ANALYSIS OF SUPERCONDUCTING COPLANAR WAVEGUIDES FOR SIS MIXER CIRCUITS

P. KITTARA, G. YASSIN AND S. WITHINGTON
Department of Physics, University of Cambridge,
Madingley Road, Cambridge CB3 OHE, UK

ABSTRACT In this paper we analyse the behaviour of the coplanar waveguide for SIS mixer circuits. In particular we present design curves for the propagation constant and the characteristic impedance of the transmission line, in addition to conduction, radiation and surface waves losses. The analysis is based on a conformal mapping method which rigorously takes into account the effect of metallisation thickness. We will show that the three loss mechanisms can be important, depending on the transmission line geometry and on frequency.

INTRODUCTION

A simple SIS chip requires three circuits: the feed circuit which couples power from the feed to the SIS device, the tuning circuit which tunes out the capacitance of the tunnel junction and the IF circuit which carries the low frequency signal. Conventionally, these circuits have been fabricated in microstrip, using Niobium in the superconducting state or Aluminium in the normal state. At frequencies above the gap however, the losses in both cases become substantial so that the noise temperature of the mixer becomes dominated by the circuit losses. Another problem that the mixer designer faces at high frequencies is that the length of the single stage stub become difficult to realise since its length becomes comparable to its own width and to the dimensions of the device. For example, at 700 GHz, taking a microstrip deposited on SiO of width w= 3μm dielectric thickness of h= 400 nm we find that the requited stub length which is requited to tune out the capacitance of a typical 1 μm^2 junction is about l=3μm. In this paper we shall present our work which addresses those difficulties, by introducing the superconducting coplanar waveguide (SCPW) as an alternative to the superconducting microstrip.

The coplanar waveguide is extensively used in printed circuit technology despite the the fact that at low frequencies and large dimensions it is known to have higher conduction losses than the microstrip. For example, a microstrip which consists of a copper strip of width w= 200μm, deposited on a substrate of dielectric constant $\epsilon = 12.9$ and thickness h= 200μm has a characteristic impedance value of Z_0= 43 Ω and an attenuation constant of $\alpha = 0.025$ db/mm. On the other hand, a coplanar waveguide which uses the same material and substrate and having a gap of s=5 μm and a central conductor width w= 40 μm has a characteristic impedance Z_0= 21 Ω and an attenuation constant $\alpha = 0.3$

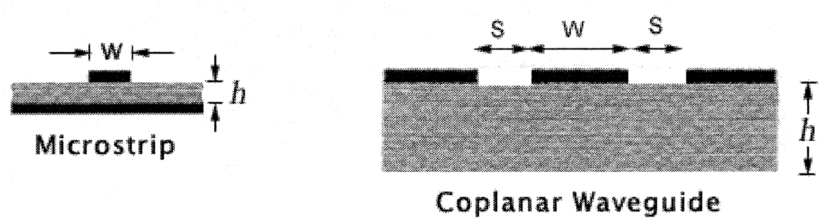

FIGURE I Geometry of the coplanar waveguide and the microstrip

db/mm. The situation however becomes completely different when SIS mixer circuits dimensions are employed. For example, a copper microstrip line at 900 GHz with $\epsilon = 5.8$, h= 0.4μm, w= 2 μm and film thickness $t = 0.4\mu$m gives Z_0= 21 Ω and an attenuation value of α = 33.9 db/mm. In comparison, a copper CPW with the same film thickness and at the same frequency, having the same dielectric constant and h= 60 μm, w=100μm, s=2 μm, yields a characteristic impedance value $Z_0 \approx 37$ Ω and a much lower attenuation constantα = 2.5 db/mm. For mixer circuits therefore, the conduction loss of the microstrip circuit is far larger than the loss of the coplanar waveguide. In addition, It is recognised that CPW have the following advantages over the microstrip:

- It offers a useful range of characteristic impedance values. As we shall see later, impedance values in the range of 50-200 Ω may be obtained for gap and central conductor width dimensions which are easy to fabricate.
- The electrical parameters are insensitive to the substrate thickness. This property is particularly important in SIS mixer circuits
- The central conductor and the ground planes are in the same plane. This means that only a single layer deposition is required, which could simplify the fabrication of the mixer chip a great deal.

