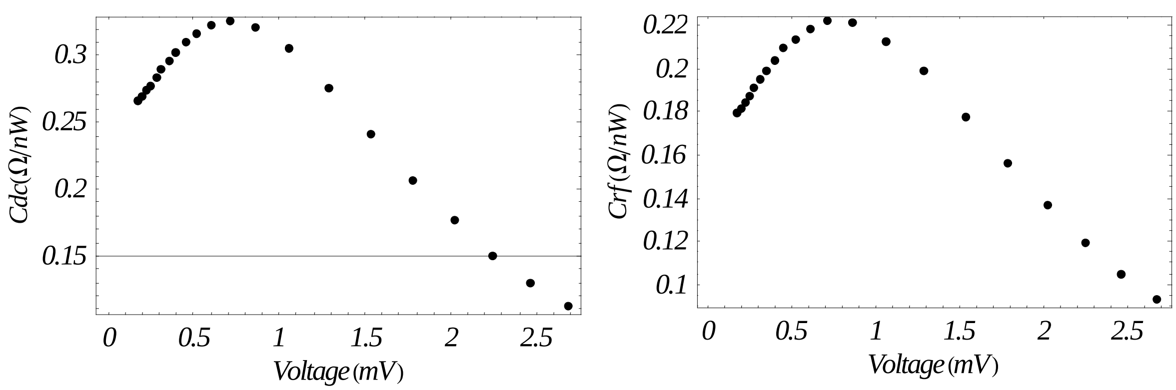

Figure 6. Calculated C_{DC} (left) and C_{RF} (right) along the 250nW absorbed LO power curve for different bias voltages.

Inserting these values in (7), the conversion gain is calculated. Figure 7(a) shows the calculated and measured conversion gain. Although the shape of the curves is similar, the calculated values are about 9 dB higher that what we actually measured. The calculated output noise (15) is a factor of 3 higher than what is measured. Figure 8(a) shows the calculated and measured output noise. On the other hand because of the high gain, the model predicts very low receiver noise temperature (16), which is shown in figure 9(a).

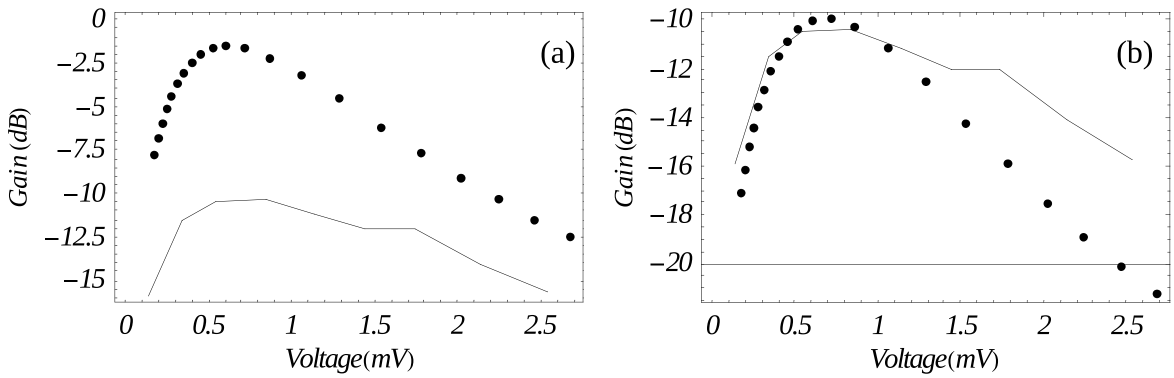

Figure 7. The measured (solid line) and calculated (dots) conversion gain for 250 *n*W absorbed LO power; (a) for high C_{RF} values as shown in figure 4; (b) for C_{RF} values reduced by a factor of three.

This means that the model cannot predict the measured curves. However a close study revealed that this is only due to the high C_{RF} values. If we reduce the C_{RF} values by a factor of three, all the measured and calculated curves fit together. The calculated and measured conversion gain, output noise and receiver noise temperature after this reduction is depicted in figures 7, 8, 9 (b) respectively.

