SUPERCONDUCTING MICROSTRIP LINE MODELS AT MILLIMETER AND SUB-MILLIMETER WAVES AND THEIR COMPARISON

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Abstract

Performance of superconductor – insulator - superconductor (SIS) tunnel junction mixers and their instantaneous input RF bandwidth are mostly depending on the integrated tuning circuit used to resonate out SIS junction capacitance; Nb-based SIS mixers operate in the frequency range of about 80 - 1000 GHz and typically use microstrip-based integrated tuning circuitries. One of the major challenges in designing the tuning circuitry for SIS mixers is accuracy of models for superconducting microstrip line (SML). Modeling gives the only tool to solve the problem of designing SML-based circuits because for such high frequencies no direct measurements of a superconducting transmission line can be made with *required* high accuracy. However, creating an accurate model for such a superconducting transmission line is a challenge by itself. In the SML, produced usually by thin-film technology, the magnetic field penetration depth is comparable with the thicknesses of the dielectric and superconductors comprising the line. As a result the electromagnetic wave is propagated not only in the dielectric media but also inside the superconducting strip and ground electrodes constituting the SML. This creates dramatic changes in the transmission line behavior that should be carefully accounted by including the superconducting material properties into the modeling. Nb superconductor, as the most commonly exploited material, was used in this study for modeling of the superconducting microstrip though the same approach would work for any different BCS superconducting material. The purpose of this paper is to introduce a new model for SML and compare it with previously suggested models and results of SML numerical simulation.

Introduction

Superconductor – insulator - superconductor (SIS) tunnel junction mixer is a dominating technology for MM and SubMM super heterodyne receivers for radio astronomy [1-3]. Presence of large intrinsic capacitance makes use of SIS-based mixers at these extremely high frequencies (80-1000 GHz) problematic; employing of an integrated tuning circuitry to resonate out SIS junction capacitance is the most convenient way to achieve ultimate performance and wide operational frequency band. Typical approach for implementing such a tuning circuitry is to integrate it on the substrate and fabricate it in the same processing steps as the SIS junction itself. The resulting tuning structures are typically a microstrip-based circuitry; the microstrip is a natural choice due to its complete compatibility with the SIS fabrication process, low RF loss and flexibility. As a result of this approach the SIS mixer tuning circuitry is being fabricated using thin-film technology and superconductors for the microstrip conductors. Accuracy of modelling for a superconducting microstrip line (SML) introduces one of the major challenges in design of tuning circuitry for the SIS mixers. The purpose of this article is an introduction of a new model for SML and its comparison with previously suggested models [4 - 8] and numerically simulated SML using 3D electromagnetic simulation package.

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of interest is 80-1000 GHz; experimental direct measurements of a superconducting transmission line at this frequency range are extremely challenging technically (the object of measurement is in a cryostat at 4 K ambient temperature) and cannot provide necessary precision required for accurate design. In the SML, produced by thin-film technology, magnetic filed penetration depth is comparable with the thicknesses of the dielectric and the conductors of the line and this produces radical changes in the transmission line performance. The energy stored in the layers of the superconducting strip and ground electrodes, where H-filed penetrates in, becomes comparable with the energy of the electromagnetic wave propagating in the dielectric and affects performance of the SML. Therefore, besides the geometry, the modelling of the SML should involve accurate simulation of the superconducting material properties and its influence on the SML characteristic impedance and the propagation constant.

Superconducting Microstrip Transmission Line

SML has a conventional microstrip transmission line geometry (depicted in Fig. 1) with both strip and ground conductors made of a superconductor material. RF and DC current conductivity in the superconducting materials is provided by paired electrons, Cooper pairs, and single electrons, quasiparticles, produced by thermal excitations or by breaking the Cooper pairs caused by the propagating RF signal with its corresponding energy of quanta. In this article we will consider the line strip and ground planes made of low- T_c BCS¹ superconducting materials. Nb superconductor is used in the model calculations through the whole paper as a material the most widely used for superconducting thin-film technology.



Figure 1. Schematic view of the superconducting microstrip line. The strip conductor thickness, t_s , ground conductor thickness t_g and the dielectric thickness h considered to be comparable with the static penetration depth of magnetic field, λ_o , while the widths are as following $W_{ground} >> W_{dielectric} >> W$.

