Gaussian Beam Analysis of Relay Optics for the SEQUOIA Focal Plane Array

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Abstract

The Large Millimeter Telescope (LMT) is a 50 m diameter millimeter-wavelength telescope being built in Mexico as a joint project between UMass and Instituto Nacional de Astrofisica, Óptica, y Electrónica (INAOE) in Mexico. The scientific goals of the LMT require high efficiency imaging over wide fields at wavelengths from 4 to 1 mm (75 to 300 GHz). Large-format focal plane arrays are being planned for the LMT. Of these, SEQUOIA (Second Quabbin Optical Imaging Array), a dual-polarization 16-pixel 3 mm wavelength focal plane array [1] has already been constructed, and has been operating at the FCRAO 14 m telescope. SEQUOIA will be installed on the LMT once the telescope has been commissioned. System-level optical and mechanical constraints results in a large focal ratio of 10.5 at the Cassegrain focus of the LMT. However, smaller focal ratios are usually more suitable for illuminating the individual detector elements of focal-plane arrays. Indeed SEQUOIA has an input focal ratio of ~ 4. A reimaging or relay optics (RO) system is required in the LMT receiver cabin to convert between these focal ratios.

A first-cut design of the RO system for SEQUOIA on the LMT was done using geometric optics principles using ray-tracing procedures available in the commercial optics package CODE V. In this paper, we present a new design for the SEQUOIA RO system based on full gaussian beam propagation considerations. The design utilizes gaussian beam modules in a software package called ASAP (Advanced Systems Analysis Package). Gaussian beam analysis in ASAP uses the principle of Gaussian Beam Decomposition (an ensemble of Gaussian beams is used to simulate coherent beam propagation). We present various test cases that verify the gaussian beam analysis in ASAP against analytical formulations. We present the final optimized RO design and various efficiency parameters.

1 Introduction

Heterodyne receivers at millimeter and submillimeter wavelengths are starting to approach the quantum limit in noise temperature. High efficiency widefield spectroscopic imaging is still a very important goal of radio astronomy. Given that we are approaching the limits in single-pixel receivers, further gains in mapping can be derived by building large format focal-plane arrays. Indeed, in the last fifteen years, rapid development has been achieved in the area of sensitive focal-plane arrays [2].

The Large Millimeter Telescope (LMT) is a 50m diameter millimeter-wavelength telescope being built atop Sierra Negra, a volcanic peak near Mexico City [3]. The LMT will have a wavelength coverage of 4 mm to 1 mm (75 - 300 GHz). The scientific goals of the LMT require high efficiency imaging over wide fields at wavelengths from 4 to 1 mm (75 to 300 GHz). Large-format focal plane arrays are being planned for the LMT. For the 3 mm wavelength band, SEQUOIA (Second Quabbin Optical Imaging Array), a dual-polarization 16-pixel focal plane array [1] has already been constructed, and has been operating at the FCRAO 14 m telescope. SEQUOIA will be installed on the LMT once the telescope has been commissioned.

Optical and mechanical considerations (like keeping the size of the secondary manageable) dictated that the final designed Cassegrain focal-ratio, $F_{CAS} = 10.5$. However, for illumination of detectors in a focal-plane array, a much smaller focal-ratio is desirable. In fact, as designed, SEQUOIA has an f-ratio $F_{SEQ} \approx 4.1$. Thus, a "reimaging" or "relay" optics (RO) system is needed to convert the focal ratio from the Cassegrain focus to that required by SEQUOIA. The scientific goals for SEQUOIA on the LMT require high efficiency imaging over the entire field-of-view (FOV). The RO package is required to contribute minimally to the system temperature, and hence large off-axis mirrors (rather than lenses) will be required to maintain high image performance over large fields.

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A first-cut design of the RO system for SEQUOIA on the LMT was done using geometric optics principles using ray-tracing procedures available in the commercial optics package CODE V [4]. In this paper, we present a new optimized design for the SEQUOIA RO system based on full gaussian beam propagation considerations. The design utilizes gaussian beam modules in the commercial optics package called ASAP (Advanced Systems Analysis Package) from Breault [5].