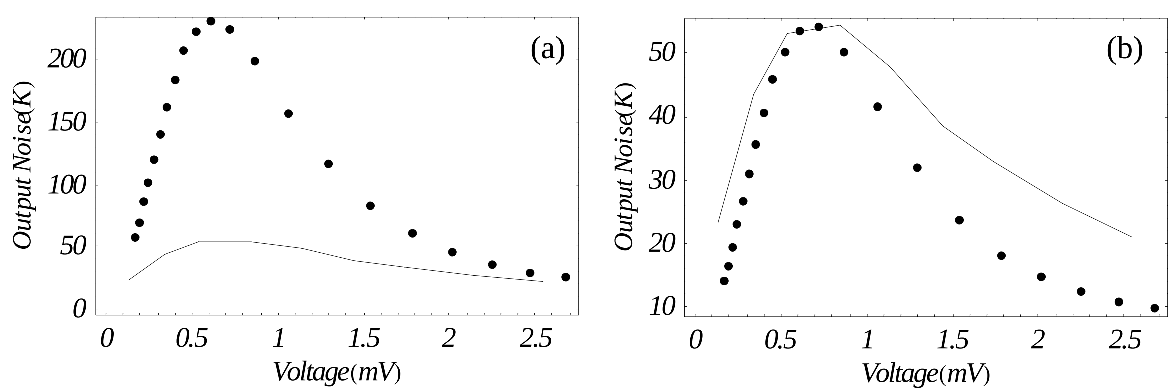

Figure 8 The measured (solid line) and calculated (dots) output noise for 250 *n*W absorbed LO power; (a) for high C_{RF} values as shown in figure 4 ; (b) for C_{RF} values reduced by a factor of three.

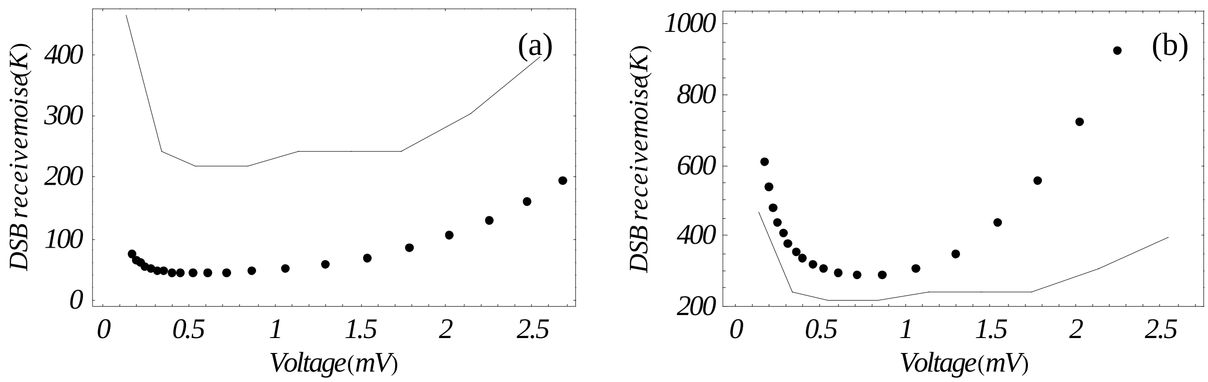

Figure 9. The measured (solid line) and calculated (dots) receiver noise temperature for 250 *n*W absorbed LO power; (a) for high C_{RF} values as shown in figure 4; (b) for C_{RF} values reduced by a factor of three.

After reducing C_{RF}, the value of $C_{RF}\, I_0^2$ at a typical optimum operating point is 0.16. Since I_0/I_{00} (I_{00} is the current on the unpumped IV curve c.f. [1,4]) is about 0.4 at the optimum point for receiver noise temperature [4], $C_{RF}\, I_{00}^2$ is about 0.7. This quantity was used as an adjustable parameter in [4], and for optimum performance was found to be about 1. This explains why the empirical "standard model" in [4] yields results in agreement with experiment close to the optimum point.

VII. Conclusion

The one-dimensional distributed model is fully capable of predicting the IV curves, conversion gain, output noise and receiver noise temperature within acceptable accuracy. However this requires applying a tuning factor to the calculated C_{RF}. This tuning factor was found to be about 0.34 to fit the measured data for device M2-1. It simply means that the resistance oscillation due to the mixing term in the RF is over estimated by calculating a small resistance change due to small change in LO power. The physics behind this factor is currently under investigation. Major progress has been made in comparison with the point-bolometer model in for example [4], in that the variation of all measurable quantities (conversion gain, receiver noise and output noise) with bias voltage and LO power is predicted very well.

Acknowledgements

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Appendix A: Johnson noise

The thermal noise from a resistance R at temperature T can be written as:

$$\left\langle v_n^2 \right\rangle = 4k_B TR, \qquad (1.A)$$

where k_B is the Boltzmann constant. In order to calculate the noise we can insert this noise source in the bolometer circuit. Figure 1.A shows this circuit. ΔR and ΔI are small variations of resistance and current respectively.

