QUASI-OPTICAL MULTIPLEXING USING REFLECTION PHASE GRATINGS.

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Abstract

Heterodyne array receiver systems for both ground based and satellite telescope facilities are now becoming feasible for imaging in the submillimetre/terahertz regions of the EM spectrum. Phase gratings can be usefully employed as high efficiency passive multiplexing devices in the local oscillator (LO) injection chain of such receivers, ensuring that each element of the array is adequately biased and that the reflected LO power level at the array is minimised. For the wavelengths of interest both transmission and reflection gratings can be manufactured by milling an appropriate pattern of slots into the surface(s) of a suitable material. Thus, the required phase modulation is produced by the resulting pattern of varying optical path lengths suffered by the incident wave-front. We report on work we are undertaking to develop all reflection quasi-optical multiplexing systems so as to reduce reflection losses at the grating and minimise the number of surfaces that can contribute to standing wave effects in the optical system. As part of this endeavour we have also developed a quasi-optical technique for analysing the inevitable degradation due to multiple reflections on transmission grating design. This analysis is based on the Gaussian beam mode technique, and a further application of this technique allows one to assess tolerance limitations on the grating.

1. INTRODUCTION

In previous papers we reported on our work on transmission Dammann gratings [1,2], one of the simplest types of grating to model and manufacture. The Dammann grating is a binary optical component consisting of a regular arrangement of slots or recesses of equal depth in a suitable transparent dielectric material such as quartz [3]. The phase grating as a whole consists of a repeated two-dimensional array pattern of basic cells. At the location of the Fourier plane in the subsequent optical system a regular two-dimensional pattern of non-overlapping beams is formed for a grating illuminated by a collimated beam. The number of basic cells illuminated by the incident beam controls the ratio of the beam width to the inter-beam spacing of the Fourier pattern, while the phase modulation function produced by a basic cell governs the peak-intensity of individual beams. For a typical imaging array the ideal is a pattern of equal intense closely spaced images of the LO feed.

In section 2 we describe an experimental procedure for prototyping and testing the equivalent reflection gratings to those transmission gratings already developed. This allows the use metal rather than quartz as the basic grating material into which slots are to be milled. One benefit of using metal is that it greatly reduces the model-manufacture-test cycle time. The technique, if used in conjunction with ferrite polarising materials, can be used to develop a complete reflection based quasi-optical multiplexing system. We can thus considerably improve standing wave performance for use in high sensitivity detector arrays.

In section 3 a Gaussian Beam Mode Analysis is applied to the problem of multiple reflections and standing waves associated with transmission phase gratings. At each interface of the grating some energy is transmitted and some is reflected. To take this effect into account an analogous modal scattering matrix approach to that applied in horn antenna modeling is used [4]. For quasi-optical systems the forward scattering matrices of a number of optical components have already been considered in the literature (e.g. [5,6]). In the full scattering matrix approach necessary to analyze standing waves, track has to be kept of both the backward and forward going components of the propagating fields. By combining a scattering matrix description of the partial reflection at each grooved face with a propagation matrix to describe the beam travelling through the grating standing wave effects are investigated. In the examples considered a Gaussian beam illuminates transmission gratings of refractive index of 1.66 and 2. In section 4 a Gaussian beam mode analysis of the effects of tolerance errors associated with the gratings is also presented.

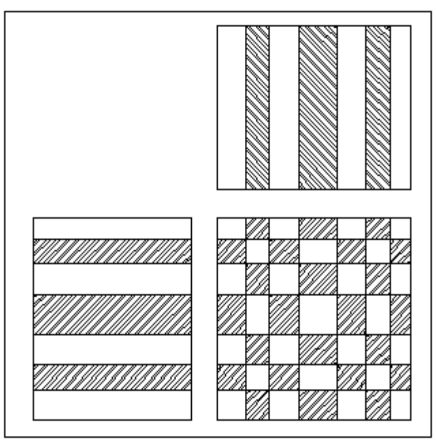
2. MANUFACTURE AND TESTING OF DAMMANN GRATINGS.

