

## **DETECTION AND INTERPRETATION OF BISTABILITY EFFECTS IN NBN HEB DEVICES**

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**ABSTRACT-** Characteristics of an unpumped NbN Hot Electron Bolometer (HEB) device, between the superconducting state and the resistive state, show that a negative differential resistance occurs in the region between the superconducting state and the resistive state. The device is potentially unstable in this region, and we have performed further observations of the periodic waveform which occurs for this case on a fast digital oscilloscope. In order to fully understand the physical processes in this region both device voltage and current need to be obtained simultaneously. We then use the established theory for bistabilities in superconductors [1] to calculate the rise time of the waveform, and the frequency of the relaxation oscillations at about 6 MHz, which we observe in a number of devices. We show that the above theory can provide a qualitative interpretation of this waveform. The calculated rise time and frequency of the relaxation oscillations do not agree very well quantitatively, however, indicating that the simplest model based solely on the thermal conduction equation is insufficient. We also suggest how this model might be modified in order to obtain better quantitative agreement.

### **I. INTRODUCTION**

Observing the Current – Voltage Characteristics of an unpumped NbN Hot Electron Bolometer (HEB) device, a negative differential resistance occurs in the region between the superconducting state and the resistive state. The device is potentially unstable in this region, and we reported some results of studying this state at the 12<sup>th</sup> Space THz Technology Symposium [2]. We have made further observations of the periodic waveform which occurs for this case on an oscilloscope. In order to fully understand the physical processes in this region both device voltage and current need to be obtained simultaneously. We then use the established theory for bistabilities in superconductors [1] to calculate the rise time of the waveform, and the frequency of the relaxation oscillations at about 6 MHz, which we observe in a number of devices. We finally discuss our results in terms of the most recent models for PHEB mixer devices.

### **II. EXPERIMENTS AND RESULTS**

The 3.5 nm thick NbN HEB device was mounted on a circuit board in a small metal box in order to shield it from outside radiation, and to minimize parasitics. A small resistor,  $r = 10 \Omega$ , was used to enable us to measure the device current. The box was connected to two coaxial cables housed in a stainless steel tube “dipstick” to allow insertion of the device into liquid helium. The structure is shown in Figure 1. Two separate voltages,  $V_1$

and  $V_2$ , can be measured by connecting each cable to one channel of a digital oscilloscope whose maximum frequency is about 1 GHz. In this situation both the voltage across the device ( $V_d$ ) and the device current ( $I_d$ ) can be measured simultaneously. This turned out to be very important for the interpretation of the data.

$$V_d = V_1 - V_2 \quad (1)$$

$$I_d = V_2 / r \quad (2)$$

The bias supply is constructed with OPAMPS and has a very low source resistance, accomplished through feedback. There are also OPAMP circuits for recording the voltage and current levels on digital multimeters, which can then be displayed on a computer through a LABVIEW program. The sampling rate is about 20 samples per second, so only very slow variations can be recorded at the bias supply. It is connected through a low-pass filter with a cut-off frequency of 2.5MHz. One typical voltage biased I – V characteristics of a HEB device is shown in Figure 2. As is well – known there are 4 different regions in the I – V curve: (1) the superconducting region; (2) a region in which slow (a few hundred Hz) oscillations occur, whose frequency basically is determined by characteristics of the OPAMP in the bias supply (slow enough to be picked up by the LABVIEW system); (3) a second negative differential resistance region where the oscillations are much faster, and can be observed on the oscilloscope; the DC voltage and current as recorded through LABVIEW are stable at any particular point in this region; we will primarily study the device behavior in region (3) in this paper; (4) a stable hotspot region, where a stable hotspot is formed and can expand when the bias voltage is increased. All recordings are stable in region (4). We have studied the bistability for a voltage bias situation.

