A hot-spot model for membrane-based HEB mixer

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ABSTRACT

Membrane-based hot electron bolometric mixers (HEB) are of specific interest for applications above 2.7 THz. There the absence of a dielectric close to the antenna reduces losses and enlarges the antenna structure. This allows the usage of large volume devices at higher RF frequencies. All these effects are beneficial to reduce the mixer noise temperature. A reduction in IF bandwidth due to the reduced cooling is by far outweighed by these effects. Here a device model for membrane-based HEB mixers is presented that takes the interaction between electrons and phonons of the film and membrane phonons as well as the phonon diffusion along the membrane into account. The model is based on a numerical solution of two nonlinear coupled heat balance equations on the superconducting strip: One relation is set up for the electron temperature and another for the phonon temperature resulting in I-V curves. The mixer conversion gain and the receiver noise temperature are obtained by applying a small signal model. The model allows comparing the performance of thick substrate-based HEB and membrane-based HEB: membrane-based HEB exhibit a conversion gain lower than in substrate-based HEB for a given LO power. However the maximum conversion gain (obtained when the hot spot is as short as possible to ensure a stable operating point) is comparable. It is clearly shown that, for the same conversion gain the LO power is expected to be reduced by a factor of 20% in the membrane-based case. Moreover, using the membrane-based HEB in a quasi-optical receiver together with a matched back-short offers the possibility to improve the LO and RF coupling efficiency.

INTRODUCTION

The submillimeter band, which may be defined as the wavelength region between 1 mm and 100 μm is of great importance to astronomy, atmospheric study, and more generally molecular spectroscopy. Hot bolometric mixers (HEB) have been accepted as the best devices for those receivers [1-3], when one seeks to detect molecular lines at wavelengths smaller than 300 μm, and when cryogeny is available. Describing and modeling the physics and the behavior of this device is critical to optimize the HEB performances. In contrast with earlier models [4], the "hot-spot model" [5] has been accepted within the last few years as the most accurate and powerful one. In this model, the mixing in a HEB is described by a time-varying normal conducting hot spot governed by a system of one-dimensional heat transport equation. For phonon-cooled bolometers, a coupled heat balance for electrons and phonons must be considered. Two major model assumptions have to be made for the large signal model: Assuming the phonon diffusion to be negligible compared to phonon escape to the substrate, leading to the localized cooling assumption, the phonon heat balance relation is reduced to an analytic equation allowing eliminating the phonon temperature from the electron heat balance. So far, all models simulated the behavior of the phonon-cooled bolometer based on a cold substrate. If we investigate the case where the substrate would be shortened from 250 micron (typical of Si wafer) to 1 micron (typical of the membrane we use [7]), the heat removal capability of the substrate will then be considerably attenuated. On top of that, as the membrane is very thin, it will be taken into account the phonon diffusion. In the usual substrate-based hot spot model, the equations are set up without the phonon diffusion effect. This paper present the comparison of the behavior of a bolometer, mounted on a thick substrate and on a thin membrane. The differences are due to the phonon-diffusion effect in the membrane, along the strip. Moreover, at the transition between the superconductor and the normal resistive metal (hot-spot), the andreev reflexion occurs. This will be expressed in the equation, by adding a factor in the electron diffusion process.

ANDREEV REFLECTION

At the border between the hot spot and the superconducting parts of the HEB bridge, electrons from the hot spot may only cross into the superconductor when there is a "partner" with suitable impulse to for a Cooper pair. This partner is provided by a formation of an electron-hole pair with appropriate impulse resulting in the transition of the initial electron plus the reflection of a hole. Only those electrons are allowed to pass into the superconductor that has an energy larger than the bandgap. This process is called Andreev reflection. Most of the electrons in the normal conductor have an energy lower than the bandgap. Therefore Andreev reflection provides a good thermal insulation of the hot spot. Recent hot spot model takes into account the Andreev reflexion [6]
The bandgap is assumed temperature independent. The andreev reflexion has to be recalculating with
The normal-supraconductor interface and the temperature dependency bandgap shape is summarized in Figure 1:

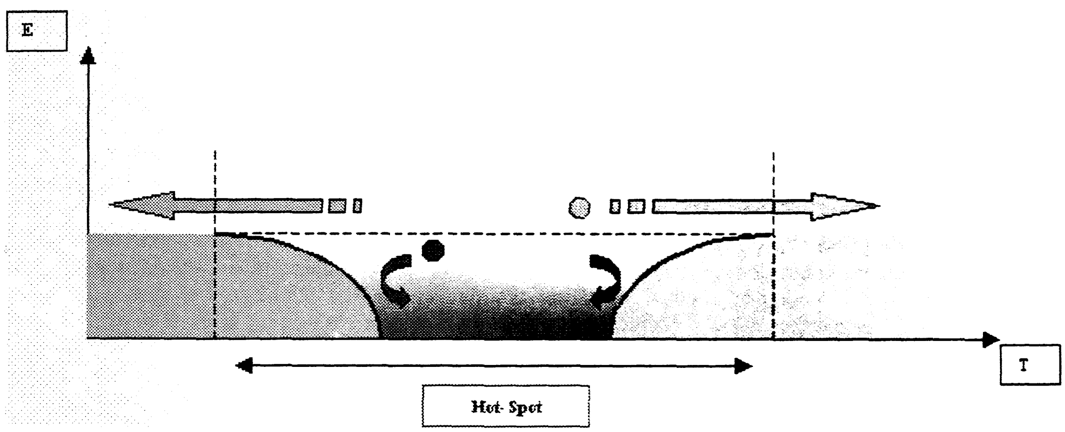


Figure 1: Schematic of a HEB bridge. The whole bridge is heated by RF, the bias heating acts only on the hot spot where superconductivity is suppressed. The electrons are cooled by phonon escape to the substrate and by outdiffusion to the pads. Outdiffusion is reduced by Andreev reflection at the hot spot boundary. We can consider the heat Andreev transmission coefficient as:

$$\alpha = \frac{\int_{\Delta}^{\infty} \sqrt{E}\, f(E) n_E(E) dE}{\int_{0}^{\infty} \sqrt{E}\, f(E) n_E(E) dE} \qquad (1)$$

with :

$$\Delta(T) = \Delta_0 \left[1 - \frac{T}{T_c} \right] \; ; \; n_E(E) = n(0).\frac{E}{\sqrt{E^2 - \Delta^2}} \; ; \; f(E) = \frac{1}{1 + e^{E/kT}}$$

As the bandgap energy is temperature dependant, we then have to integrate equation (1) over the temperature, in order to find the Andreev reflexion:

$$\beta = \frac{\int_{0}^{Tc} \int_{\Delta(T)}^{\Delta(T+\delta T)} \sqrt{E}\, f(E) n_E(E) dE.dT}{\int_{0}^{\infty} \sqrt{E}\, f(E) n_E(E) dE}$$

It is numerically found an Andreev reflexion of 12 %. This Term will be taken into account for the electron diffusion in the film, in the equation (8), by a factor 0.12.

HOT-SPOT MODEL

A hot electron bolometer is a submicronic, ultra-thin (2-3 nm) superconducting micro-bridge, in which the resistive state can be modulated by photon irradiation. The total radiation power, which causes resistance change, can be written as:

$$P(t) = P_{LO} + 2\sqrt{P_{LO}P_S}\cos(\omega_{IF}t)$$

The resistivity depends on the electron temperature. In traditional resistive phonon-cooled bolometers, incident radiation is absorbed by electrons which strongly interact with the lattice atoms. The absorbed energy is therefore quickly transformed into lattice vibrations (phonons). Then, the whole bolometer medium (film and substrate) is being heated up. As illustrated in Fig. 2, there are basically two ways for the heat to be removed from electrons : electron diffusion into the normal-metal electrodes via the film layer ('' diffusion-cooling '')[3], or scattering via phonons in the substrate layer ('' phonon-cooling '') [1-2].

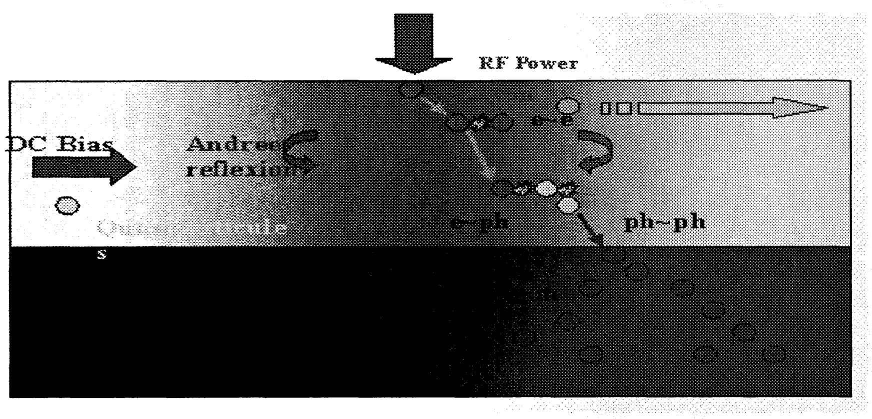


Figure 2: Hot Electron Bolometer is a radiation sensor. Energy absorbed is distributed in the electron subsystems. Heated electrons exchange energy via electron diffusion or via phonon scattering.

The energy removal mechanism can be described in an energy exchange system between electrons of the film, electrons and phonons in the film and between film and substrate phonons. In the substrate-based HEB mixer case, the substrate acts as a heat reservoir. Fig. 3 explains the energy transfer mechanism between the subsystems. The RF power heats up electrons in the film. Those electrons can diffuse in the film, and interact with phonons in the films. The phonons in the film can then interact with phonons of the substrate and remove heat. By describing the energy exchange between the 3 subsystems « electrons », « film phonons » and « substrate phonons», we can derive the heat-balance equation for a bolometer (7).