2 Requirements and Design Concept of RO

The operational band for SEQUOIA is 85–115 GHz. On the LMT, at 100 GHz, the Gaussian beam-waist at the Cassegrain focal plane is $\omega_{0,CAS} = 0.22[T_e(dB)]^{1/2}F_{CAS}\lambda = 22.43$ mm, where we assume that the antenna edge taper, T_e (dB)= 10.5. Figure 1 shows a view of the gaussian beams launched from the throats of the SEQUOIA feed-horns. The far-field beam pattern of the SEQUOIA square feedhorns have been measured, and they were found to be gaussian in nature to 98% accuracy. The fitted $\omega_{0,SEQ}$ of this beam pattern = 8.7 mm. So, we will need a RO system with magnification of $M = \frac{22.43}{8.7} = 2.6$ to transform the SEQUOIA focal-plane gaussian beam-waist to the Cassegrain focal-plane beam-waist requirement.

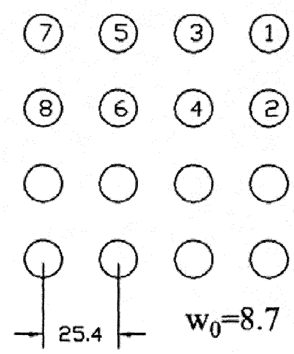

Figure 1: (a) SEQUOIA focal-plane array layout. The size and location of the gaussian beams launched by the 16 SEQUOIA feedhorns is depicted. The numbers correspond to the beams that were simulated.

Other requirements of the RO system are: (a) the FOV must be wide enough to cover the entire 4×4 array of SEQUOIA beams, (b) the antenna performance over the FOV must have minimal gain variation, (c) off-axis reflecting surfaces should be used to minimize any noise temperature penalties that could be incurred by refracting RO systems (d) the Gaussian beam coupling coefficients to the desired Cassegrain focal-plane illumination pattern should be uniform and as high as possible across the entire FOV across the entire receiver bandwidth of SEQUOIA.

One elegant solution for the design of a RO for a broadband receiver system is the so-called "Gaussian beam telescope" (GBT) as outlined in [6]. In a GBT, the pair of focusing elements are separated by a distance equal to the sum of their focal lengths, ($f_1 + f_2$). Figure 2 shows a schematic representation of a GBT. The magnification factor of the GBT is $M = w_{o,out2}/w_{o,in1}$, where $w_{o,out2}$ is the output beam-waist of the second mirror, and $w_{o,in1}$ is the input beam-waist of the first mirror. The output beam-waist location after the second mirror is given by,

$$d_{o2} = \frac{f_2}{f_1}\left(f_1 + f_2 - \frac{f_2}{f_1}d_{i1}\right) \tag{1}$$

where d_{i1} is the distance to the input beam-waist at the first mirror. It can be seen that if $d_{i1} = f_1$, then the location of the output beam-waist is wavelength independent, and is given by, $d_{o2} = f_2$. The size of the output and input beam-waists are proportional to wavelength as desired.

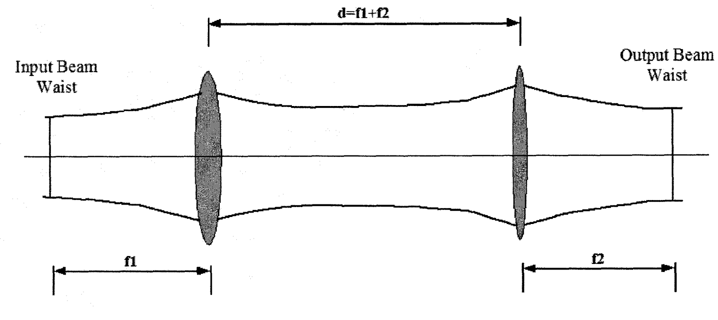

Figure 2: (a) A Gaussian Beam Telescope, consisting of two focusing elements, with focal lengths, f_1 and f_2, separated by the sum of their focal lengths.

While, these desirable features of the GBT make it suitable for this RO application, it must be noted that the derivation is strictly valid only for the beam propagating on-axis. For the case of a focal-plane array of feedhorns, the GBT conditions cannot be simultaneously met by the central beam *and* off-axis beams. The resulting efficiencies will have to be calculated using non-analytic techniques.