The existing SML models [4 - 8], including the one being suggested in this paper, provide different ways for calculating the characteristic impedance and propagation constant for superconducting microstrip line; close formula equations based on conformal mapping or approach based on transmission line lump element circuit and/or superconductor surface impedance are used in the SML modelling. We are interested in the microstrip lines where the dielectric thickness is *comparable* with magnetic field penetration depth, the London penetration depth, λ_0 , as well as with the thicknesses of the strip and ground superconductors forming the line. These particular relations between the strip, ground and dielectric thicknesses reflect typical geometry of the superconducting microstrip line produced by a thin-film technology used

¹ BCS Theory: J. Bardin, L.N. Cooper, J.R. Schrieffer, "Theory of Superconductivity", Phys. Rev. 1957

for, e.g., SIS mixer fabrication [9, 10]. In this article we compare the accuracy of different models for calculations of the line characteristic impedance, propagation constant and its dispersion. Additionally we use a numerical simulation with High Frequency Structure Simulator, HFSS [11], representing the superconducting material conducting properties via its surface impedance as proposed in [12].

RF Conductivity of a Superconductor

In contrast with conventional conductors, the DC and RF currents in superconducting materials are carried mainly by dual-electron carriers, Cooper pairs, without any conducting loss (supercurrent); a lossy part of the current is carried by quasiparticles (single electrons) generated by thermal excitations and interaction of electromagnetic field and the Cooper pairs. The presence of energy gap in the electron density of states, $\Delta = \Delta(T)$ ($e\Delta \approx 1.5$ mV for Nb material), limits the ability of the Cooper pairs in superconductors to carry the RF current at frequencies closer to the gap frequency, $F_g = 2e\Delta/h$, where e is the electron charge, h is the Plank's constant and F_g is the gap frequency; above F_g the conducting properties of the superconductor become close to that of a conventional conductor.

The magnetic component of the field penetrates into superconductor within a characteristic depth defined as the static London penetration depth, λ_o , for DC current. Actual magnetic field penetration depth, λ , has noticeable frequency dependence for frequencies even well below the gap frequency and, as the frequency approaches the gap frequency of the superconducting material, λ becomes highly frequency dependent [13 - 15]. The microstrip line with superconducting electrodes becomes material-dispersive, and above the gap frequency the line has increased losses [16, 17] due to breaking of Cooper pairs and an increased appearance of conducting loss in the strip and ground electrodes.

The Mattis – Bardeen theory [18] of the skin anomalous effect describes the behaviour of a superconductor vs. frequency in terms of complex conductivity σ :

$$\sigma = \sigma_1 - j \cdot \sigma_2 \tag{1},$$

where the real and the imaginary components, σ_1 and σ_2 , translate directly into the normalelectron and Cooper-pair currents in a superconductor. The applicability of the Mattis - Bardin theory to specific superconducting materials is discussed in [16, 17]. In integral form the equations for σ_1 and σ_2 , are as the follows [16]:

$$\frac{\sigma_{1}}{\sigma_{n}} = \frac{2}{\hbar\omega} \int_{\Delta}^{\infty} \left[f(\varepsilon) - f(\varepsilon + \hbar\omega) \right] \times \frac{\varepsilon^{2} + \Delta^{2} + \hbar\omega\varepsilon}{\sqrt{(\varepsilon^{2} - \Delta^{2})} \cdot \left[(\varepsilon + \hbar\omega)^{2} - \Delta^{2} \right]^{1/2}} d\varepsilon + \dots$$

$$\dots + \frac{1}{\hbar\omega} \int_{\Delta}^{\hbar\omega - \Delta} \left[1 - 2f(\hbar\omega - \varepsilon) \right] \times \frac{\hbar\omega\varepsilon - \Delta^{2} - \varepsilon^{2}}{\sqrt{(\varepsilon^{2} - \Delta^{2})} \cdot \left[(\hbar\omega - \varepsilon)^{2} - \Delta^{2} \right]^{1/2}} d\varepsilon$$
(2),