Despite the above obvious advantages, the SCPW is not commonly used in mixer circuits. Even at frequencies above the gap, the very lossy microstrip has been preferred. In fact it has been reported that in the few cases where attempts were made to use SCPW , the results were poor as a result of large losses. We attribute this problem to the fact that the design of these circuits was not based on rigorous theoretical procedure which took into account all the loss mechanisms. In this paper we shall describe design equations which take into account the various loss mechanisms in SCPW in particular radiation and surface waves losses. We will show that the contribution of each to the total loss is strongly dependent on the geometry.

METHOD OF CALCULATION

Comparison of various methods

Full wave analysis of CPW is very hard since this requires the calculation of the current density over the conductors for arbitrary film thickness and as a function of frequency. This is particularly true in the case of mixer circuits where the metallisation thickness is comparable to some of the transmission line parameters such as the CPW gap. The need to include metallisation thickness in the basic formulation arises from the fact that the current density near an infinitely thin metal edge diverges to infinity as $r^{\frac{1}{2}}$ where r is the distance from the edge. Consequently the loss over a conductor with infinitely thin edges is unbound. To avoid laborious numerical procedures which add little to the accuracy of the final results we make use of the TEM approximation which requires that the cross section dimensions of the transmission line to be small compared to the wavelength . This allows us to derive analytical expressions for the characteristic impedance and the propagation constant for arbitrary film thickness, using a quasi-static approach. Methods that use this technique proceed usually as follows:

- Use Schwartz-Christoffel conformal mapping to map the CPW with thick metallisation in the z-plane to a CPW geometry with infinitely thin metallisation in the Z_1 plane. As a result of the symmetry, it is sufficient to work with half the cross section and consequently the resulting geometry in the Z_1 plane is a slot line with finite assymetric ground planes.
- The electrical parameters of the slotline in the Z_1 plane can now be found rigorously using standard computational techniques. However, following Heinrich approach (Heinrich, 1993), a quasi-TEM method combined with reasonable approximations is used to calculate the capacitance and inductance of the slotline using analytical formulas. Those expressions extend over several pages, hence are too long to include in this paper. The reader is therefore referred to the cited reference. The accuracy of Heinrich expressions is good as long as we satisfy the quasi-TEM condition:

  $$\nu << \frac{c}{\sqrt{\epsilon_{eff}}}\frac{1}{w+2s}; \qquad h > 2(w+2s) \tag{1}$$

  where c is the speed of light in vacuum. Taking a cross section dimension of 50μm we find that $\nu \ll 3$ THz. It should be added however that when the gap is much smaller than the strip width and is comparable to the film thickness this condition could be relaxed since the fields will be confined to the gap and do not fringe much into the dielectric.

  It is also worthwhile noticing that a main difference between the expressions given in this paper and others is that here,the influence of the thickness is incorporated in the formulation of the problem while in other approximate treatments the effect of the thickness is taken as a correction to the zero-thickness case. We therefore believe that Heinrich expressions are more accurate, in particular for very thick metallisation as it is the case in mixer circuits.
- Whence the inductance and characteristic impedance are known the conduction loss is calculated using Wheeler's incremental method.
- After deriving expressions for the modal parameters, the superconducting values can then easily be calculated by replacing the normal conductivity with the complex conductivity given by Mattis-Bardeen equations.