It is possible that the device in its negative resistance region can be stabilized by adding a series resistor  $r_{stab}$ . The circuit will become stable when increasing  $r_{stab}$  to make the total resistance become positive as we showed earlier [2].

$$r_{stab} > \left| \frac{dV_d}{dI} \right| \quad (3)$$

The corresponding I – V curve is shown in Figure 3 that includes a curve without the stabilizing resistor to make a comparison.

In our paper at the 12<sup>th</sup> STT [2] we recorded the device current by measuring the voltage across a series resistor inside the voltage bias supply. Because of the slow response rate of the OPAMP in the voltage supply, the current appeared to be almost constant with a percentage change of only about 1%. This time we measure the device current directly, as shown in Figure 1, allowing us to record the instant current value for the HEB device. Figure 4 shows the simultaneous measurement of both voltage and current. The voltage waveform is almost the same as before. It has a very fast rise (rise time about 12 ns) and then performs periodic relaxation oscillations. The amplitude of the voltage peaks indicates that the device is in a totally normal state at the peak of the oscillation. We can show this by multiplying the average current (about 0.2 mA for the data in Figure 4) by the full normal

resistance ( $R_N = 150 \, \Omega$ ) of the device. We find a voltage of 30 mV, which agrees with the peak voltage amplitude in Figure 4. When the relaxation oscillations die out, the device returns to superconducting, as indicated by the voltage across it being zero. The corresponding current during the relaxation oscillations used in this estimate is the average current in the instability region. The current also shows (weaker) relaxation oscillations. When the oscillations vanish, and the device is temporarily back to being superconducting, the current gradually increases, reaches the critical value, and the cycle repeats. The repetition rate of the waveform increases with increasing bias voltage, as shown in Figure 5.

### III DISCUSSION

We now discuss the measurement results in terms of a one – dimensional hotspot model.

#### *The Superconducting State*

It has been claimed that the main mechanism which determines the critical current in the superconducting state of an HEB device is vortex pinning. As long as the pinning force is larger than the Lorentz force, the device remains in the superconducting state. But our calculation concludes that there are no vortices available in the NbN film of an HEB device of a typical size. For example, in the simplest case, we consider a strip with width  $W = 5 \mu\text{m}$ ; length  $L = 1 \mu\text{m}$ ; thickness  $d = 4 \text{ nm}$ ; and total current  $I = I_C = 500 \mu\text{A}$ . The  $x$ -direction is along the device width. The  $H$  field in the  $x$ -direction (perpendicular to the current direction) is more or less independent of  $x$ , except for at the edges. Although the field has a singularity at the edges, this singularity is of no consequence when we integrate the field in order to obtain the total flux. We can then estimate that  $B_x = \mu_0 H_x = \mu_0 I / 2W = 1.2 \times 10^{-4} \text{ Vs/m}^2$ , and the total flux  $\Phi = 3 \times 10^{-16} \text{ Vs}$ . However, a vortex must contain a minimum flux of one flux quantum, which is  $\Phi_0 = 2 \times 10^{-15} \text{ Vs}$ . *Thus vortices can not exist in this strip.* The critical current is probably determined by depairing of the Cooper pairs rather than by vortex pinning.

#### *Instability Region*

In earlier studies it has been found that the superconductivity can be destroyed by Joule self – heating even when the transport current is much lower than the critical current [1],[3],[4]. The cooling and heating mechanisms obey the heat balance equation:

$$Q(T) = W(T) \quad (4)$$

$$W(T) = \frac{h(T)}{d} (T - T_0) \quad (5)$$

Here  $Q(T) = jE(T)$  is the power of the Joule heat released per unit volume.  $W(T)$  is the specific power of heat transfer to the coolant kept at a bath temperature  $T_0$ ,  $h(T)$  is the heat transfer coefficient,  $d = A/P$ ,  $A$  is the area, and  $P$  is the perimeter of the cross section of the specimen. A bistability occurs when this equation holds for several temperatures, as shown