and

$$\frac{\sigma_2}{\sigma_n} = \frac{1}{\hbar\omega} \int_{\Delta-\hbar\omega,-\Delta}^{\Delta} \left[1 - 2f(\varepsilon + \hbar\omega) \right] \times \frac{\varepsilon^2 + \Delta^2 + \hbar\omega\varepsilon}{\sqrt{(\varepsilon^2 - \Delta^2) \cdot \left[(\varepsilon + \hbar\omega)^2 - \Delta^2 \right]^{1/2}}} d\varepsilon$$
(3),

where T is the temperature [K], σ_n is the conductivity of a superconductor just above the critical temperature T_c , $\Delta = \Delta(T)$ is the energy gap parameter [eV], $f(\varepsilon) = 1/(1 + exp(\varepsilon/kT))$ is Fermi

function, $\omega = 2\pi f$ is the angular frequency, k is Boltsman's constant and h is the reduced Plank's constant. The first integral of σ_1 represents conduction of thermally excited normal electrons, while the second integral of σ_1 introduces generation of quasiparticles by incoming radiation. The lower limit on the integral for σ_2 becomes $-\Delta$ when the frequency exceeds the gap frequency.

To complete the description of a superconductor we need the relation between the physical parameters of a superconducting material [16]:

$$\lambda_o = \sqrt{\frac{h}{\pi \mu_o \sigma_n \Delta}} \tag{4}$$

where μ_0 is the permeability of vacuum. The specific surface impedance Z_s per square (unit area) of the superconducting film with thickness d is expressed as follows [16]:

$$Z_s(\omega) = (j\omega\mu_o/\sigma)^{1/2} \coth((j\omega\mu_o\sigma)^{1/2}d) = R + jX$$
(5).

For magnetic and electric field penetration depths, λ and δ_r respectively, we then can write the follow expressions [17]:

$$\lambda = \frac{X}{\omega \mu_o} \qquad (6), \ \delta_r = \frac{R}{\omega \mu_o} \qquad (7).$$

The equations (1-6) describe the behaviour of the magnetic field penetration depth vs. frequency and the equations (1-5, 7) give the skin depth vs. frequency dependence.



Figure 2. σ_1 (solid line) and σ_2 (dot line) components of Nb superconducting film, calculated based on Mattis-Bardin theory. In the modelling the Nb material parameters were used as follows: $\sigma_n = 1.739 \text{ x } 10^{-7} [\Omega^{-1} \text{m}^{-1}]$, $T_c = 8.1$ [K], $\lambda_o = 8.5 \text{x} 10^{-8}$ [m]; the calculation was made for 4 K physical temperature.



Figure 3. δ (solid line) and λ (dot line) penetration depth of E and H components of the electromagnetic field *correspondingly* for Nb superconducting film, calculated based on Mattis-Bardin theory (the values are normalized by London penetration depth, λ_0). In the modelling the Nb material parameters were used as above; the calculation was made for 4.2 K physical temperature. Already at 200 GHz, less than 30% of the gap frequency, the magnetic field penetration depth, λ , is about 5% larger than the static London penetration depth, reaching maximum of about 1.47 λ_0 .

References [13, 19] present approximation for the frequency dependence of the magnetic penetration depth in a close-form equation. Concluding, the dispersion in a superconducting microstrip transmission line has two different contributions, i.e., the modal dispersion as in any conventional microstrip line *and* dispersion due to the frequency dependence of the superconducting material properties.

Microstrip Superconducting Line Models with Uniform EM Field

Swihart [20] analyzed the SML for the case of uniform electromagnetic field across the dielectric, *i.e.*, with $W_{ground} \gg W \gg h$ (see Fig. 1) and found solution for the Maxwell equations in that case, with the line characteristic impedance and the propagation constant expressed as the follows^{*}:

$$Z_{o} = \frac{120\pi}{\sqrt{\varepsilon_{r}}} \cdot \frac{h}{W} \cdot \sqrt{\left[1 + \frac{\lambda_{o} \operatorname{coth}(\frac{t_{s}}{\lambda_{o}}) + \lambda_{o} \operatorname{coth}(\frac{t_{g}}{\lambda_{o}})}{h}\right]} = Z_{p} \times Sw^{-0.5}$$
(8),