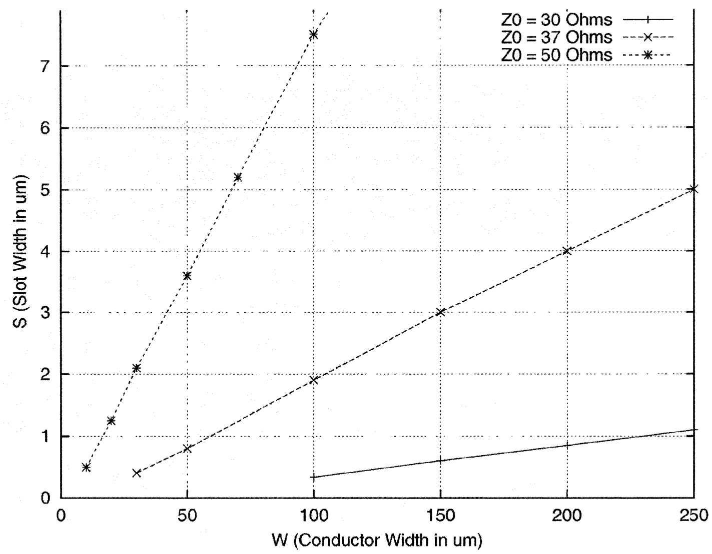

FIGURE II Design curves at 700 GHz for a Nb CPW in the normal state for three characteristic impedance values. The substrate dielectric constant ϵ_r=3.8, t= 400 nm and h= 60 μm

Finally, the electrical parameters computed by Heinrich theory are compared with those given by Whitaker *et al* (Whitaker, 1988), (Gupta, 1996) and the commercial package HP-EEsof. Whitaker *et al* uses approximate expressions based on conformal mapping and corrected by for the film thickness (Gupta, 1996). To make the expressions apply beyond the TEM limit they use a frequency correction factor which includes imperial parameters. We cannot therefore comment on the applicability of this correction for all CPW dimensions. We also found that HP-EEsof which uses similar formalism applies in the TEM interval only.

Calculation of the characteristic impedance

We start our computation by showing a design curve for three characteristic impedance values corresponding to a Niobium CPW (Fig. 2). The chosen film thickness, substrate height and Z_0 values are typical to those found in mixer circuits. We would like to emphasise that superconductivity (unlike in microstrip lines) does not alter those values significantly. This is in contrast to microstrip geometry where the penetration into the strip, substantially modifies the modal parameters of the transmission line (Yassin and Withington, 1995). Another important observation which we can learn form Fig. 2 is that it is difficult to obtain low impedance values for convenient strip and gap dimensions. For example, to obtain a 38 Ω, for a gap s = 2μm we need a strip thickness w=100 μm which is too wide. Lower impedance values can only be obtained for much narrower gaps which increases the fabrication complexity. Values of 50Ω and above on the other hand are easily obtained. This is shown in Fig 3 where we plotted the impedance as a function of the strip width for several gap values.
The dependence of the propagation constant and the characteristic impedance for a CPW in the normal state on frequency are plotted in Fig. 4 and Fig. 5 respectively. It can be seen that our computations agree well with those given by

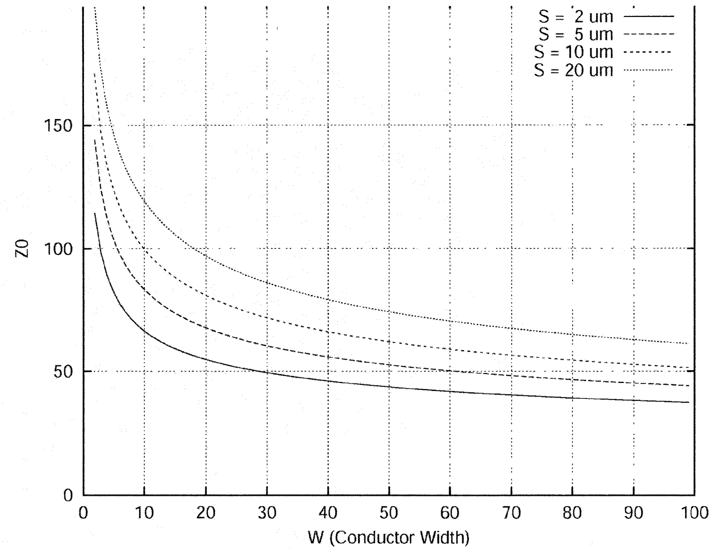

FIGURE III Characteristic impedance values for a CPW as a function of the strip width and gap. ϵ_r=3.8, t= 400 nm and h= 60 μm

Whitaker *et al* for frequencies below 1 THz. Above this frequency, the results start to depart slowly as a result of the frequency correction factor used by Whitaker. The equivalent plots for the superconducting state are very similar to those in Figs. 4,5 , hence are not plotted below.