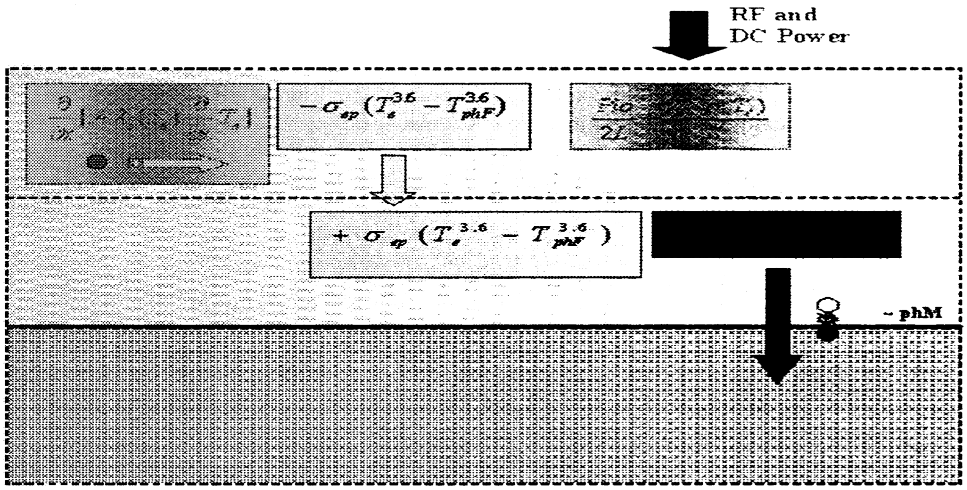


Figure 3: The power exchange subsystems.

λ_e and λ_p are called electron and phonon thermal conductance and $_{ep}$ is called electron-phonon coupling efficiency.

$$\frac{\partial}{\partial x}\left[\lambda_e(T_e)\frac{\partial}{\partial x}T_e\right]-\sigma_e\left(T_e^{3.6}-T_p^{3.6}\right)+\frac{P_{LO}}{2L}+\frac{I_0^2\rho(T_e)}{S}=0 \qquad (7)$$

$$\sigma_e\left(T_e^{3.6}-T_p^{3.6}\right)=\sigma_p\left(T_p^4-T_{substrate}^4\right)$$

Solving these equations for electron temperature Te, assuming a certain electron temperature dependence of the resistivity, gives the I-V curves, mixer gain, and noise temperature curves for substrate-based HEB mixers.

HOT-SPOT MODEL ON MEMBRANE

The HEB is built on a substrate with a thin membrane on it (this will allow us to investigate new quasi-optical injection techniques and designs, for instance to construct compact HEB heterodyne 2D arrays at higher frequencies [8]). The substrate is then removed and only the membrane remains, as seen on Fig.4:

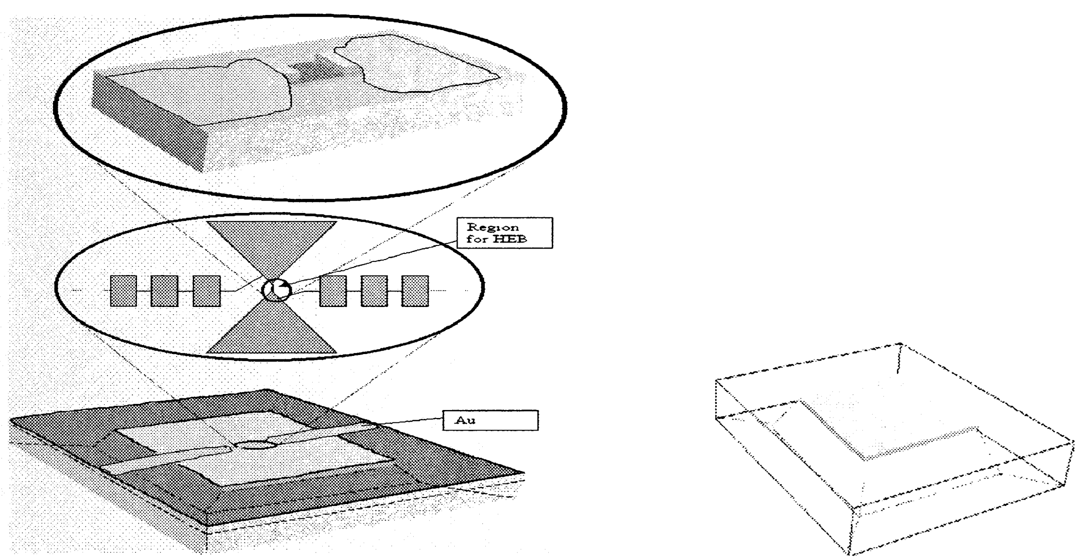
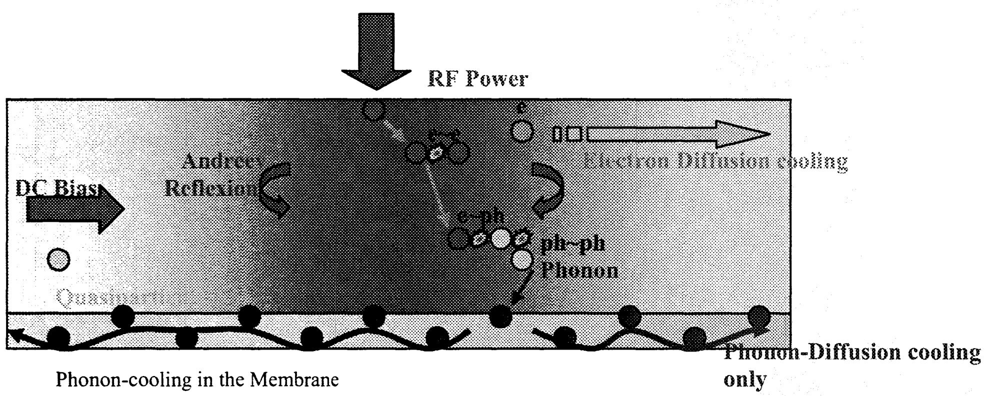
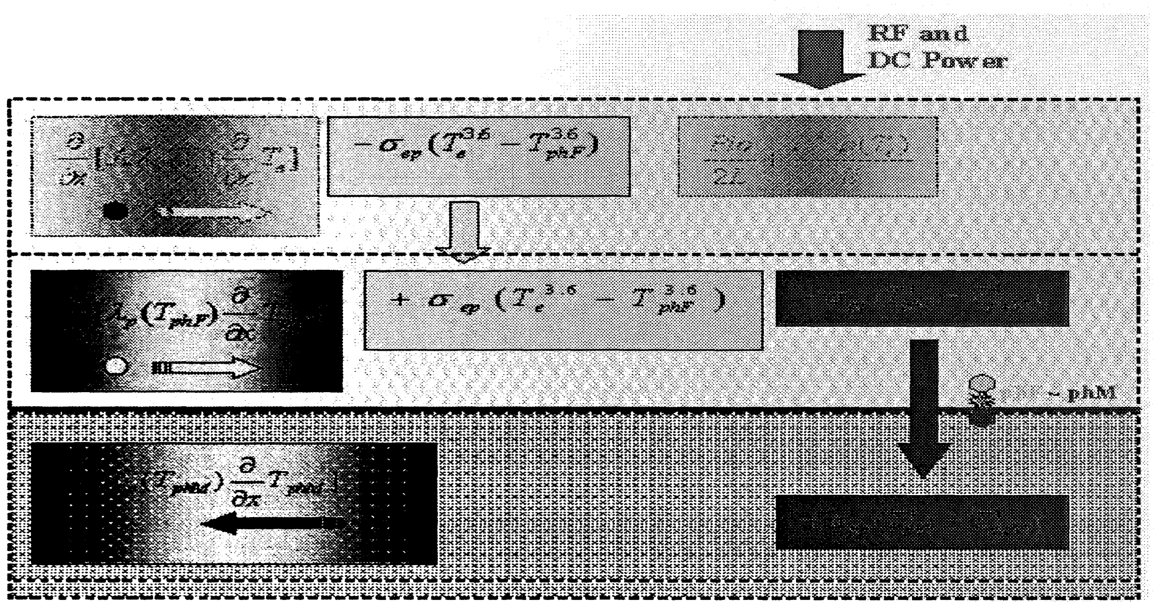


Figure 4 : membrane-based HEB configuration

In the membrane-based HEB device, the substrate has been considerably thinned from 250 micrometers to 1 micrometer. The result can be intuitively seen immediately: the heat reservoir provided by a thick substrate is suppressed and the heat will be mainly removed in the membrane by phonon diffusion along the membrane. The phonons temperature in the membrane will then increase and heat up back the phonons in the film. This effect will increase the electron temperature and then the resistance and the mixer properties.



Equation 7 will then be considerably changed, since it will be taken into account the disappearance of the energy removal by the substrate reservoir and the effect of the phonon diffusion along the membrane, which leads to the subsystems and to equation 8:



$$\begin{cases} \dfrac{\partial}{\partial x}\left[A.\lambda_e(T_e)\dfrac{\partial}{\partial x}T_e\right] - \sigma_{ep}(T_e^{3.6} - T_{phF}^{3.6}) + \dfrac{P_{LO}}{2L} + \dfrac{I_0^2\rho(T_e)}{S} = 0 \\ \dfrac{\partial}{\partial x}\left[\lambda_{phF}(T_{phF})\dfrac{\partial}{\partial x}T_{phF}\right] + \sigma_{ep}(T_e^{3.6} - T_{phF}^{3.6}) - \sigma_{pp}(T_{phF}^4 - T_{phM}^4) = 0 \\ \dfrac{\partial}{\partial x}\left[\lambda_{phM}(T_{phM})\dfrac{\partial}{\partial x}T_{phM}\right] + \sigma_{pp}(T_{phF}^4 - T_{phM}^4) = 0 \end{cases} \quad (8)$$

Assuming λ proportional to d/L, d being the thickness of the material in which the phonon is diffusing, and L the nanobridge length, we obtain:

$$\frac{\lambda_{phF}}{\lambda_{phM}} = \frac{F}{M}$$

F being the film thickness and M the membrane thickness.
Taking $\lambda_{ph} = \lambda_e/10$, λ_{phF} and λ_{phM} x independent, and assuming T_{phM} and T_{phF} of approximately the same shape and magnitudes:

$$\lambda_{eff} = \frac{\lambda_e}{10}(1 + M/F)$$

Then equations (8) become:

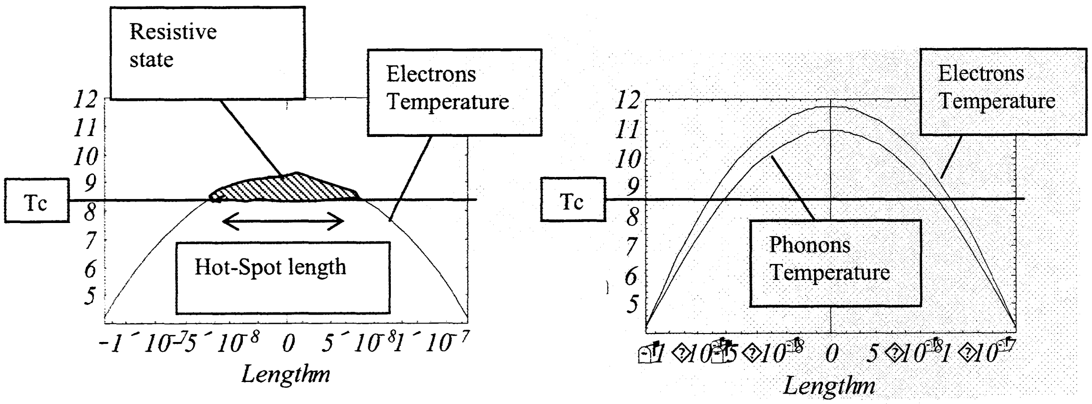
$$\begin{cases} \dfrac{\partial}{\partial x}\left[\lambda_e(T_e)\dfrac{\partial}{\partial x}T_e\right] - \sigma_{ep}(T_e^{3.6} - T_{phF}^{3.6}) + \dfrac{P_{LO}}{2L} + \dfrac{I_0^2\rho(T_e)}{S} = 0 \\ \dfrac{\partial}{\partial x}\left[\lambda_{p,eff}(T_{phM})\dfrac{\partial}{\partial x}T_{phM}\right] - \sigma_{ep}(T_e^{3.6} - T_{phM}^{3.6}) = 0 \end{cases} \quad (9)$$

In order to compare the results with the same model and to see the effect of the phonons diffusion along the membrane, we will see the substrate as a very thick membrane of 300 μm. The thickness of the membrane will be taken as 1 μm, and other parameters as follow:

Parameter	$\lambda_e(T_e)$	σ_{ep}	Rn	δT
Value	$6.10^{-18}.T_e^3$	5.6.10-4	50	0.1
Dimension	Wm/K	$W/(mK^{3.6})$	Ω	K

Membrane effect

Solving the two nonlinear coupled heat balance equations (9) derived for the membrane-based HEB case, we obtain on Fig. 5 the film electron temperature profile (a) and the film phonon temperature profile (b). The part of the superconducting film, which becomes normal due to heated electrons, is called the " hot spot ".

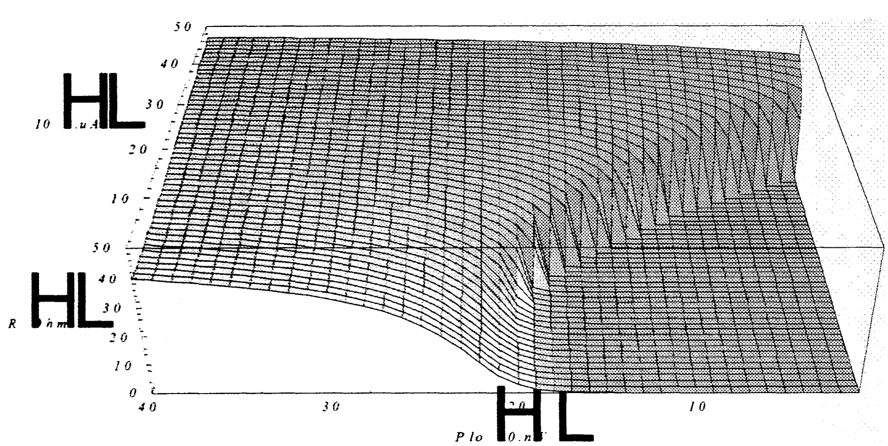


a **Figure 5** b

It is clearly shown in fig.5 that the minimization of the substrate, which usually acts as an energy acceptance reservoir, now increase the electron temperature profile of the film. In clear, the phonon will heat up the electron in the film. Indeed, the superconducting nanobridge will need less microwave power irradiation to be driven into the resistive state. Then, the sensitivity of the membrane-based HEB mixer should be higher.