$$\beta_{o} = \frac{2\pi}{\Lambda_{o}} \cdot \sqrt{\varepsilon_{r}} \cdot \left[1 + \frac{\lambda_{o} \operatorname{coth}(\frac{t_{s}}{\lambda_{o}}) + \lambda_{o} \operatorname{coth}(\frac{t_{g}}{\lambda_{o}})}{h}\right]} = \beta_{p} \times Sw^{-0.5}$$
(9),

$$Sw = 1 + \frac{\lambda_{o} \operatorname{coth}(\frac{t_{s}}{\lambda_{o}}) + \lambda_{o} \operatorname{coth}(\frac{t_{g}}{\lambda_{o}})}{h}$$
(9a),

where Λ_o is the free space wavelength, Z_P and β_P are the characteristic impedance and the wavefactor of the line with the same geometry, uniform field distribution and made of a perfect conductor, the rest of the parameters in the equations (8, 9) and (9a) are dimensional parameters of the superconducting microstrip line (Fig. 1).

The equations (8) and (9) have a very simple physical interpretation: the electromagnetic field penetrates in the strip and ground superconducting electrodes of the microstrip transmission line at the effective depth of λ_0 , the London penetration depth, extending the thickness where the electromagnetic wave propagates beyond the thickness of dielectric, h, into the both superconductors. Similar approach to a microstrip line made from a normal conductor with the dielectric thickness comparable with the skin depth of the strip and ground conductors is presented in [21]. Swihart solution assumes that the magnetic field penetration depth is the static London penetration depth, λ_0 , independent on the frequency of the electromagnetic wave propagating inside the microstrip line.

With the same assumption about the field uniformity in the SML, Kautz [16] has applied the theory of Mattis and Bardeen, taking into account that the behaviour of the superconducting material depends on frequency and, at frequencies close to the superconducting material gap frequency, the conducting properties of the superconductor are described by anomalous skin effect with surface impedance of the superconductor presented via complex conductivity. The SML impedance, the propagation constant and loss were calculated for a wide band of

^{*} Here and through the entire paper we assume that the strip and ground electrodes are made from the same type of superconducting material.

frequencies. However, the both models neglect the fringing component of the electromagnetic field, which contributes substantially giving error up to 30% of the line characteristic impedance and the wave propagation constant in practical SML.

Line Models Including Fringing EM Field and a New SML Model

The behaviour of a superconducting microstrip transmission line taking into account fringing fields was analyzed by a number of authors [4-8]. Two approaches are used to obtain characteristics of the superconducting microstrip line. The first approach is, as for a conventional microstrip, a solution of Maxwell equations for given geometry and boundary conditions, using e.g., conformal mapping [7], and, as a result, deriving close form equations for SML impedance and wave-propagation factor. Another approach to obtain the impedance and the wave propagation factor in SML uses surface impedance of the superconductors comprising the line, derived by employing the Mattis-Bardin theory, and include this impedance in the calculations of the line characteristic impedance and the wave-factor via taking into account geometry factor, i.e., the RF current distributions in the strip and ground electrodes, e.g., in [5, 6, 8]. Interestingly, the result obtained by Swihart, who assumed the uniform field, shows that the superconducting material contribution is factorized in a form of a multiplier for both the characteristic impedance and wave-factor, equation 8, 9 and 9a, compare to a perfect-metal based line; similarly for the SML with negligible loss and the fringing field, taken into account, the factorizing takes place as well, e.g., [8].

In order to introduce a new model for the SML we would like to consider the influence of a superconducting material on the characteristic impedance and the propagation constant of a superconducting transmission line via analyzing a lump model equivalent circuit of transmission line, depicted in Figure 4. Let us assume a microstrip transmission line using a perfect conductor with radiation and dielectric material losses being negligibly small. The line equivalent parameters, L and C, take into account the geometry and the fringing field. Similar circuit can be used for SML, having the same geometry, with additional circuit components describing the superconductors, Fig.4 (2).



Figure 4. Equivalent circuit of a short piece, dx, of a lossless microstrip transmission line (1) made of a perfect conductor: L is inductance and C is capacitance, per unit of length extracted using, e.g., [22]. Equivalent circuit for dx long superconducting transmission line (2 & 3): L is inductance and C is capacitance (the same as for the perfect conductor line) per unit of length. Z_{ss} and Z_{sg} are impedances of the strip and ground respectively, per unit of length describing contribution of the superconductor.