in Figure 6, where there are three intersection points between  $Q(T)$  and  $W(T)$ . The step shape of the  $Q(T)$  curve is due to the stepwise increase of the resistivity  $\rho(T)$  around the critical temperature  $T_C$ . The stability of the three points 1, 2, and 3 with respect to a small perturbation is determined by the following criterion:

$$\frac{\partial W}{\partial T} > \frac{\partial Q}{\partial T} \quad (6)$$

From the above equation we can see that the states corresponding to the phases  $T_1$  and  $T_3$  are stable, and the state corresponding to phase  $T_2$  is unstable. There is a simple method to decide the condition of stability that introduces the Stekly parameter  $\alpha$  defined as:

$$\alpha = \frac{\rho_n j_C^2 d}{h(T_C - T_0)} \quad (7)$$

where  $\rho_n$  is the resistivity in the normal state, and  $j_C$  is the critical current at the bath temperature. It turns out that the differential conductivity can be expressed by  $\alpha$  as:

$$\sigma(E) = (1 - \alpha) \rho_n^{-1} \quad (8)$$

The condition for obtaining the above equation is  $j_C \gg E \rho_n^{-1}$  which is satisfied in thin superconducting films. For example, the typical value of  $j_C$  in our NbN devices is around  $1 - 3 \times 10^6$  A/cm<sup>2</sup>, and  $E \rho_n^{-1}$  is about  $10^5$  A/cm<sup>2</sup> for NbN thin film material. The differential conductivity becomes negative when  $\alpha > 1$ , which indicates that there is a thermal instability in the resistive state. Obviously, the upper limit of the current density in order to get the thermal instability is  $j_C$ , while the lower limit  $j_m$  can be calculated by assuming that the temperature of the superconductor at  $j = j_m$  is  $T_C$ ; then  $j_m$  can be expressed as:

$$j_m = \left[ \frac{h(T_C) T_C}{\rho_n (T_C) d} \right]^{1/2} \left[ 1 - \frac{T_0}{T_C} \right]^{1/2} = j_C \alpha^{-1/2} \quad (9)$$

Here  $j_m$  depends on the characteristics of the superconductor and the coolant parameters, and can be much lower than the critical current  $j_C$ . Now the transport current density should be in the range  $j_C \alpha^{1/2} < j < j_C$  in order for  $Q(T)$  and  $W(T)$  to have three intersection points. Reference [1] shows that the lower limit is actually  $j_p = \sqrt{2} j_m$ . Outside this range the superconductor material will be in one of the stable states. In the case of NbN HEB devices, the value of  $\alpha$  is usually much larger than 1. For example, typical values for a device such as the one used to obtain the data in Figures 3 and 4, are  $\rho_n = 2 \times 10^{-4}$   $\Omega$ cm,  $j_C = 2 \times 10^6$  A/cm<sup>2</sup>,  $d = 5 \times 10^{-5}$  cm,  $h = 20$  W/cm<sup>2</sup>K,  $T_C = 10$  K,  $T_0 = 4.2$  K. Substituting the above values in equation (7) we get  $\alpha = 287 \gg 1$ . The lower limit for the current,  $I_p$ , is estimated to be 30  $\mu$ A,

whereas  $I_c = 400 \mu A$ . The bistability is thus potentially very strong. We can interpret the behavior of NbN HEB devices that we observed in Figure 4 in terms of propagation of switching waves, which describes the movement of the domain walls between the two stable states: superconducting and normal states. The domain wall moves in one direction when transforming from one stable state to the other, or vice versa, depending on the value of  $j$ . The velocity  $c$  of the propagation can be expressed as in [5]:

$$c(i) = (1 + 0.561\alpha^{-1.45}) (M - 1) / \sqrt{M} \quad (10)$$

$$M = \left( \alpha i^2 - 1 + \frac{i}{2} \right) / \left( 1 - \frac{i}{2} \right) \quad (11)$$