Hot-spot length comparison

As we've just seen, less irradiation power is needed in case of a membrane. We plotted on Fig.6 the Hot-spot length versus P_{lo} and P_{dc}



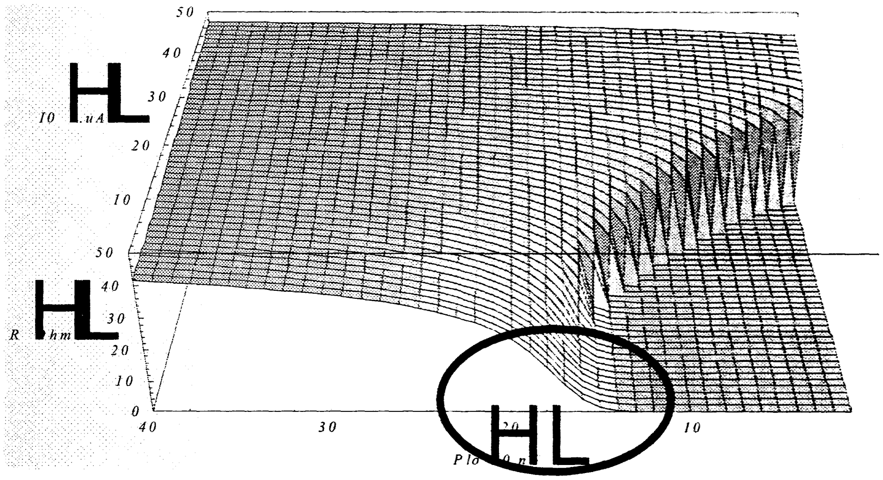
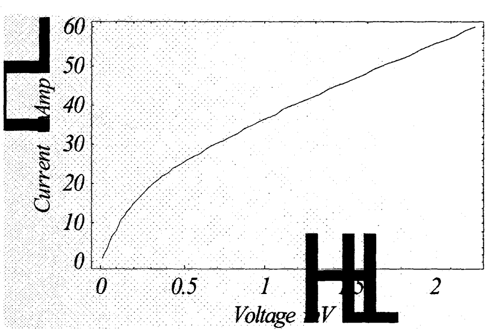


Figure 6: comparison of Hot-spot length, with respect to substrate or membrane cases.

Here is shown that the hot-spot start to be formed at less P_{lo} and P_{dc}, as we expected. While the HEB built on substrate show a hot-spot start at around 220 nW, the membrane-based HEB show it at 170 nW.

I-V curves comparison

The results presented show the model works well with the addition of the phonons diffusion term.
I-V curves are obtained in both the thick substrate and membrane (see Fig. 7 and 8). As expected, since the hot-spot will be formed "earlier" with the membrane, less LO power (Plo) will be needed at fixed DC power (Pdc), in the membrane-based HEB case, to produce the same pumped I-V curve.



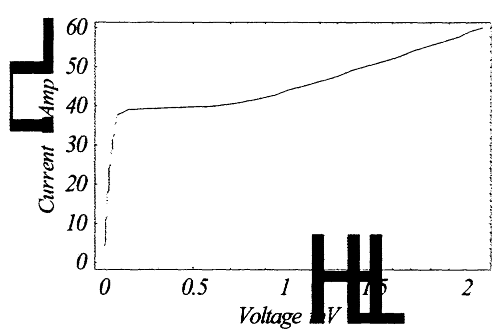


Figure 7: a) IV, 220 nW P_{lo}, thick substrate b) I-V, 190 nW P_{lo}, thick substrate

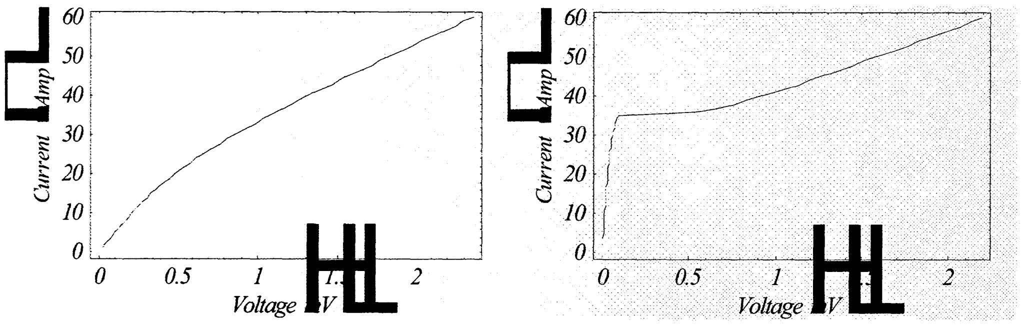


Figure 8: a) IV, 190 nW P_{lo}, membrane b) I-V, 150 nW P_{lo}, membrane

Then, theoretically, less LO power will be needed to pump the I-V curve for the HEB on membrane and reach the optimum curve.

Gain curves comparison

The Gain(V) curves depend as well on the Hot-spot formation. Gain curves have been plotted in Fig.9, for both the thick substrate and the membrane cases.