The Dammann grating, whether produced as a transmission or reflection grating, is a binary phase modulating structure which acts on the incoming wavefront to produce phase steps of 0 or π in the propagating beam. When imaged in the far field the desired result of this phase modulation is to produce an array of equi-intense images of the input field. This array of images can then be used, for example, to efficiently couple local oscillator energy into a multi-pixel heterodyne receiver. Algorithms have been developed to find the optimum phase transition points within a basic unit grating cell for a design to produce arrays of 2,3,4 and 5 output beams of equal intensity [1].

In the case of a transmission grating the modulation is achieved by arranging grooves in the surface of some transparent dielectric material such that the wave-front passing through the ungrooved portions suffer a phase delay of π, with respect to the rest of the wave-front phase. This is achieved by setting the groove depth to $\lambda/2n$, where n is the refractive index of the grating material.   Multiple reflections within the grating can cause losses and upset the distribution of power between the beams. These problems can be minimized if the grating thickness is chosen to be resonant at the wavelength of interest, i.e. $N\lambda/2n$ where N is odd. Of course the grating needs to be resonant both at the top and bottom of the grooves and this can only be achieved by choosing materials of the correct refractive index. Quartz is a good candidate for transmission gratings because it is low loss and has a refractive index of approximately 2. The effect when n is not optimum is discussed in section 3, in which multiple reflections in transmission gratings is analysed.

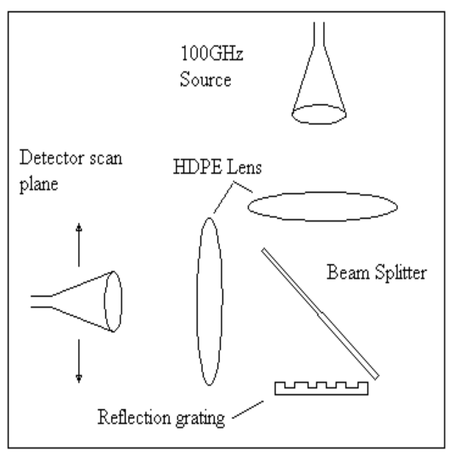
Conceptually a 2 D array can be obtained by passing the beam through two successive 1 D gratings rotated about the beam axis by 90 degrees with respect to each other. In reality this is achieved by forming each 1 D grating on opposites sides of a slab of quartz. This method is suited to quartz because the grooves are machined using a diamond grinding wheel, which can create long grooves easily but is not suitable for producing square sided pockets in a surface, as is required to produce a 2 D grating on one surface.

A reflection grating works on the same principles as the transmission grating. The phase transition points are the same as for the transmission grating but the groove depths need to be $\lambda/4$ to produce the required $\lambda/2$ phase delay in the reflected beam. Of course the grating needs to create the 2-D modulation on one surface and the drawing in *FIG. 1* shows how this is achieved.



Combining two 1-D Patterns.

FIG. 1.



Reflection Grating Test Set-Up.

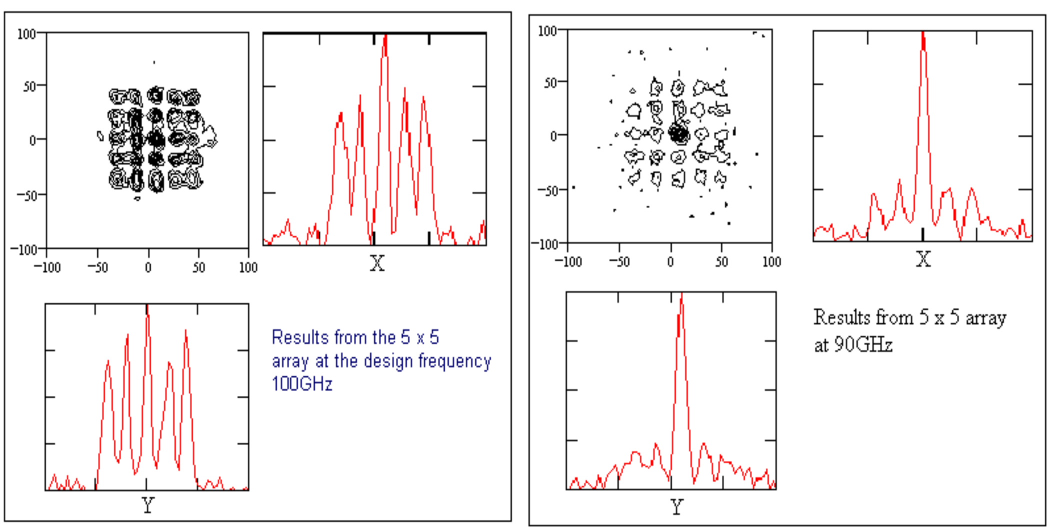
FIG. 2.