If the frequency of the electromagnetic wave propagating through the line is well below the gap frequency (about 650 GHz for Nb material used as example here) the RF conducting loss in the superconductor is negligible (Fig. 3), thus the impedances Z_{ss} and Z_{sg} should be pure imaginary and *inductive*. Representing these impedances, Z_{ss} and Z_{sg} , as a product of the specific line inductance impedance, ωL , we can write the follows:

$$Z_{ss}, Z_{s\sigma} \Longrightarrow \omega L \times (SB^2 - 1)$$
(10),

where SB represent a correction factor for the specific inductance of the perfect conductor line and comprises both geometrical [23] and superconducting material contributions. Calculating the characteristic impedance and the wave factor of the transmission lines represented in Fig. 4 and taking into account that the series impedance and shunt admittance of the transmission line in Figure 4, (1), are $Z=\omega L$ and $Y=\omega C$ and correspondingly $Z=\omega L-(Z_{ss}-Z_{sg})$ and $Y=\omega C$ for SML for the equivalent circuit shown in Figure 4, (2 & 3), we arrive to the following equations:

for the perfect conductor microstrip line the characteristic impedance Z_P and wave factor β_P are:

$$Z_{P} = \sqrt{\frac{\omega L}{\omega C}} = \sqrt{\frac{L}{C}} \quad and \quad \beta_{P} = \sqrt{\omega^{2} L C} = \omega \sqrt{L C} \quad (11);$$

for the superconducting microstrip line the characteristic impedance Z_S, taking into account equation (10):

$$Z_{S} = \sqrt{\frac{\omega L + \omega L \times (SB^{2} - 1)}{\omega C}} = SB \times \sqrt{\frac{L}{C}} = SB \times Z_{P}$$
(12)

and the wave-factor β_S :

$$\beta_s = \sqrt{(\omega L + \omega L \times (SB^2 - 1)) \times \omega C} = \sqrt{\omega^2 SB^2 \times LC} = SB \times \omega \sqrt{LC} = SB \times \beta_p \quad (13),$$

where L and C are the equivalent inductance and shunt capacitance of the line based on a perfect conductor.

The correction factor SB depends, in general, on: *i*. the surface impedance of the strip and ground electrodes constituting the microstrip line, see equations (1 - 5), *ii*. the geometrical factor of the field penetration including the RF current distribution effects and the fringing field.

G. Yassin and S. Withington [7] introduced a penetration factor, χ , that is a parameterized Hfield penetration in the superconductors of microstrip line and is a function of the line geometry. The calculations of χ , made in [7] for different geometries of the SML (Fig. 5), show that when the SML strip conductor is relatively thin, t_s/h \leq 0.1, and narrow, W/h \leq 7, the penetration factor χ is larger than unit, indicating difference in the field penetration compare to the case of the uniform field distribution when $\chi \rightarrow 1$.

Practical considerations and limitations of thin-film technology narrow and constrain the range of attainable SML geometries. In case if *no* planarization process is used for fabrication of SML, e.g., Nb-material based transmission line with SiO or SiO₂ dielectric, we should consider appearance of steps with the height equal to the thickness of the dielectric layer; in order to avoid physical and electrical breaks in the strip conductor the latter should be thicker than the height of the step.

Taking into account RF conducting properties and the H-field penetration depth, the follow empirical formula can be introduced for obtaining the strip thickness, t_s , for a given dielectric thickness h:

$$t_s \ge h + (1.5 \dots 2.0) \times \lambda_o$$
 (14).



Figure 5 from the ref. [7]. The penetration factor χ is shown as a function of normalized strip width for various strip thicknesses: t/h = 1.0, 0.5, 0.1, 0.01, 0.001 and 0.0001.