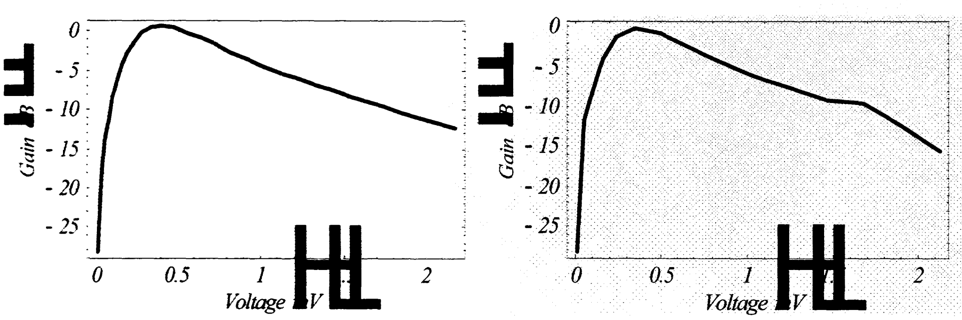


Figure 9: a) substrate, 220 nW Plo b) membrane, 173 nW Plo

It is shown again that similar curves are obtained for different LO powers, due to the membrane effect. Indeed the hot spots are approximately identical but are formed at different power irradiations, hence giving identical results for different LO powers.

Simulation of the behavior of the gain curves while increasing the irradiation power is described in Fig. 10.

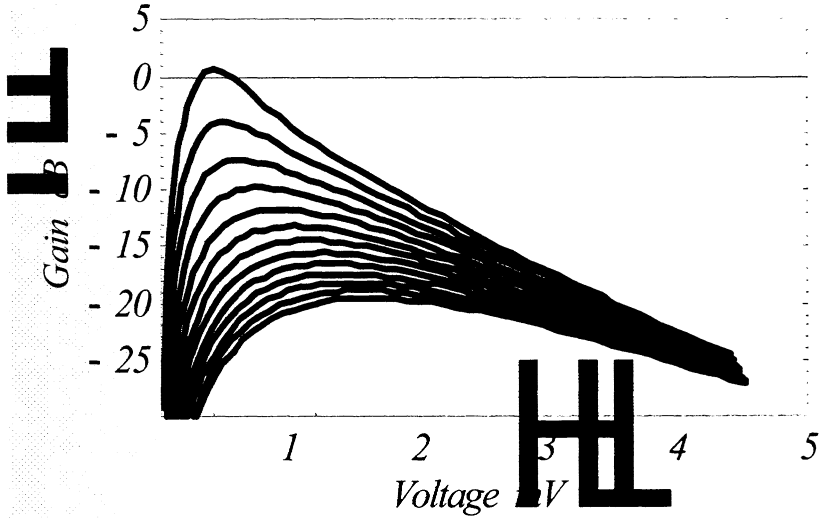


Figure 10: Gain curves for increased LO power

As we can see, the maximum of the gain curve decrease as we increase the LO power. We can see this effect on Fig. 11 for the membrane on the thick substrate and then are able to compare the too curves.

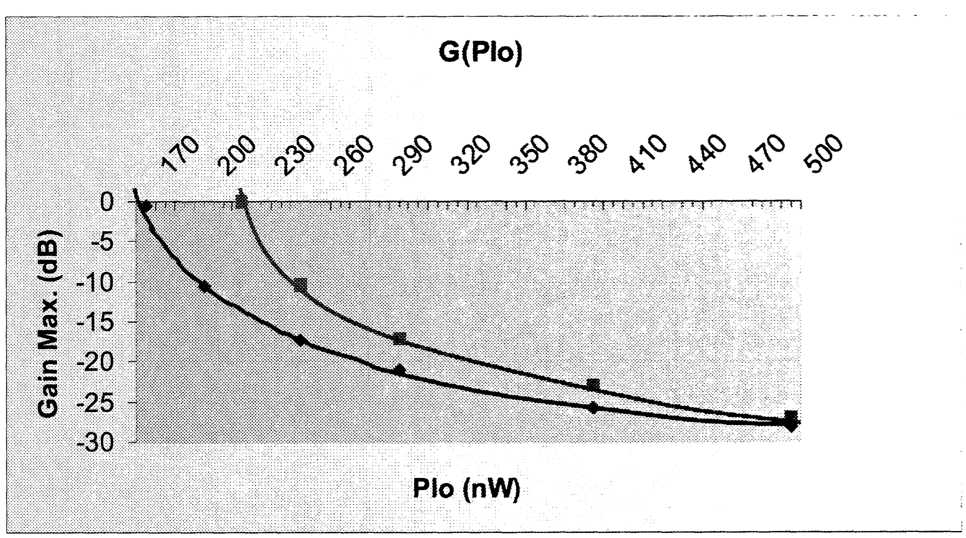


Figure 11: maximum gain Vs Plo for membrane, maximum gain Vs Plo for substrate

The two curves present approximately the same behavior, again due to the fact that all those curves are mainly hot-spot length dependant. Nevertheless, they converge for high Plo.

For a fixed gain, less power will be needed in the membrane case. It means that the substrate cools two much the film, below 220 nW of Plo, to see a hot-spot to form. But, as the phonons in the membrane, due to the diffusion along the membrane, heat up the electrons in the films, the hot-spot will be able to form at 170 nW, showing as well a gain at this irradiation power.

From the curves of fig. 11, we can calculate the difference of Plo needed to obtain the same maximum gain, with respect to different membrane thichnesses. (we find for instance a difference of Plo of 4% between a membrane of 1 μm and a substrate of 250 μm)
Further calculation are depicted in Fig. 12.