The shaded squares show where $\lambda/4$ pockets need to be formed in the surface. Notice the white squares where two grooves overlap. No metal need be removed here, because two overlapping grooves cause a phase shift of 2π which is equivalent to zero.

The resulting cutting pattern is more complex than that of the two sided transmission grating but it can be easily cut in aluminium sheet using a CNC milling machine. The milling machine can cut the pattern for a 5 x 5 reflection grating in about 2 hours. When compared with the 2 or 3 months required to have a quartz grating manufactured by a specialist optics company, it can be seen that using reflection gratings can greatly accelerate the design, build, test-cycle.

Simple reflection gratings need to be illuminated with a beam at normal incidence to prevent shadowing of the grooves, and this means that the test set-up is a little more complex than the standard inline 4-f configuration required by the transmission grating. The test set-up is shown in *FIG. 2*. The test facility consists of a Fourier 4-*f* optics set-up with a conical horn antenna feed driven by a variable frequency Gunn oscillator operating over the 90 to 105GHz range. In testing the system HDPE lenses are also used as the focussing elements because they produce less aberration in the beam pattern compared to the 90 degree throw off-axis ellipsoidal mirrors used previously [2]. The reflection grating is placed in the Fourier plane of the first lens and illuminated by a normally incident beam. The phase modulated reflection is coupled out to a computer controlled raster scanned detector through a large aperture beam splitter. The use of the beam splitter is not ideal because even with a perfect 50/50 split, only 25% of the initial beam power is directed towards the scanning detector. A better option would be to use a ferrite polarisation rotator to maximise the efficiency of the system, but for the purpose of grating testing at 100GHz where there is plenty of power available this is not a problem.

Several one and two dimensional gratings have been manufactured and tested using this system. A 5 x 5 reflection grating was cut and tested and the resulting patterns are shown in *FIG. 3*. The results are in good agreement with the software model.



Results for the 5 × 5 array at the centre frequency and beyond the band limit.

FIG. 3.

The results at 90GHz show that the grating still produces the 5 x 5 array but that the pattern is dominated by the central peak. We believe that this is caused by a combination of standing waves in the test setup and the degradation in grating performance at or near the band edges.
The bandwidth and manufacturing tolerances are closely related. A 10% change in operating frequency at 100GHz is equivalent to a change in the optimum groove depth of 0.08mm. In [1] we showed that the useful bandwidth of a transmission grating is approximately ±10% so a dimensional error of this size would render the grating useless. In terms of manufacturing tolerances, this level of accuracy is easily achieved in aluminum, and not so easily in quartz, but care must be taken to achieve good surface flatness over the entire grating area. It is interesting to note that the effect of groove depth tolerance errors is doubled in a reflection grating, where the phase shift is achieved by the two way trip in and out of the groove.