As has been demonstrated in the geometric optics modeling [4] of a proposed RO package for the LMT, if we use the GBT system for the RO, and postulate a few more conditions, then the only free parameters in the design of the SEQUOIA RO system are the focal-length f_1, and the incident angle γ_1 at the first mirror. The conditions that reduce the number of free parameters in the design are: (1) the input SEQUOIA beam-waist and the output Cassegrain beam-waist at the output of the GBT are exactly one focal-length away from the respective mirror, (2) the phase centers of the incident and reflected Gaussian beams are the geometrical foci of the generating ellipse of each elliptical mirror of the off-axis mirrors used in the GBT, and (3) the two incident angles at each mirror, $\gamma_1 = \gamma_2 = \gamma$. Given the required magnification, $M = 2.6$, it is possible to optimize for the best f_1 and γ that will improve the efficiency of the RO system. Such an exercise was done purely using geometrical ray-tracing in [4]. In this paper, we will carry out a similar exercise using a commercial software package that has the ability to treat gaussian-beam propagation.

3 Comparison of Commercial Optics Packages

Many commercial optics software packages are available for the design and analysis of optical designs. Most of them, however, are not well-suited for application to the far-infrared to millimeter-wavelength regime. Of these software packages, the following have show varying levels of success for modeling millimeter-wave optical systems: ASAP, GLAD, CODE V, ZEMAX and GRASP. Table 1 shows a comparison of the various packages.

While GRASP appears to be the best commercially available package, it is quite expensive. GLAD is a good compromise but it is not cheap either. We decided to use ASAP as it has a free academic license program. We ran a suite of tests to determine the applicability of ASAP for gaussian beam propagation calculations. The resulting tests are summarized in the next section, and the final ASAP analysis of the SEQUOIA RO system is provided in § 5.

4 Overview of ASAP and Gaussian Beam Propagation Tests

Advanced Systems Analysis Program (ASAP) from Breault Research Inc. is a ray tracing program with surface/structure modeling for incoherent or coherent beam propagation. The Gaussian beam is a solution to the

Table 1: **Comparison of Optics Design Packages**

Package	ASAP	GLAD	ZEMAX/CODE V	GRASP
Underlying Theory	Gaussian Beam Decomposition & Complex Ray Tracing	Angular Spectrum Techniques	Ray Tracing & Diffraction calculation only at exit pupil	Physical Optics & Geometric Theory of Diffraction
Ability to Model Sources	Using Gaussian beamlets	Arbitrary Source Fields	Only Point Sources / Single Gaussian Beams	Standard horns & arbitrary fields
Available Fields	Through Synthesis & Decomposition	At all planes (both polarizations)	Only at output plane	At all planes (both polarizations)
Speed	Complex Ray Tracing – Fast	Diffraction Calculations – slower	Fast	Slow for aberration Calculations
Optimization	No - but Perl-scripting possible	Yes	Yes	No
Cost	Free academic license	$5k	<$2k	$20k

paraxial wave equation. The propagation of a Gaussian beam is well understood, and easily characterized by a few simple parameters. In ASAP, each gaussian beam is represented in turn by five rays – one base ray and four "parabasal" rays (see Figure 3a). In performing Gaussian beam propagation, ASAP performs complex ray tracing of all five of these parabasal rays. The next important principle in ASAP's gaussian beam analysis is its Gaussian Beam Decomposition. Here any complex field is represented as a superposition of individual Gaussian beams (called beamlets). Figure 3b shows the principle of ASAP's Gaussian beam decomposition (GBD) as applied to an example optical system. Complex source fields can be well-modeled using GBD. In general, Gaussian beams have a form that comes close to the spatially localized, non-diverging ideal. The angular divergence of their wavefront normals is the minimum permitted by the wave equation for a given beam width. The energy of the beam is concentrated primarily along the propagation axis, and falls off rapidly with radial position. This in turn allows Gaussian beams to perform localized sampling of optical surfaces. This is important especially for surfaces with higher-order structure, such as off-axis mirrors. Usually, GBD is done for the input fields before being incident on off-axis mirrors, thereby increasing the accuracy of the treatment. The accuracy in calculation of the resulting fields is always a tradeoff between accurate sampling of fields and optical surfaces versus violating the paraxial approximation (as can happen when the beam-waists of the individual beamlets approach the wavelength). When using ASAP to analyze millimeter and submillimeter wavelength quasi-optical systems, the user has to have a rather careful understanding of the underlying theory and assumptions in ASAP. One of the main limitations of ASAP's treatment is that when aperture dimensions (or object spatial frequencies) are near to, or below the operating wavelength, the method tends to break down, and may produce spurious results.