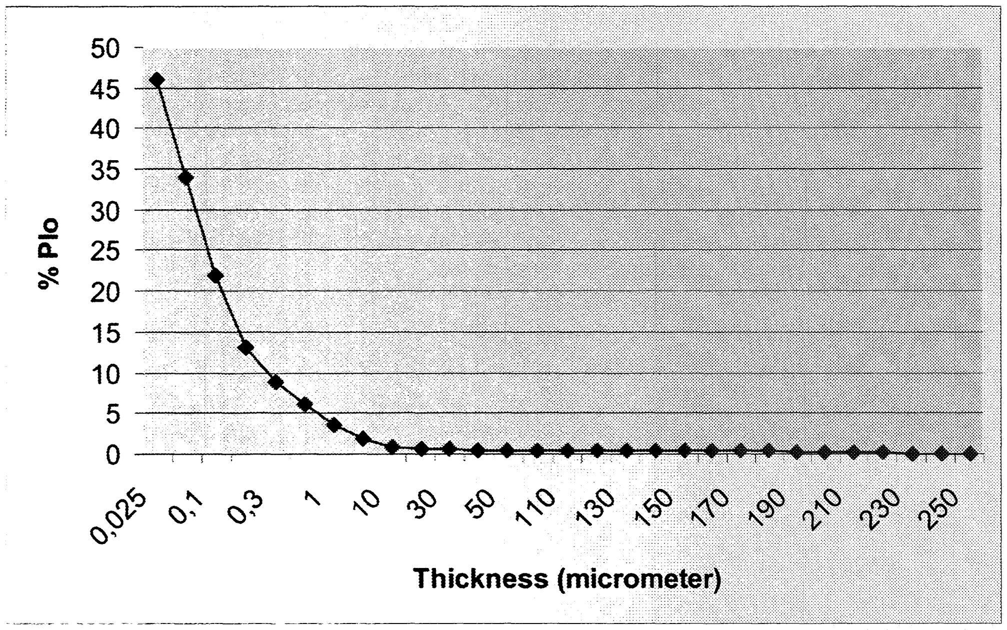
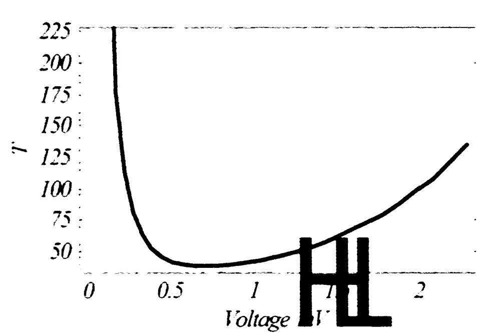


Figure 12: %Plo gained between substrate and membrane

A percentage of 20 % could be achieved with a membrane thickness of 0.25 μm, which seems to be processable. To summarize, irradiation power can be gained by reducing the membrane thickness below 1 μm. As the cooling mechanism will be less efficient, the IF bandwidth should be affected (since the time taken by the substrate to remove the heat from the film will be shorter). Above this limit, a phonon-cooled HEB should work as well as on a normal substrate.

Noise Temperature curves comparison

We want here to check the model behavior by simulating the noise temperature curves for both cases. We expect to find same curve for different Plo. Indeed, that result is similar that finding the noise temperature higher in the membrane case for a fixed Plo .
Those assumptions are verified on Fig.13.



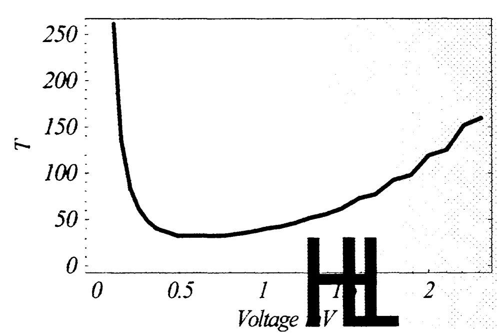


Figure 13: a) 250 nW, thick substrate b) Membrane, 195 nW

CONCLUSION

On the basis of our simulations, we can foresee a normal behavior for a phonon-cooled HEB based on a thin membrane, provided that the membrane thickness is not smaller than 1 µm. For those thicknesses, even the IF bandwidth shouldn't been affected, since the membrane still removes the heat as fast as would a thick substrate. Below 1 µm, however, the IF bandwidth might be affected. Nevertheless, mixer sensitivity will then improve and less LO irradiation power will be needed to reach the same gain and noise temperature, compared to the thick substrate case. This is a worthy consideration since LO power generation remains a technological challenge at the very high (above 1 THz) frequencies where HEB mixers are to be used increasingly.
Moreover, the absence of dielectric close to the antenna permits to avoid the use of a lens and to investigate new quasi-optical injection techniques and designs, such as planar diffractive optics or focusing mirrors, for instance to construct compact HEB heterodyne 2D arrays at higher frequencies. Smaller optics loss is also beneficial with respect to improving the overall HEB receiver noise temperature.

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