3. MULTIPLE REFLECTIONS IN TRANSMISSION GRATINGS

Analysing reflections in a quasi-optical system requires that the appropriate scattering matrices associated with a grating and all the other components involved be calculated. The quasi-optical system as a whole or any component is represented by a single scattering matrix [S] with the reflection and transmission characteristics determined by the equation:

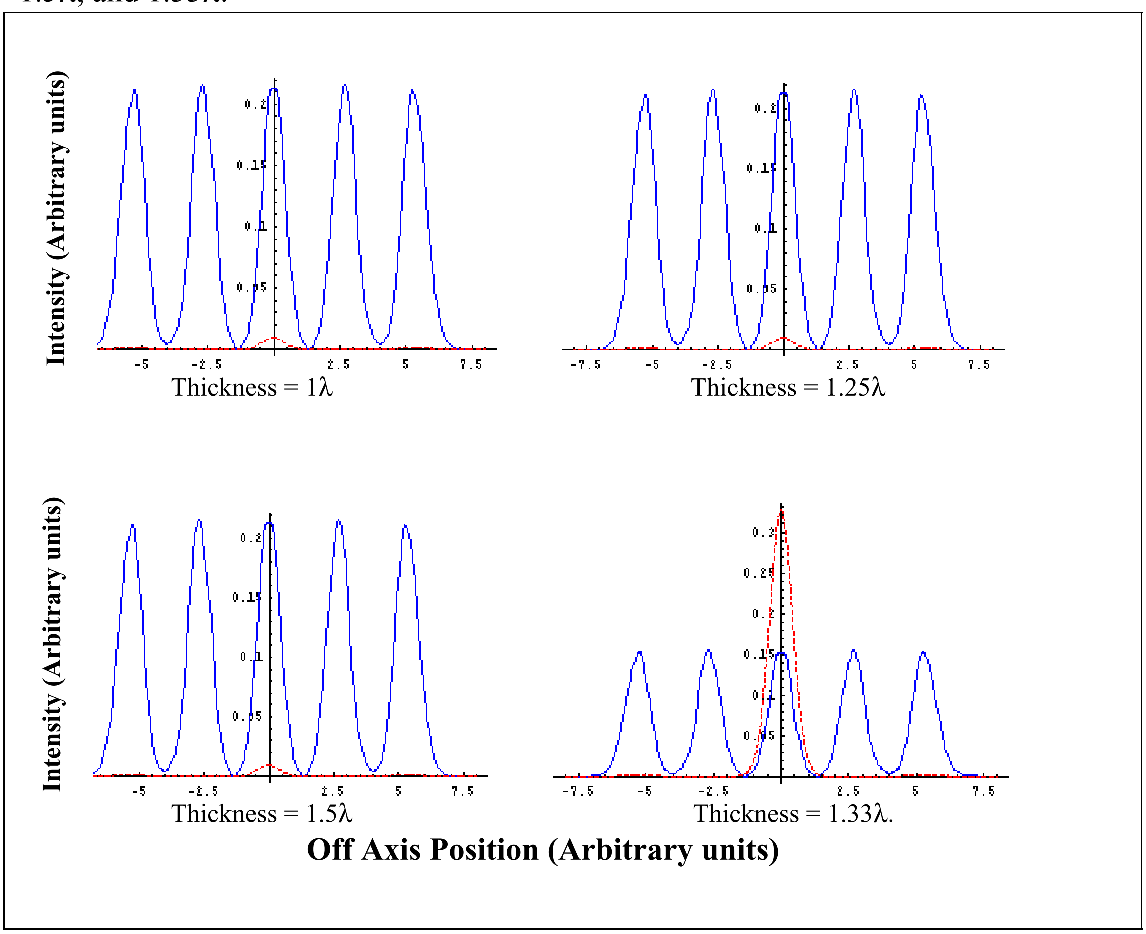
$$\begin{bmatrix}[B]\\ [D]\end{bmatrix} = \begin{bmatrix}[S_{11}] & [S_{12}]\\ [S_{21}] & [S_{22}]\end{bmatrix}\begin{bmatrix}[A]\\ [C]\end{bmatrix}.$$