Maximum thickness of the dielectric layer is limited typically due to the restrictions of lift-off process allowing of about 350 nm thick SiO and slightly thinner SiO₂; with the aim of achieving a thicker dielectric two consequent depositions can be performed, e.g. [24], giving maximum thickness of about 450 – 650 nm. Taking into account typical impedances we should attain, the thickness of dielectric is to be more than 100 nm. These considerations define a range of the SML dielectric thickness 100 nm $\leq h \leq 650$ nm and, for Nb film with $\lambda_0 \approx 85$ nm, corresponding range of the strip thickness 230 nm $\leq t_s \leq 780$ nm.

Processing, involving optical and E-beam lithography, gives different accuracies in resolving linear dimensions: with E-beam lithography it is possible to achieve the width of strip well below 1 μ m and thus reach a higher line impedance for a given thickness of dielectric, while for repeatable results with optical lithography the strip width W \geq 5 μ m would be an appropriate choice. Bringing these additional considerations to define the ranges of parameters used to calculate penetration factor χ [7], we find out that the strip normalized thickness is of $1.2 \leq t_s/h \leq 2.3$ and the strip normalized width is of $7.7 \leq W/h \leq 50$ for optical lithography. The lattes lead us to the conclusion that the SML line with geometries of interest has the field penetration factor χ , that is *very close to the line with uniform field distribution* ($\chi \cong 1$).

Based on that fact and referring the equations (8, 9), (11, 12) and (9a), we can write a new equation for the correction factor SB:

$$SB = \sqrt{1 + \frac{\lambda(\omega) \cdot \coth(\frac{t_s}{\lambda(\omega)}) + \lambda(\omega) \cdot \coth(\frac{t_g}{\lambda(\omega)})}{h}} \quad and \quad \lambda(\omega) = \frac{X(\omega)}{\omega\mu_o}$$
(15).

In the equation (15) we bear in mind the Swihart solution for static (frequency independent) London penetration depth, λ_o , as in equation (9a), and use similarity of the equation (8, 9) with the equations (12, 13), suggesting that $SB \propto \sqrt{Sw}$, which should be exact for W>>h. Taking into account that the H-field penetration depth is a *frequency dependent quantity*, as it is clear from the equation (6), and considering that $\lambda(\omega)$ is a slow function of the frequency (Figure 2), we use the frequency dependent $\lambda(\omega)$ in the Sw factor, equation (9a), assuming the Swihart solution to be valid for every single frequency. Summarizing, the proposed new model takes into account the fact that contribution of the superconducting state of the strip and ground conductors to the

microstrip line performance can be represented in the form of the SB multiplier factor, equation (15). The SML characteristic impedance and the propagation factor can be obtained from those of a microstrip line with the same geometry based on ideal conductor via scaling them by the SB factor.

The presented model was successfully used for a number of SIS mixer projects [15, 19, 24 - 26] demonstrating its accuracy. However, the accuracy was judged indirectly, via the achieved performances of the SIS mixers. In this paper we would like to evaluate the suggested model further by competitive comparison with the previously proposed SML models and 3-D electromagnetic field numerical simulation.

Numerical Simulation of SML with HFSS

The SML was simulated using 3-D electromagnetic field numerical simulation package, HFSS by Agilent. We used approach suggested by A.R. Kerr [12] where a superconductor of finite thickness T is represented by two infinitely thin conducting planes (Fig. 6 below) with surface impedance calculated based on the transmission line theory, the surface impedance of the actual superconductor and the fact that the two surfaces interact via the penetration of the magnetic field (the thickness T is assumed to be comparable with the magnetic field penetration depth λ). The equation for surface impedance in the HFSS simulation was slightly modified for term β , page 7, reference [12]; the modifications introduced in the equation reflect the fact that the magnetic field penetration is frequency dependent item. Therefore, the surface impedance of the superconductor of the thickness T, having its bulk surface impedance Z_s(ω), equation (5), is represented as the follows:

$$Z_{x}(\omega,T) = Z_{s}(\omega) \times \left[1 - \frac{j\omega\mu_{o}T}{2 \cdot Z_{s}(\omega)} + \sqrt{1 + \left(\frac{j\omega\mu_{o}T}{2 \cdot Z_{s}(\omega)}\right)^{2}}\right]$$
(16)

Based on the equation [16], the ground plane is represented in HFSS as an infinite conducting plane with the surface impedance $Z_{ground}(\omega) = Z_x(\omega, t_g)$, Figures 1, 6. Employing the same approach to the strip we should represent it as rectangular structure with conducting planes having the surface impedance of the horizontal planes $Z_{strip_h}(\omega) = Z_x(\omega, t_s)$ and the vertical plane surface impedance, edges of the strip, $V_{strip_v}(\omega) = Z_x(\omega, W)$, as depicted in Figure 6.