Calculation of conductor losses

In Fig. 6 we compare the conduction losses of a SCPW when the modal values are computed by the three methods as a function of frequency. We also include the loss for a superconducting microstrip with the same metallisation (Niobium) and dielectric constant, having a width of 2μm and a substrate height of 400 nm (Yassin and Withington, 1995). It can be seen that in the frequency range below 1THz, there is close agreement between Whitaker *et al* and our methods both below and above the superconducting gap. In addition it is also shown that the conduction loss of the microstrip above the gap is substantially higher than that of the CPW. Notice that at frequencies above the gap, the losses of the SCPW become equal to those computed for the normal CPW. This is of course to be expected since at frequencies above the gap, Cooper pair-breaking becomes dominant.

RADIATION AND SURFACE WAVE LOSSES

In addition to the dielectric and ohmic losses, coupling of power to radiation and surface waves in coplanar lines contributes substantially to the total loss. those losses depend strongly on frequency and geometry. For thick substrates, radiation losses are dominant since the CPW radiates efficiently into the dielectric. For very thin substrates, radiation losses are small and surface waves are cutoff. In this case, conduction losses are dominant. As the substrate thickness increases, surface waves start to propagate resulting in steeply increasing loss as

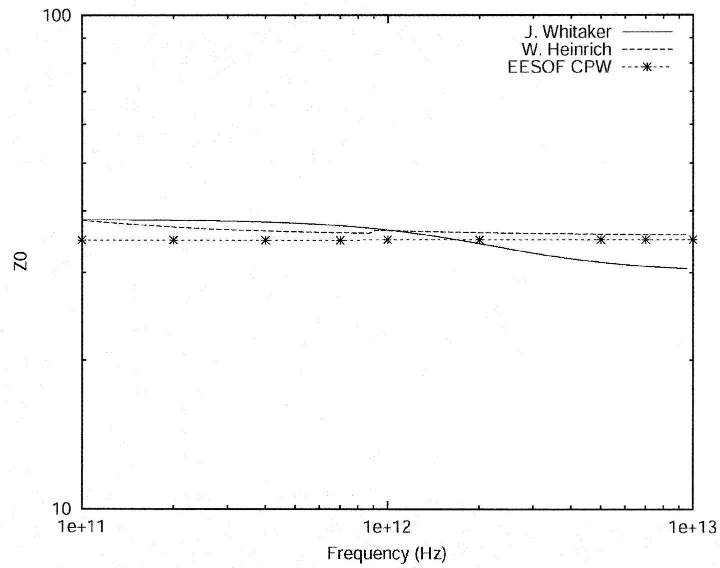

FIGURE IV Characteristic impedance as a function of frequency for a Nb CPW. ϵ_r=3.8, t= 400 nm, w= 100 μm and h= 60 μm