Here  $c(i) = v/v_h$ ,  $v_h = L/t_h$ , where  $L$  and  $t_h$  are the characteristic thermal length and time, and  $i = j/j_c$  is the dimensionless current density. The  $c$  value as a function of dimensionless current  $i$  is plotted in Figure 7. We also estimate  $v_h = 10^4$  cm/sec. Our interpretation of the device behavior in region (3) can then be summarized as follows:

1) After  $j_c$  has been exceeded, a normal domain spreads with an initial normalized velocity  $c = 24$ . The expansion will slow as the resistance grows, and the current decreases, until the normal domain reaches the contacts. The average velocity during the initial expansion can be estimated to be  $10 \times v_h = 10^5$  cm/s. The initial velocity is so high that it is close to the sound velocity in the film, which should act as an ultimate speed limit. The estimated time for a domain to cover half the length of the device is about .5 ns, a very short time, while we measured a value of 12 ns. The initial rise time is, however, also influenced by the response time of our measurement system, which we checked by a measurement using a fast (6 ns rise time) pulse generator substituting for the HEB device. The rise time was found to be 15 ns which confirms that we could not have measured the rise time of the HEB device waveform if it had been as fast as .5 ns.

After the initial transient, the voltage and current oscillate around an average current of about 200  $\mu A$ . From the bistability theory, we would expect that for  $j > j_p$ , the domain would grow with increasing velocity until it reaches the contacts, and  $R = R_N$ . In region (3) there is not enough power available to sustain a stable hot spot, and the current has meanwhile decreased below  $j_p$ , which reverses the sign of the velocity, and the domain then decreases in size; the current subsequently increases above  $j_p$  again, leading to the domain oscillating back and forth until the device goes superconducting, etc.. The frequency of these relaxation oscillations can be found by estimating the time required for the domain to expand through about half the length of the device, and back again. We assume that the oscillations are sinusoidal. Also, the measured average current is 200  $\mu A$  rather than the expected value of 30  $\mu A$ , based on  $j_p$ , and  $\alpha = 290$ . The velocity curve for this larger average current value has also been plotted in Figure 4. It corresponds to  $\alpha = 10$ . If we assume that the oscillations occur around this current point, and use the measured peak current amplitude from Figure 4, we find a relaxation oscillation frequency of about 60 MHz, with a period of 16 ns. Instead, we measure the period of the oscillation to be roughly 160 ns, or a frequency of 6 MHz. In addition to the higher than expected average current, this seems to give us pretty clear evidence that one must include further physical

effects in the calculation of the propagation speed than the purely thermal effects (lateral thermal conduction and heat transfer to the substrate) which were included in the above calculation. Possible such effects are:

- (a) The contacts are likely to be involved in the thermal balance, since the normal domain periodically reaches these. The contacts would add considerable heat capacity, and could have this effect.
- (b) Recent modeling of phonon-cooled HEB devices while under LO illumination has shown bistable regions which may correspond to the bistability discussed here [6]. It has also been argued that the hotspot should be defined in terms of the critical current as a function of position in the device, rather than the electron temperature [7]. The latter paper also showed that Andreev reflection at the superconducting/normal boundary should have an influence on the temporal response of the device. A very preliminary calculation [8] of the oscillation relaxation frequency yielded 30 MHz, which is closer to the measured value than our simplified estimate above.
- (c) The thermal healing length in the above calculation is about half the length of the HEB device, which will introduce some further error to the simple estimate of the relaxation oscillation frequency, which is based on an infinitely long strip.

We want to emphasize that the relaxation oscillation frequency has been determined to be independent of the external circuit, and must be related to the physics of the device itself. The theory may be better fitted by measuring a longer device, an experiment which we are now planning to perform.

2) The device is superconducting during a portion of the repetitive behavior measured in Figure 4. During this time no power can thus be dissipated. The current rises driven by the voltage supply, and we interpret the delay in reaching  $I_c$  as due to the effect of the time constant determined by the inductance in the LP-filter and the cable, together with resistance  $r$ . The measured inductance is 20  $\mu\text{H}$ , which yields a time constant of about 2  $\mu\text{s}$ , which agrees reasonably well with our measured waveform. This also explains that the repetition rate of the waveform increases with increasing DC voltage, as shown in Figure 5. Note that the duration of the part of the repetitive waveform when the relaxation oscillations occur, is more or less independent of the voltage.