Figure 6. Simulation of the SML in finite-element solver (HFSS). The strip is simulated as a box structure with conducting planes having different surface impedances for vertical and horizontal planes.

SML Model Comparison

In order to establish common reference for the model comparison we used Nb-material based SML with the material properties as follows: the normal state conductivity, $\sigma_n = 1.619 \times 10^7 [\Omega^{-1} m^{-1}]$, critical temperature, $T_c = 8.7$ [K], London penetration depth, $\lambda_o = 8.5 \times 10^{-8}$ [m] and ambient temperature T = 4.2 K. This combination of the material parameters gives the superconducting gap Δ (4.2) = 1.377 meV at 4.2 K. For modelling we have chosen SiO₂ as the dielectric (ϵ_r =3.74) with several thicknesses of 150, 250, 450 and 650 nm. The penetration depth, λ_o , is a superconducting material constant of an absolute value for a given material; in order to preserve this fact we used absolute thickness of the dielectric material in the modelling, as above, and the strip width of 2, 4, 6 and 10 µm for each thickness of the dielectric, rather than usual relative ratio of the dielectric thickness, t/h, and the strip width, h/w. With the thicknesses of the dielectric and the widths of the strip as above we go slightly beyond the area of usual SML-based tuning circuitry for SIS mixers as discussed before and therefore could check how the suggested new model is compared to others with the wider range of geometries. In the modelling, the strip and the ground plane were assumed having the same thicknesses calculated according to the equation (14).

The starting point for every presented below modelling results is: the line geometry, as in Figure 1, the superconducting material parameters, as above, and, if required, the surface impedance calculated for every thickness of the Nb material according to the equations (1-7) and for the Nb material parameters listed. The modelling was done using MathCAD [27] program to implement the models from references [4, 7]. In order "to align" W.H. Chang model, [4], and make it more representative in the comparison with more recent ones, we introduced frequency dependence of the H-field penetration depth $\lambda = \lambda(\omega)$ and used it *instead* of λ_o as was in the original model. The SML model used in Caltech [5, 6] was realized in the *SuperMIX* software package [28] and the code was used to model the described above SML configurations. We have chosen to plot slow-wave factor or the wavelength ratio, $\Lambda_{\text{free}_space}/\Lambda_{\text{in}_line}$, in order to characterize the propagation in the SML for different models.

Discussion

The results of the simulations show that more or less all the models converge towards HFSS simulations. The most spread in the modelling results is in the slow-wave factor while the results on the characteristic impedance calculations show more consistency. As expected, introducing the frequency dependent H-field penetration produces drastic changes in the SML impedance and the slow-wave factor, $\Lambda_{\text{free_space}}/\Lambda_{\text{in_line}}$, those become up to 30% higher for thin dielectric (150 nm) around the gap frequency compared to, e.g., @100 GHz.

The new model produces results, which are very close to HFSS simulations and the model presented by G. Yassin & S. Withington for all SML geometries, while require relatively simple calculation procedures as described above especially if one of the approximation functions for $\lambda(f)$ is used. Presence of a noticeable discrepancy between Yassin & Withington model on one side and HFSS on another side for thin dielectric, e.g., Figure 22 and 30, brings us to the conclusion that further adjustments can be made for HFSS approach using more realistic distribution of the surface impedance at the edges of the strip. The discrepancy leaves open the question about a reference model.



Figure 7. SML impedance for the $2\mu m$ wide strip on **650 nm** thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with $\lambda(f)$.



Figure 9. SML impedance for the 4 μ m wide strip on 650 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with λ (f).



Figure 8. SML slow-wave factor for the 2 μ m wide strip on 650 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with λ (f).



Figure 10. SML slow-wave factor for the 4 μ m wide strip on 650 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line SuperMIX, solid line with rectangulars - Chang with $\lambda(f)$.