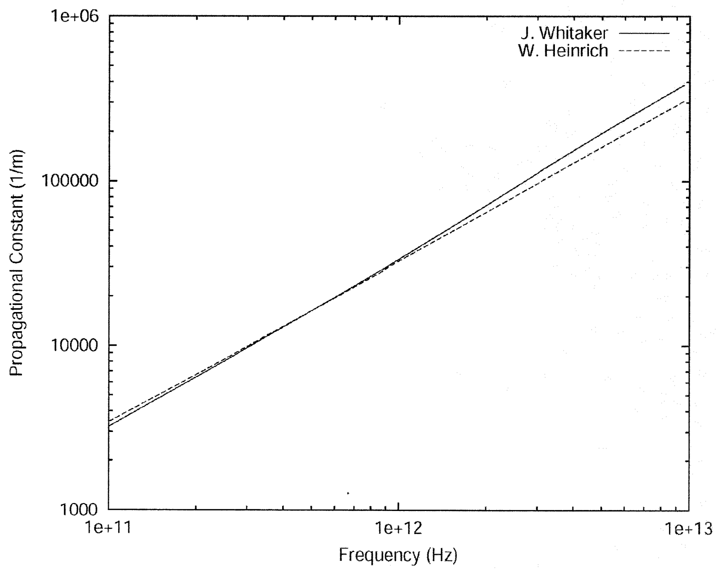

FIGURE V propagation constant as a function of frequency for a Nb CPW. ϵ_r=3.8, t= 400 nm and h= 60 μm, w= 100 μm

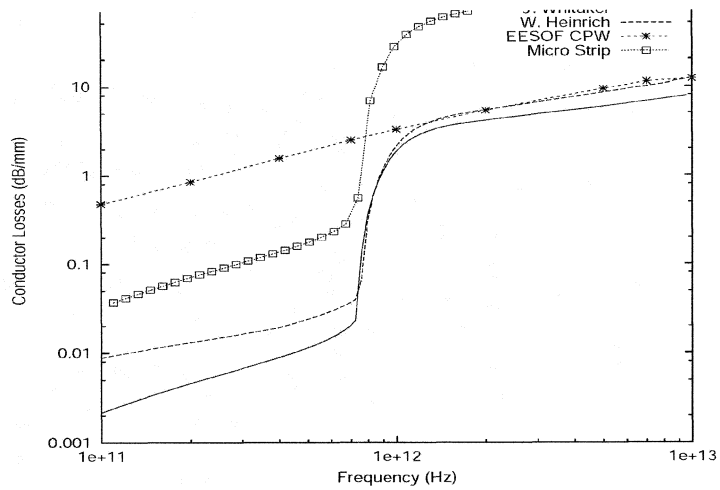

FIGURE VI Conduction losses as a function of frequency for for a superconducting Nb CPW and a microstrip. For the CPW: ϵ_e=3.8, t= 400 nm and h= 60 μm , w= 100 μm. For the microstrip: ϵ_r=3.8, t= 400 nm w=2 μ and h= 400 nm

shown in Fig. 8.

Thick Substrate

The radiation into the dielectric occurs because the phase velocity of the waves propagating along the transmission line is larger than the velocity in the dielectric. The radiation is emitted in a semi-cone of an angle ϕ_{rad} given by

$$\cos(\psi_{rad}) = \frac{k_z}{k_d} \tag{2}$$

where k_z is the CPW propagation constant and k_d is the propagation constant in the dielectric. Using the reciprocal method and quasi-static approximation, it can be shown that the attenuation coefficient is given by (Rutledge, 1983)

$$\alpha_{rad}^{CPW} = \frac{58.7(1-1/\epsilon_r)^2}{K(k)K'(k)\sqrt{1+1/\epsilon_r}} \frac{(w+2s)^2}{\lambda_d^3} \qquad \text{dB/m} \tag{3}$$

where k = s/(w+2s) and K(k) is the elliptic integral.

It can easily be seen that the radiation loss is proportional to the square of the total line width and to the cube of the frequency. At very high frequency, the quasi-static approximation breaks down and full wave analysis becomes necessary (Phatak, 1990).