3) In region 2, we also observe slower oscillations, at frequencies of the order of hundreds of Hz, which can be changed by adding elements to the external circuit. Substituting a simple bias supply consisting of a battery and a potentiometer, eliminates these lower frequency oscillations, and only the relaxation oscillations are observed. These are thus clearly the most fundamental phenomenon observed in the negative resistance region.

*Consequences for device behavior in active mixer operation*

The model we have used to estimate the rate at which a hot-spot domain can vary its size could also be used to analyze device models in the stable state, where LO power is applied to bring the differential conductivity to positive values. The modeling required for actual HEB mixer operation is likely to be much more complex, as shown in papers by Merkel et al. [6,7]. The advantage of our approach is that it is possible to test the models for the bistable regions and for the temporal response of hotspots with relatively simple measurements. Our experiments also detect higher frequencies in the GHz range, when the HEB device is measured in the bistable region through a low-noise cooled HEMT amplifier.

#### **IV. CONCLUSION**

In this paper we measured the voltage and the current of the NbN HEB device simultaneously in the bistability region. We conclude that no vortices can exist in a typical NbN HEB strip. The transition between the superconducting state and the normal state was interpreted qualitatively as the propagation of a switching wave. The calculated relaxation oscillation frequency based on the domain velocity is about ten times higher with the measured result. We expect that use of a more detailed model of the device will result in closer quantitative agreement with measurements in the future. Also, measurements on longer devices, which we plan to carry out, should facilitate better agreement between theory and experiment.

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[8] H.F. Merkel, private communication.

## FIGURES

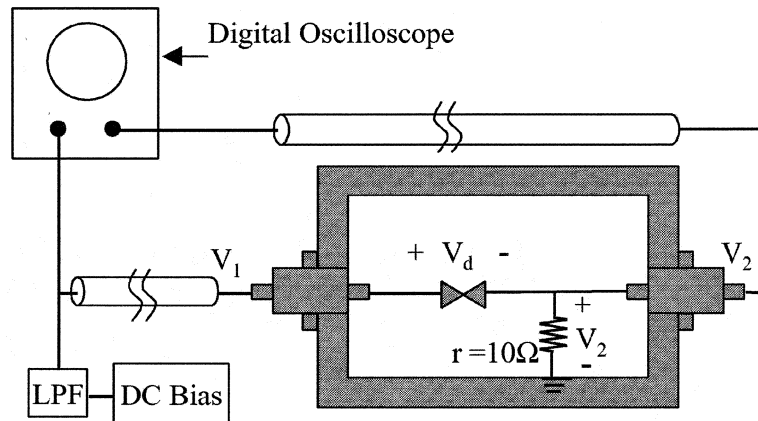


Figure 1: Experimental Setup for the current and voltage measurement of the NbN HEB device