[A] and [B] are vectors containing the forward and reflected mode coefficients, A_n and B_n, respectively, looking into the system at the input side. [C] and [D] are vectors of the mode coefficients, C_n and D_n, of all the modes looking into the system at the output plane.
In the case of a grating there is a partially reflected wave at each of the free space/dieletric interfaces. The reflected and transmitted electric fields are given by the Fresnel equation for normal incidence: $E_{refl} = \rho E_{inc}$, where $\rho = (n_1 - n_2)/(n_1 + n_2)$, and $E_{trans} = \tau\, E_{inc}$, where $\tau = 2\, n_1 / (n_1 + n_2)$. In these equations n_1 and n_2 are the refractive indices of the media for the incident and transmitted radiation, respectively. The reflected field E_{refl} can be written in terms of the modes travelling in the negative z direction $E_{refl} = \Sigma_n B_n \psi_n^-$. Since E_{inc} itself is written as a sum of modes travelling in the positive z direction $E_{inc} = \Sigma_n A_n \psi_n^+$, then the B_n can be derived from the scattering relationship: $B_n = \Sigma_m S_{mn} A_m$, where $S_{mn} = \rho \int_A (\psi_n^-)^* \psi_m^+ \, dx$. Since it is assumed that the modes travelling in the negative z direction have the same waist position and waist radius as those travelling in the positive z direction: $S_{mn} = \rho \int_A (\psi_n^+)^* \exp(-ikr^2/R)\, \psi_m^+ \, dx$, integrated over the grating surface. The quadratic complex exponential term represents the fact that the reflected wave suffers a sign change of its phase front

radius of curvature on reflection, or in other words that the reflected field continues to diverge after reflection.

In the following example the above theory is applied to the case where the incident Gaussian beam is transformed into 5 × 5 beams at the output Fourier plane of the grating. The gratings considered have refractive indices of 1.66 and 2.00 and the cell parameters were set to those appropriate values (see [1]). In general to accurately describe the phase variation introduced on the incident beam by the grating a large number of higher order modes are needed in any Gaussian beam mode analysis. However, one can increase the accuracy with a limited mode set by careful choice of the beam width parameter W (and not setting it equal to the beam width of the incident Gaussian). In fact, the best choice is determined by the scale of the structure on the grating rather than the width of the incident Gaussian so that fewer modes are needed to describe the grating more accurately. The incident Gaussian field must then of course be expanded in terms of a best choice mode set.

The transmitted and reflected far-field patterns of a grating with refractive index 2 and 1.66 are shown in figures 4 and 5, respectively, for a thickness value of 1λ, 1.25λ, 1.5λ, and 1.33λ.