Thin Substrate

Surface waves losses occur when elementary sources on thin substrates couple to surface waves. Those losses become important when the substrate is too thin for radiation losses but thick enough to make the propagation constant of the surface waves larger than that of the transmission line. When this happens, the

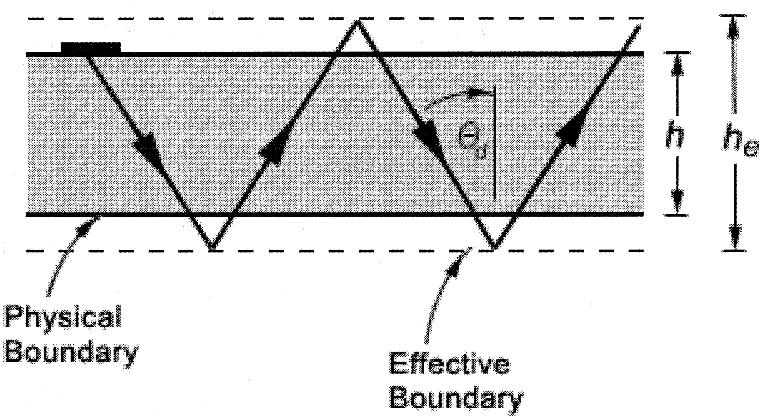

FIGURE VII Ray optics diagram for propagation in a dielectric waveguide.

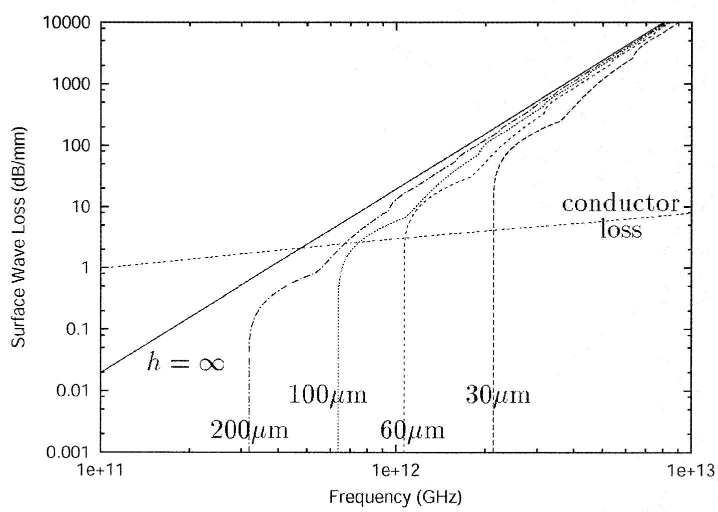

FIGURE VIII Radiation (solid line) and surface wave lossesfor a CPW with $s = 2\mu$m,$w = 100\mu$m and $\epsilon_r = 3.8$. Surface waves losses are plotted for several substrate thicknesses. Condctor losses are computed for Nb in the normal state.

transmission line begins to lose power at a rate which is determined by the angle ψ, given by:

$$\cos(\psi_{SW}) = \frac{k_z}{\beta} \tag{4}$$

where k_z is the propagation constant of the line and β is the surface waves mode propagation constant. Notice that the turn-on frequency is larger than the usual cut-off frequency of the dielectric slab guide.

Using reciprocity and quasi-static approximation for the impedance and field distributions, the attenuation coefficients due to surface waves can be written as (Rutledge, 1983)

$$\alpha_{SW}^{TE} = \frac{149.6}{\sqrt{1+1/\epsilon_r}} \frac{\sin(\psi_{SW}^{TE})\cos^2(\psi_{SW}^{TE})\sin(\theta_d)\cos^2(\theta_d)}{h_e^{TE}KK'}(\frac{w+2s}{\lambda_d})^2 \quad \mathrm{dB/m} \tag{5}$$

$$\alpha_{SW}^{TM} = \frac{149.6}{\sqrt{1+1/\epsilon_r}} \frac{\sin^3(\psi_{SW}^{TM})\sin(\theta_d)}{h_e^{TM}KK'}(\frac{w+2s}{\lambda_d})^2 \quad \mathrm{dB/m} \tag{6}$$

where θ_d is the angle of incidence on the dielectric-air interface as in Fig. 7 and h_e is the effective guide thickness given by

$$h_e = h + 2\delta \tag{7}$$

where h is the actual dielectric thickness and δ is the apparent ray penetration in the air region on the total internal reflection and can be calculated from

$$\delta^{TE} = \frac{1}{\sqrt{(\beta^{TE})^2 - k_0^2}} \tag{8}$$

$$\delta^{TM} = \frac{1}{\sqrt{(\beta^{TM})^2 - k_0^2}} \frac{1}{[(\frac{\beta^{TM}}{k_d})^2 - (\frac{\beta^{TM}}{k_0})^2 - 1]} \tag{9}$$

where $\beta = k_d \sin(\theta_d)$ is the dielectric guided propagation constant.