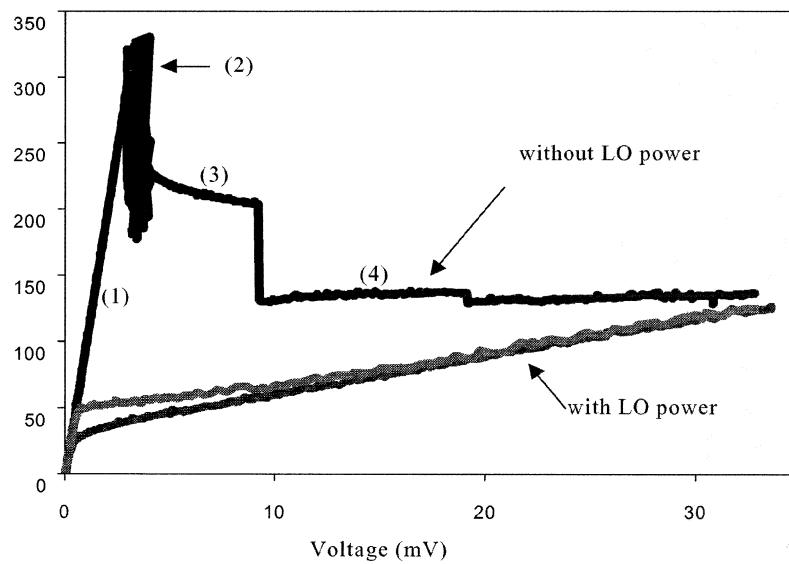


Figure 2 Typical voltage biased I – V characteristics of a HEB device



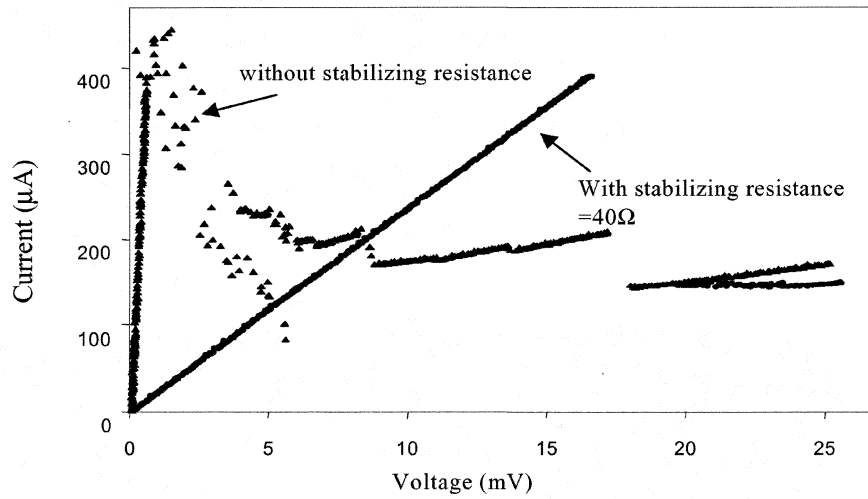


Figure 3: I –V characteristics of a HEB device with stabilizing resistor

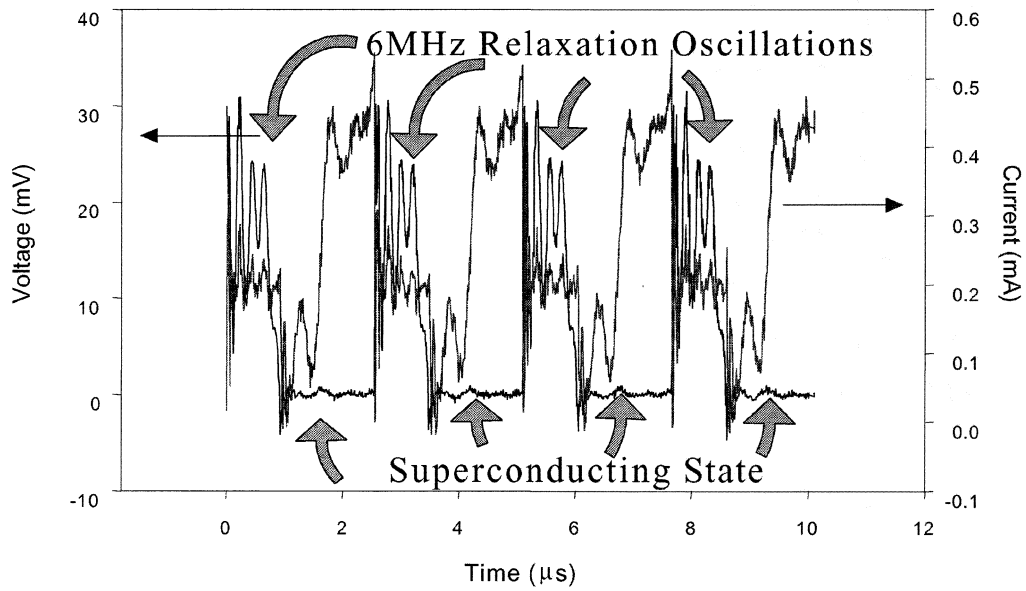


Figure 4: Voltage and Current waveform in the bistability region of the HEB device