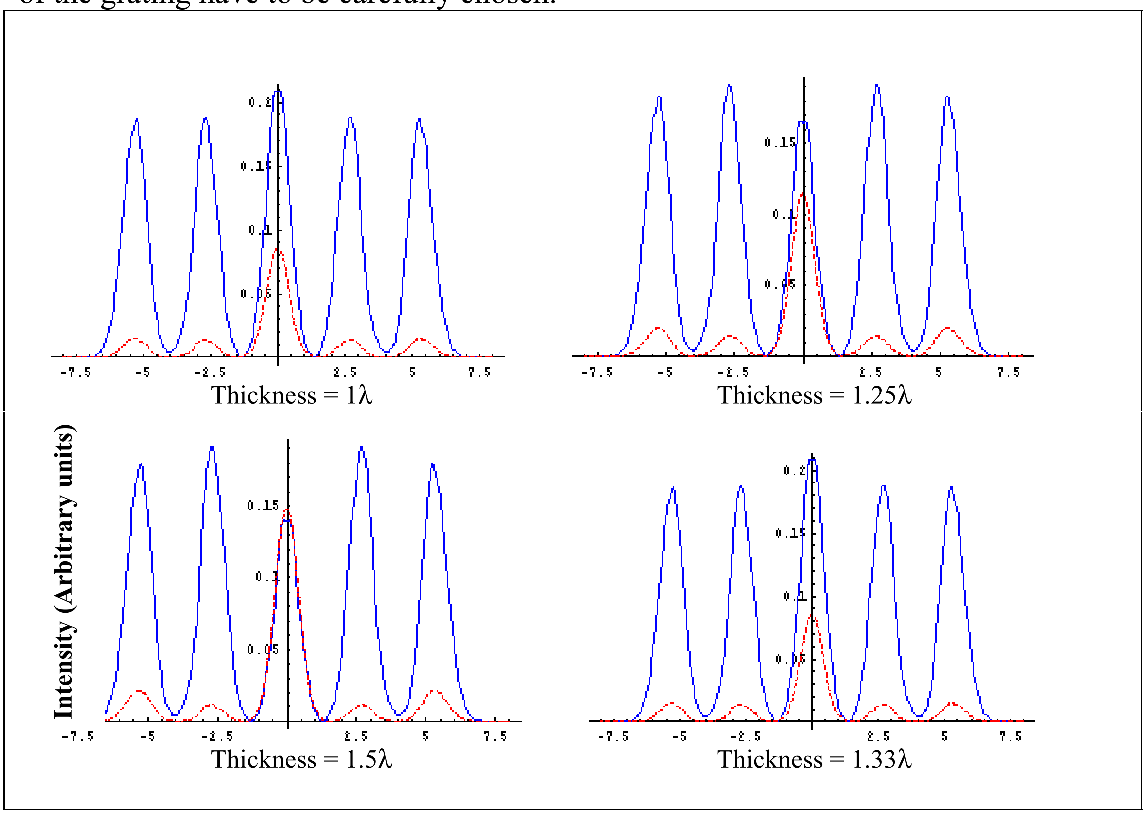
Cross section of beam patterns of 5x5 grating (n = 2) of thickness 1λ, 1.25λ, 1.5λ, and 1.33λ.

FIG. 4.

The overall reflection and transmission coefficients for a slab of dieletric of thickness t are given by $r = \dfrac{\rho + (-\rho)e^{2i\beta}}{1+\rho(-\rho)e^{2i\beta}}$ and $t = \dfrac{\tau\tau e^{i\beta}}{1+\rho(-\rho)e^{2i\beta}}$, where $\beta = 2\pi n t / \lambda_0$

For maximum and minimum transmission in the grating $e^{2i\beta}$ will either be zero or one. This means that both the refractive index n and the thickness of the grating must be chosen correctly to insure maximum transmission.

It can be seen that the transmission peaks for the grating of refractive index 2 are all equal intensity while for the grating of refractive index of 1.66 the transmission peaks are of unequal intensity. This is due to the contribution to the overall field from the wave that has been multiply reflected within the grating and does not add with the same phase lag as the straight through beam. What this clearly indicates is that for idealised grating operation both the refractive index of the material and the thickness of the grating have to be carefully chosen.



Cross section of beam patterns of 5x5 grating (n = 1.66) of thickness 1λ, 1.25λ, 1.5λ, and 1.33λ.

FIG. 5.

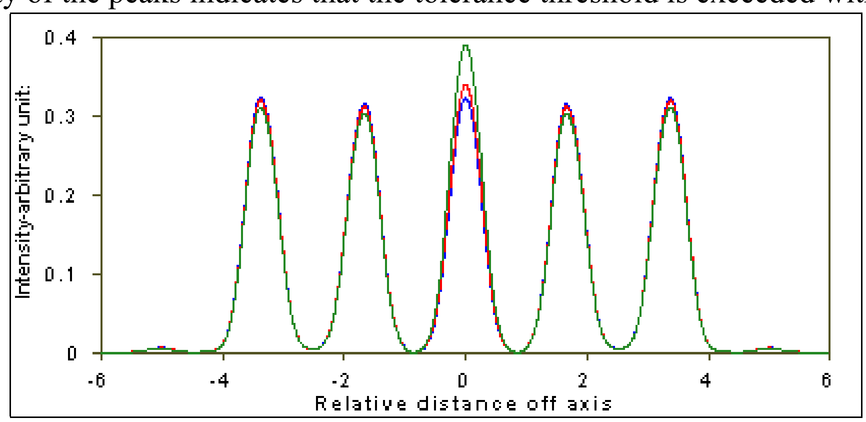
4. MODE ANALYSIS OF GRATING TOLERANCES.