Figure 11. SML impedance for the 6 μ m wide strip on 650 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses – [7], dash-dot line – SuperMIX, solid line with rectangulars – Chang with λ (f).



Figure 13. SML impedance for the 10 μ m wide strip on 650 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses – [7], dash-dot line – SuperMIX, solid line with rectangulars – Chang with λ (f).



Figure 12. SML slow-wave factor for the 6 μ m wide strip on 650 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with $\lambda(f)$.



Figure 14. SML slow-wave factor for the 10 μ m wide strip on 650 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with λ (f).



Figure 15. SML impedance for the $2\mu m$ wide strip on **150 nm** thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with $\lambda(f)$.



Figure 17. SML impedance for the 4 μ m wide strip on 150 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with λ (f).



Figure 16. SML slow-wave factor for the 2 μ m wide strip on 150 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with $\lambda(f)$.



Figure 18. SML slow-wave factor for the 4 μ m wide strip on 150 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses [7], dash-dot line - SuperMIX, solid line with rectangulars – Chang with λ (f).



Figure 19. SML impedance for the 6 μ m wide strip on 150 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses – [7], dash-dot line – SuperMIX, solid line with rectangulars – Chang with λ (f).



Figure 21. SML impedance for the 10 μ m wide strip on 150 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses – [7], dash-dot line – SuperMIX, solid line with rectangulars – Chang with λ (f).



Figure 20. SML slow-wave factor for the 6 μ m wide strip on 150 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with $\lambda(f)$.



Figure 22. SML slow-wave factor for the 10 μ m wide strip on 150 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with $\lambda(f)$.



Figure 23. SML impedance for the $2\mu m$ wide strip on **250 nm** thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with $\lambda(f)$.



Figure 25. SML impedance for the 4 μ m wide strip on 250 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses – [7], dash-dot line – SuperMIX, solid line with rectangulars – Chang with λ (f).



Figure 24. SML slow-wave factor for the 2 μ m wide strip on 250 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with $\lambda(f)$.



Figure 26. SML slow-wave factor for the 4 μ m wide strip on 250 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with $\lambda(f)$.



Figure 27. SML impedance for the 6 μ m wide strip on 250 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses – [7], dash-dot line – SuperMIX, solid line with rectangulars – Chang with λ (f).



Figure 29. SML impedance for the 10 μ m wide strip on 250 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses – [7], dash-dot line – SuperMIX, solid line with rectangulars – Chang with λ (f).



Figure 28. SML slow-wave factor for the 6 μ m wide strip on 250 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with λ (f).



Figure 30. SML slow-wave factor for the 10 μ m wide strip on 250 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with $\lambda(f)$.



Figure 31. SML impedance for the 2µm wide strip on **450 nm** thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses – [7], dash-dot line – SuperMIX, solid line with rectangulars – Chang with $\lambda(f)$.



Figure 33. SML impedance for the 4 μ m wide strip on 450 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with λ (f).



Figure 32. SML slow-wave factor for the 2 μ m wide strip on 450 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with $\lambda(f)$.



Figure 34. SML slow-wave factor for the 4 μ m wide strip on 450 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with $\lambda(f)$.



Figure 35. SML impedance for the 6 μ m wide strip on 450 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses – [7], dash-dot line – SuperMIX, solid line with rectangulars – Chang with λ (f).



Figure 37. SML impedance for the 10 μ m wide strip on 450 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses – [7], dash-dot line – SuperMIX, solid line with rectangulars – Chang with λ (f).



Figure 36. SML slow-wave factor for the 6 μ m wide strip on 450 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with $\lambda(f)$.



Figure 38. SML slow-wave factor for the 10 μ m wide strip on 450 nm thick SiO₂. Legend: solid line - HFSS, dot line - the new model, dot line with crosses - [7], dash-dot line - SuperMIX, solid line with rectangulars - Chang with λ (f).

Interestingly, the new model that initially doesn't take into account increasing loss in the superconductors around and above the gap frequency anyway produces *correct* values of the impedance and the slow-wave factor as compared to the models, which take use of the surface impedance of the superconductor and therefore include these losses. This opens possibility to use the suggested model even in the *wider* frequency band by adding conventional multiplicative loss factor [29].

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