Assuming a point bolometer at temperature T_0 and resistance R_0, the noise equivalent source is:

$$\langle v_n^2 \rangle = 4k_B T_0 R_0 \tag{2.A}$$

The total dissipated power in the bolometer is:

$$P_0 + \Delta P = (R_0 + \Delta R)(I_0 - \Delta I)^2 + v_n (I_0 - \Delta I) \tag{3.A}$$

Note that the dissipated power in the noise source is actually dissipated in the bolometer as well and it is of great importance to take it into account. Ignoring this term leads to a discrepancy between the results when the voltage noise source is replaced by an equivalent current noise source.

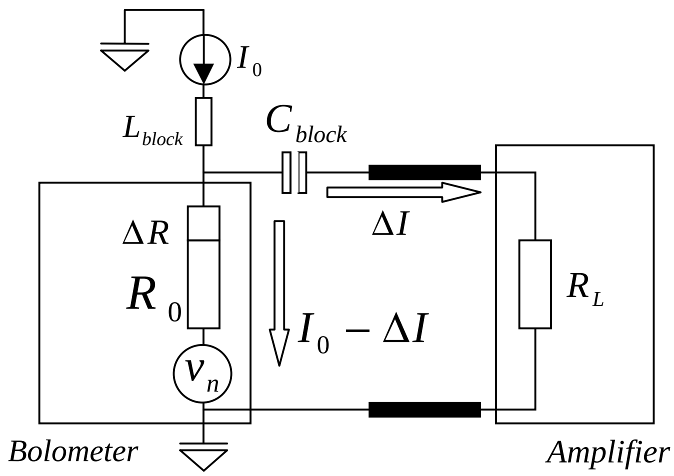

Figure 1.A. Bolometer circuit with equivalent Johnson noise source

Expanding the right side of (3.A) and ignoring the second order small signal terms, (3.A) is simplified to:

$$P_0 + \Delta P = R_0 I_0{}^2 - 2I_0 R_0 \Delta I + \Delta R I_0{}^2 + v_n I_0 \tag{4.A}$$

Separating the small signal and large signal parts yields:

$$\Delta P = -2I_0 R_0 \Delta I + \Delta R I_0{}^2 + v_n I_0 \tag{5.A}$$

since $\Delta P = C_{DC} \Delta R$, we can replace ΔP and solve (5.A) for ΔR which results in:

$$\Delta R = \frac{-2C_{DC} I_0 R_0 \Delta I + C_{DC} v_n I_0}{1 - C_{DC} I_0^2} \tag{6.A}$$

Now it is possible to calculate the voltage across the bolometer:

$$V_0 + \Delta V = (I_0 - \Delta I)(R_0 + \Delta R) + v_n \tag{7.A}$$

Separating the large and small signal here too gives:

$$\Delta V = -\Delta I R_0 + I_0 \Delta R + v_n = \Delta I \, R_L \tag{8.A}$$

where the small signal voltage ΔV is the IF voltage across the load. Solving for ΔI and replacing ΔR from (6.A) yields:

$$\Delta I = \frac{v_n}{(R_0 + R_L)\left(1 - C_{DC} I_0^2 \frac{R_L - R_0}{R_L + R_0}\right)} \tag{9.A}$$

The output noise power is simply:

$$P_{Jn}^{out} = R_L \left\langle \Delta I^2 \right\rangle = \frac{R_L \left\langle v_n^2 \right\rangle}{(R_0 + R_L)^2 \left(1 - C_{DC} I_0^2 \frac{R_L - R_0}{R_L + R_0}\right)^2} \tag{10.A}$$

Inserting (2.A) in (10.A) results in:

$$P_{Jn}^{out} = \frac{R_L \cdot 4 k_B B T_0 R_0}{(R_0 + R_L)^2 \left(1 - C_{DC} I_0^2 \frac{R_L - R_0}{R_L + R_0}\right)^2} = k_B B T_{Jn}^{out} \tag{11.A}$$

where T_{Jn}^{out} is the output Johnsson noise.

$$T_{Jn}^{out} = \frac{4 R_L R_0 T_0}{(R_0 + R_L)^2 \left(1 - C_{DC} I_0^2 \frac{R_L - R_0}{R_L + R_0}\right)^2} \tag{12.A}$$

The equivalent noise temperature at the input of the mixer is obtained by dividing the output noise by the conversion gain given in (7). The double side band (DSB) equivalent noise temperature is a factor of two smaller than the SSB one. So the expression for the DSB Johnson noise temperature at the input becomes:

$$T_{Jn,DSB}^{in} = \frac{R_0 T_0}{I_0^2 \cdot C_{RF}^2 \cdot P_{LO}} \tag{13.A}$$

This result is in agreement with the expression derived by taking another approach [2], and also with the general results for bolometers derived originally by Mather [11].