An investigation of the effect of errors in the width and depth of the slots making up one of the faces of the grating was undertaken using Gaussian Beam Mode Analysis. The phase shift difference for the distinct binary paths through the grating, depends on the groove depth, h and the refractive index of the grating material, n. The phase shift and groove depth are related by:

$$\Delta\phi = \frac{2\pi h(n-1)}{\lambda_0} = \pi \ ,$$

where λ_0 is the wavelength of the incoming radiation in a vacuum and Δn is the difference between the refractive index of the grating material and the surrounding medium.

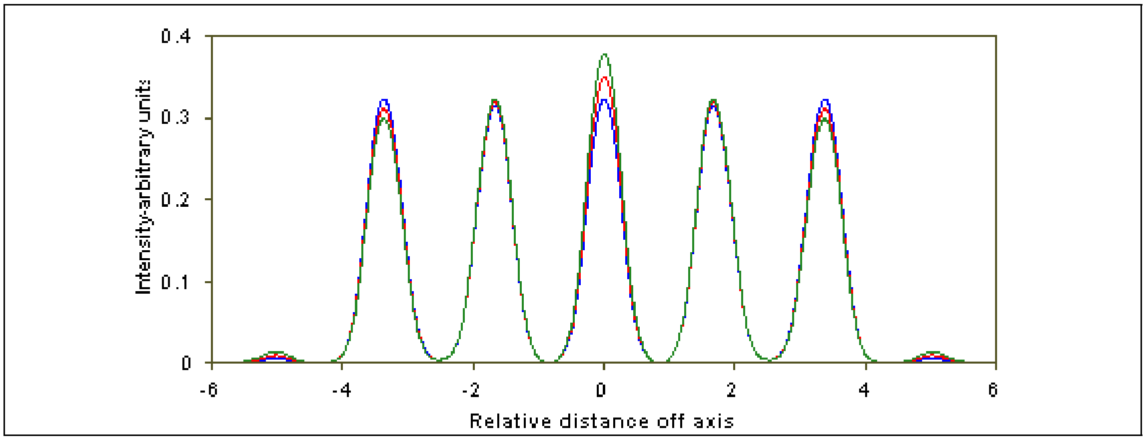
The cell depth or equivalently phase, was changed with addition of a small deviation, δ to the phase shift term. Thus, setting $\delta=0$ the phase change remains π corresponding to a groove depth of $\lambda/4$ for a reflection grating and the expected symmetric array of equi-intense images are obtained. The resulting field patterns for $\delta = \pi/16, \pi/8$ are superimposed on the ideal case for comparison in Figure 6. A phase change of $\pi/8$ illustrates the change in relative peak intensity and the deterioration of the uniformity of the peaks indicates that the tolerance threshold is exceeded with $\delta>\pi/8$.



The output field patterns for grating with phase errors of $\delta = \pi/16, \ \pi/8$ are superimposed on the ideal case for comparison.

FIG. 6.

Similarly by modifying the groove widths from the ideal values by approximately 0.2% of the cell width quite a substantial reduction in the efficiency of the operation of the grating as a multiplexor is observed. The results are shown in Figure 7.



Reduction in ideal grating operation for case where cell widths are varied.
FIG. 7.

5. CONCLUSION.

In this paper we have described some recent work on the manufacture and testing of reflection phase gratings. The analysis of reflection gratings using Gaussian Beam Mode Analysis was described and standing wave effects were shown to have a deleterious effect on ideal grating function unless the refractive index and thickness of the grating are carefully chosen. It may be possible to combine the ease of manufacture of metal gratings with a moldable powder[7] of the correct refractive index to produce transmission gratings with optimised characteristics. We have also summarised some modal modelling work we have carried out into assessing grating tolerances.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the support of Enterprise Ireland in this research programme. Special thanks to Mr. David Watson for his precision engineering skills and expertise in CNC milling of the gratings.

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