For the interfering zigzag plane waves to form a mode, the phase lag after reflection from top and bottom and return to the top must be a multiple of 2π :

$$-2k_d h\cos(\theta_d) + 2\phi = 2m\pi, \qquad m = 0, 1, 2... \tag{10}$$

$$\left.\begin{array}{l} \phi^{TE} = 2\tan^{-1}\left[\frac{\sqrt{\sin^2(\theta_d)-1/\epsilon_r}}{\cos(\theta_d)}\right] \\ \phi^{TM} = 2\tan^{-1}\left[\frac{\sqrt{\epsilon_r^2 sin^2(\theta_d)-\epsilon_r}}{\cos(\theta_d)}\right] \end{array}\right\} \tag{11}$$

where ϕ is the phase shift due to total internal reflection at dielectric-air interface (Ramo, 1994).

To calculate the attenuation we first find the angle of incidence θ_d for each mode, by solving the above simultaneous equations. From that the propagation constant and eventually the loss coefficient can be calculated. Clearly the total loss is the superposition of the loss contributions of all propagating surface waves modes. Note that TE_0 and TM_0 substrate modes can exist for very thin substrates.

CONCLUSIONS

A summary of coplanar waveguides losses discussed in this paper are computed in Fig. 8 and may be summarised as follows:

- For very thick substrates, the CPW is dominated by radiation losses in the frequency range relevant to high frequency SIS mixers.
- Surface waves loss is transformed into radiation loss as the substrate becomes very thick (see Fig. 8).
- The turn-on frequency of surface waves is a strong function of the substrate thickness. Whence turn-on occurs, surface waves losses become dominant.
- It is possible to find geometries where radiation and surface wave losses are negligible. For example, consider the CPW described in Fig. 8. Assuming that our operation frequency is 1THz, we choose a substrate thickness which is less than 60 μm to minimise radiation and surface wave losses, hence make our mixer performance limited by conduction losses (which are small for coplanar waveguides)

REFERENCES

Heinrich, W. "Quasi-TEM description of MIMIC coplanar lines including conduction loss effects," *IEEE Trans. Microwave Theory Tech.* MTT–41, no. 1, pp. 45-51, 1993

Whitaker J F et al"Propagation model for ultrafast signals on superconducting dispersive striplines", *IEEE Trans. Microwave Theory Tech.* MTT–36, pp. 277-285, 1988.

Yassin, G. and S. Withington, "Electromagnetic models for superconducting millimeter–wave and submillimeter–wave microstrip transmission line," *J. Phys. D: Appl. Phys.*, vol. 28 pp. 1983–1991, 1995.

Rutledge D B, Neikirk DP and Kasilingham D P, *Infrafed and Millimeter Waves,* vol 10, chapter II. Button K J,Ed. New York:Academic,1983.

Phatak D S, Das N K and Defonzo A P, "Dispersion fharacteristics of optically excited coplanar striplines: fomprehensive full-wave analysis," *IEEE Trans. Microwave Theory Tech.* MTT–38, no. 11, pp. 1719-1730 , 1990.

Ramo S, Whinnery J R and Duzer T V, *Fields and Waves in Communication Electronics* pp. 274-315, 1994.

Gupta K C *et al., Microstrip Lines and Slotlines.* Artech House, 1996.

Frankel M.Y.*et al.,* "Terahertz attenuation and dispersion characteristics of coplanar transmission lines", *IEEE Trans. Microwave Theory Tech.* MTT–39, no. 6, pp. 910-916, 1991.