Appendix B: Thermal fluctuation noise

From the point bolometer model [1,2] the output thermal fluctuation (TF) noise is written as:

$$T_{TFn}^{out} = \frac{I_0^2 \cdot R_L}{(R_0 + R_L)^2 \cdot \left(1 - C_{DC} I_0^2 \cdot \frac{R_L - R_0}{RL + R_0}\right)^2} \left(\frac{\partial R}{\partial T_e}\right)^2 \cdot \frac{4 T_e^{\ 2}}{c_e V} \tau_e \tag{1.B}$$

where V is the bolometer volume, c_e is the electron thermal capacity, and τ_e the electron relaxation time.

The double side band receiver noise temperature due to the TF noise is obtained by dividing the TF output noise (1.B) by two times the gain (7):

$$T_{TFn,DSB}^{in} = \frac{1}{C_{RF}^2 \cdot P_{LO}} \cdot \left(\frac{\partial R}{\partial T_e}\right)^2 \cdot \frac{T_e^{\ 2}}{c_e V} \tau_e \tag{2.B}$$

The total resistance fluctuation is the sum of all the resistance fluctuations of each small segment of the bolometer due to temperature fluctuations. Thus *dr*, the resistance of such a segment with length *dx* is written as:

$$dr = \frac{\rho\left(T_e(x)\right)dx}{A} \tag{3.B}$$

Note that the resistivity profile $\rho\left(T_e(x)\right)$ is obtained from the electron temperature profile $T_e(x)$ from (5).

the total resistance of the bolometer is obtained by:

$$R = \int_{-L}^{L} dr = \int_{-L}^{L} \frac{\rho\left(T_e(x)\right)}{A}.dx \tag{5.B}$$

The resistance fluctuation of a small segment of the bolometer can be written as:

$$\frac{\partial\ dr(x)}{\partial T_e(x)} = \frac{\partial \rho\left(T_e(x)\right)}{\partial T_e(x)} \cdot \frac{dx}{A} \tag{6.B}$$

From (6.B) and (2.B) the thermal fluctuation noise temperature from this small part is derived as:

$$\begin{aligned} dT_{TFn,DSB}^{in} &= \frac{1}{C_{RF}^2 \cdot P_{LO}} \cdot \left(\frac{\partial\, dr}{\partial T_e}\right)^2 \cdot \frac{T_e^{2}(x)}{c_e A \cdot dx} \tau_e \\ &= \frac{1}{C_{RF}^2 \cdot P_{LO}} \cdot \left(\frac{\partial \rho\left(T_e(x)\right)}{\partial T_e(x)} . \frac{dx}{A}\right)^2 \cdot \frac{T_e^{2}(x)}{c_e A \cdot dx} \tau_e \\ &= \frac{1}{C_{RF}^2 \cdot P_{LO}} \cdot \left(\frac{\partial \rho\left(T_e(x)\right)}{\partial T_e(x)}\right)^2 \cdot \frac{T_e^{2}(x)\tau_e}{c_e A^3} dx \end{aligned} \tag{7.B}$$

Now if we assume that all these local temperature fluctuations are uncorrelated, the total noise temperature is simply the integral of all the small contributions.

$$\begin{aligned} T_{TFn,DSB}^{in} &= \frac{1}{C_{RF}^2 \cdot P_{LO}} \int_{-L}^{L} \left(\frac{\partial \rho\left(T_e(x)\right)}{\partial T_e(x)}\right)^2 \cdot \frac{T_e^{2}(x)\tau_e}{c_e \cdot A^3} dx \\ &= \frac{1}{C_{RF}^2 \cdot P_{LO}} \int_{-L}^{L} \frac{\rho_n^2 e^{-2\cdot\left(\frac{T_e - T_c}{\delta T}\right)}}{\delta T^{2} \cdot \left(1 + e^{-\left(\frac{T_e - T_c}{\delta T}\right)}\right)^4} \cdot \frac{T_e^{2} \cdot \tau_e}{c_e \cdot A^3} \cdot dx \end{aligned} \tag{8.B}$$

we have also performed this calculation by assuming different values for the correlation length of the TF noise. The result is essentially unchanged, provided that ($\partial\rho/\partial T_e$) varies smoothly. The above assumption of an infinitesimal correlation length thus is justified, unless the correlation length becomes comparable to the length of the bolometer.