4.1 Tests of Gaussian Beam Propagation in ASAP

4.1.1 Free Space Propagation

In a suite of tests to determine the propagation behavior of single gaussian beams (without any decomposition), single gaussian beams of varying beam waist radii at the same wavelength (λ = 3mm) were launched, and the beam is propagated to different test positions, and the resultant waist radii as predicted by ASAP is compared to the theoretical expectation. ASAP does a very good job with free-space propagation of Gaussian beams and there is less than 0.2% error up to 20 m of distance from the beam waist even for beams with ω_0 = 8mm. At larger beam waist radii, the errors are even smaller.

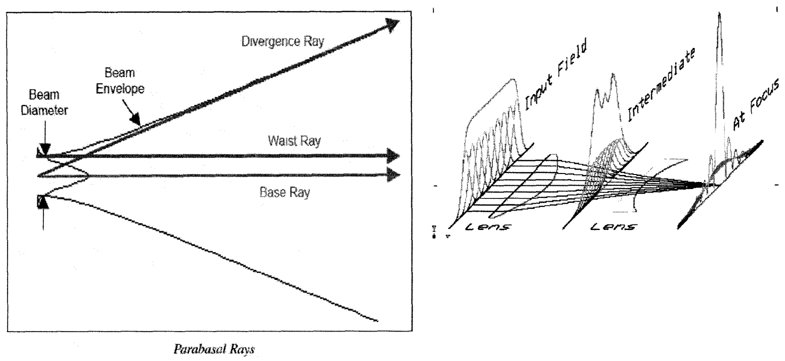

Figure 3: (a) ASAP Representation of Gaussian Beam. Parabasal rays are paraxial with respect to the base ray. Only one each of the 4 paraxial rays (2 waist rays and 2 divergence rays) are shown in the figure, the other 2 are out of the plane of the paper. If the divergence of the Gaussian beam is too rapid, the beam does not stay gaussian as it propagates, and ASAP keeps track of this violation, and duly reports it. (a) ASAP Propagation using Gaussian Beam Decomposition. The arbitrary input field is represented using a superposition of individual Gaussian beamlets. Complex ray tracing of the parabasal rays of the individual beamlets is performed, followed by reconstituting the field at the output detector plane.

4.1.2 Reflection off of Flat Mirror

In the next test, we probed the effect of reflection of a gaussian beam as it reflects off a flat mirror. Figure 4a shows a plot of the variation of coupling efficiency of the ASAP predicted beam to the theoretically expected perfect gaussian beam at varying distances from the flat mirror. Gaussian beams with 25, 12, and 8 mm beam waist radii are launched 300 mm away from the flat mirror. The incident angle of the beam is 45 degree. Coupling efficiency variation as a function of the propagation distance from the flat mirror is shown. It is seen that beyond a distance of 10 m from the flat, with gaussian beams that approach the wavelength (here $\lambda = 3$mm), ASAP starts to become inaccurate. However, with beam widths which are several times the wavelength, the errors are tolerable.

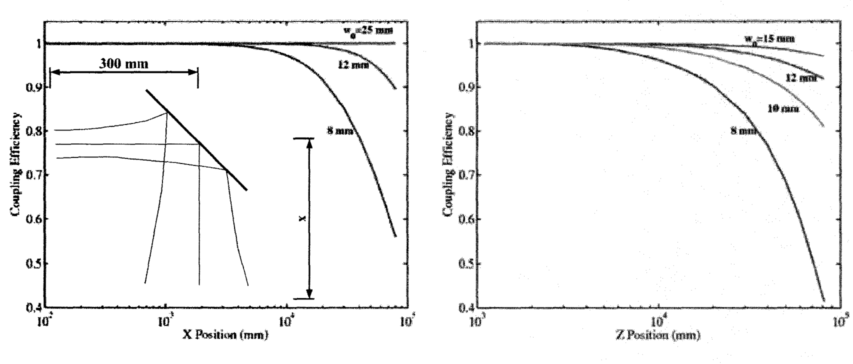

Figure 4: (a) Variation of Coupling efficiency with propagation distance after reflection from a flat mirror. A single gaussian beam of beam width 25, 12 or 12mm is launched 300 mm from a flat mirror oriented at 45° with respect to the incident beam. The X-axis represents the distance x from the mirror.