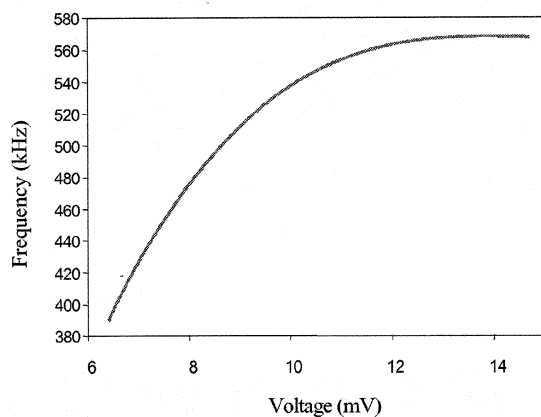


Figure 5: Repetition frequency as a function of the bias voltage

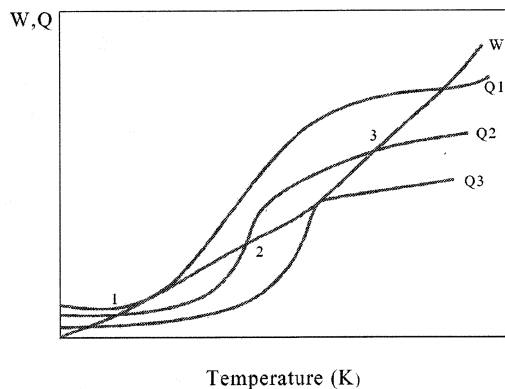


Figure 6: W and Q curves as a function of temperature

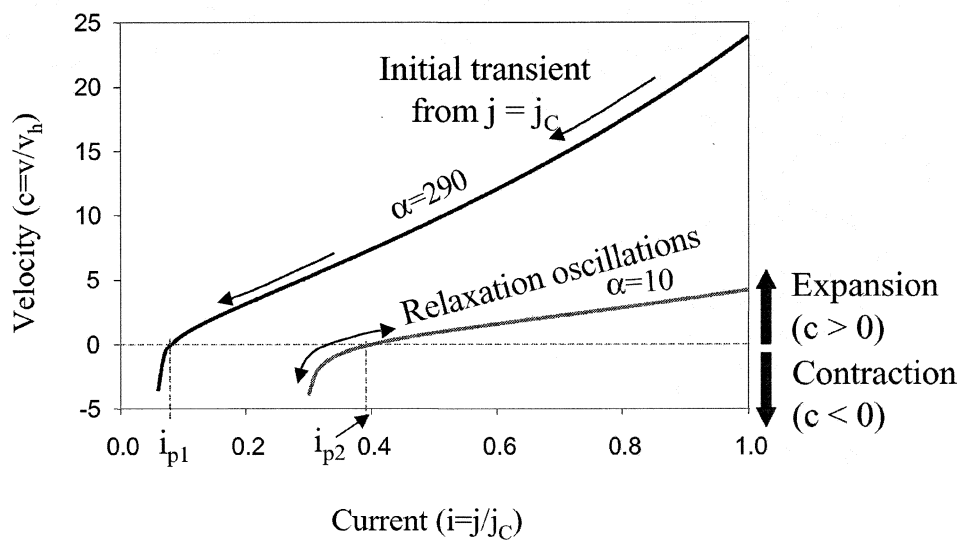


Figure 7:  $c(i) = v/v_h$  as a function of  $i=j/j_c$  for  $\alpha = 290$  and  $\alpha=10$