4.1.3 Tests of Decomposition

In ASAP, decomposition allows us to define a new set of individual beams to represent the same field as the original beams. While more beamlets will accurately sample the original field, each beamlet becomes smaller and smaller in waist size. When the beam waist size of the individual beamlets approach the wavelength of operation, the principle of gaussian beam propagation breaks down. In general, as the beam waist radius gets smaller than a few times λ, the accuracy suffers. This can be seen in Figure 4b, where a single gaussian beam is allowed to propagate in free-space, then decomposed at 1000 mm. The subsequent field is allowed to propagate and coupling efficiency to a fundamental Gaussian beam mode at various distances are plotted. When the original gaussian beam is several times λ, decomposition and subsequent propagation is accurate. At $\omega_0 = 8$ mm, the individual beamlets become small enough at decomposition, that the decomposed field does not do a good job of representing the field, as seen by the decreasing coupling efficiency with distance.

4.1.4 Gaussian Beam Coupling Tests

In the SEQUOIA RO system, we launch approximately gaussian beams from the 32 SEQUOIA feedhorns to the Cassegrain focal-plane. At the CFP these beams should meet the requirements of the field that is necessary to properly illuminate the secondary mirror. This involves optimizing the coupling efficiencies at each location at the CFP, where the SEQUOIA beam that propagates through the RO system is coupled with an ideal gaussian beam at the CFP for each pixel. In an effort to determine how well ASAP calculates coupling efficiencies between gaussian beams, we ran three tests. These tests have analytical formulations [6]. Imperfect coupling can occur between two aligned beams, if they have mismatched beam radii and/or radii of curvature. Figure 5a shows the result for two beams which are axially aligned, but with differing beam waist radii. It is seen that the ASAP prediction of coupling efficiency agrees with the analytical result.

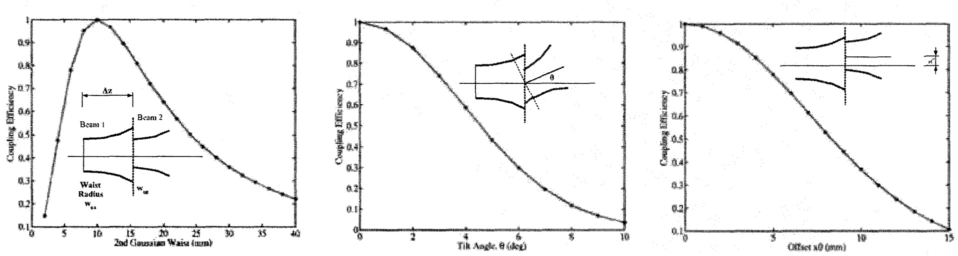

Figure 5: Gaussian Beam Coupling Tests. In all three plots, the solid line shows the analytical expectation from [6], and the starred-symbols show the ASAP analysis. (a) Coupling efficiency variation for axially aligned beams with offset beam waist radii. Beam 1 coming from the left is a $\omega_0 = 10$mm gaussian beam. Coupling efficiency variation with beam waist radii of the second beam is plotted. (b) Identical waists, tilted beams. Here, both beam 1 and 2 have the same waist radii (10 mm), but beam2 is tilted with respect to beam 2 with an angle of θ degrees. The variation of coupling efficiency with θ is plotted. (c) Identical beams, offset waist locations. Here both beam 1 and 2 have the same waist radii (10 mm), but the center of beam2 is offset by a distance x_0 with respect to that of beam 1.

Figure 5b shows the result for *tilted beams*, and Figure 5c shows the result for *offset beams*. Again, it is seen that ASAP prediction is inline with the analytical formalism, showing the ASAP's coupling integral calculations are accurate.

5 Gaussian Beam Analysis of SEQUOIA RO

Having tested ASAP for a variety of test cases, we concluded that ASAP works well except when the beam waist approaches the wavelength of operation. In the SEQUOIA RO system, we need to expand the size of the Gaussian beam waists from 8.7 mm at the SEQUOIA focal plane to 22.43 mm at the CFP (see §2). The errors in decomposing the input gaussian beam field will be more than at the CFP, as the beam keeps expanding through the RO system. It is expected that errors can be minimized by careful decomposition in front of the first mirror, after the initial beam from the feed horn has expanded in size.

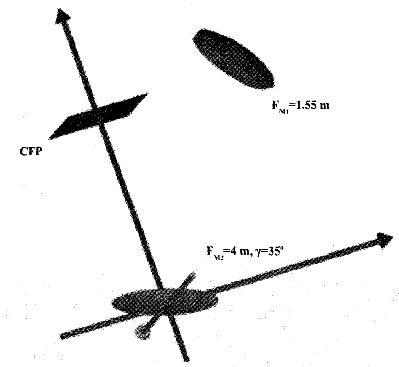

Figure 6: SEQUOIA RO System. M1 and M2 are the two elliptical mirrors that form the Gaussian Beam Telescope. SEQUOIA is located in front of M1. The incident angle of the beams at both mirrors are equal, and is given by $\gamma = 35°$.

Figure 6 shows the final optimized layout of the final adopted SEQUOIA RO system. As has been outlined in § 2, the final design of the RO system can be expressed in terms of two free parameters – the focal length of one of the mirrors of the Gaussian Beam Telescope (GBT) and the incident angle in each of the mirrors, γ.

ASAP does not have a built-in optimizer module. However, the latest version of ASAP comes equipped with the ability to use Perlscript [7], a perl-based programming language, which permits the use of ASAP commands from within the perlscript, whilst maintaining the power of an interpreted programming language with powerful procedural and object-oriented programming techniques.

In the optimization procedure, we followed the following recipe. A large perlscript macro was written and called from within ASAP. The second mirror, M_2 is always located at the origin (see Figure 6). For each focal length f_1 and incident angle, γ, we determine the rest of the parameters of the RO system as described in § 2. We then perform a geometric ray trace of the top 8 beams from the SEQUOIA feed horns to determine the approximate phase center positions and direction cosines after the first reflection from M_1. After this procedure, full coherent optics calculations are performed on all 8 beams with two decompositions, first before M_1, and then before M_2 (with careful checks along the way to ensure appropriate sampling of the fields and non-violation of the paraxial approximation). The beams are propagated to the CFP, and the coupling integrals are computed for all 8 beams. A global error figure is determined with all 8 beams to the ideal condition of unity gain of the system, and this error figure is minimized by successive iterations. The whole procedure takes a whole day of iterating on a modern computer, but eventually an optimized solution is reached. The design presented in Figure 6 is close to this optimal design, and the resulting coupling integrals from this design are presented in Figure 7. The design was optimized at 3 mm wavelength, but verified for the whole SEQUOIA band.

In the final optimized design, the focal lengths of M_1 and M_2 are 1.55 m and 4 m respectively, with an angle of incidence at each mirror given by $\gamma = 35°$. The SEQUOIA focal plane is thus located 1.55 m from M_1, and the 8.7mm beam waists from the feedhorns have expanded already to $\sim$ 170mm by the time they reach M_1. Reasonably acurate decomposition was possible for such a beam. The resulting beam efficiencies

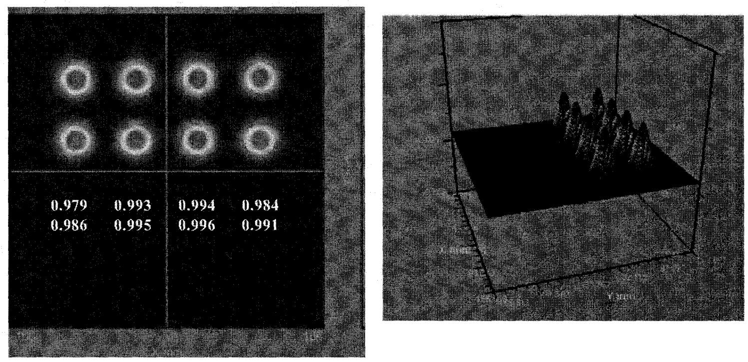

Figure 7: Output Field of 8 pixels at the CFP. The top 8 beams of SEQUOIA are launched from the SEQUOIA focal plane through the RO System, and plotted at the CFP. The coupling efficiencies to the required gaussian beams at the CFP for the 8 pixels are noted in the lower half of the figure. The figure on the right shows the isometric surface plot of the SEQUOIA beams at the CFP.

show less than 3% degradation in efficiency even for the edge pixels (see Figure 7).

6 Conclusions and Future Work

A large relay optics system has been designed for the 3mm SEQUOIA focal plane array on the LMT. Full gaussian beam analysis has been used to design the system using Breault Research Inc.'s ASAP software. In the future, we intend to perform a complete tolerance analysis of the location and orientation of the mirrors that form the RO system, and also determine cross-polarization losses in the system.

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