Embedding the Quantum Mixer Theory into a Time Domain Field Solver

Douglas W. Henke, Poman P. M. So, Stéphane M. X. Claude, and Wolfgang J. R. Hoefer

***Abstract*—To completely account for the quantum-mechanical features of the *superconductor-insulator-superconductor* (SIS) junction, Tucker's quantum mixer theory must be used. The standard approach is to perform the computations in the frequency domain. To determine the large signal waveform resulting across the junction, a time domain formulation has also been used in the *voltage update method* (VUM). Even using this method, the nonlinear diode current (computed using the time domain equations) is related to the rest of the circuit in the Fourier domain. An iterative procedure enables the correct junction voltage to be found.**

This paper proposes a variation to the VUM such that all calculations are performed in the time domain, in a time-stepping fashion, enabling the theory to be implemented into a time domain field solver. The main advantages of performing simulations within the time domain are the ability to process arbitrary time signals applied to the nonlinear junction and to generate complete information on the mixing products over a wide frequency range. It is recognized that a noise analysis is not feasible within the time domain and would be treated separately.

***Index Terms*—Quantum mixer theory, time domain analysis, superconductor-insulator-superconductor devices**

I. Introduction: Time Domain Quantum Mixer Theory

THE time domain formulation of the quantum mixer theory has been presented in [1]-[3]. Time domain theory has already been used for certain aspects of simulation, e.g., determining the local oscillator (LO) voltage waveform established across the junction using the *voltage update method* [4],[5]. However, a full embedding of the mixer theory into a time domain 3D electromagnetic field solver has not yet been done. While the current and voltage waveforms can be modeled in the time domain, it is not feasible to predict the noise, and so this would need to be treated separately.

Abstract received February 24, 2005. This work was supported in part by the National Research Council of Canada at the Herzberg Institute of Astrophysics through research contributions under the D. C. Morton Fellowship, and by the Natural Science and Engineering Research Council of Canada (NSERC).

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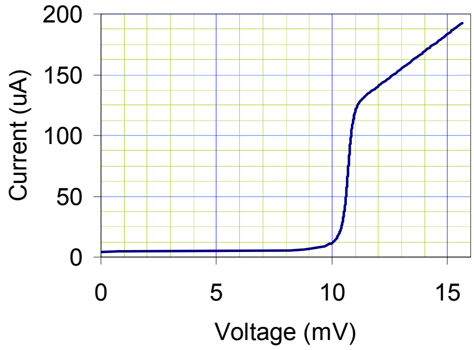
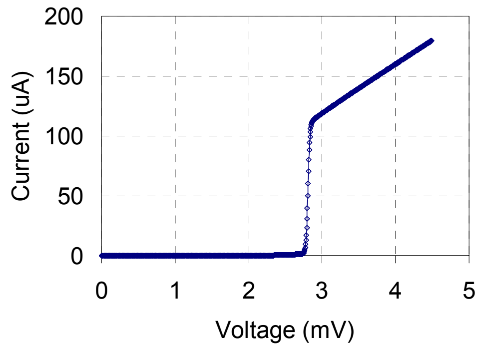
The following summarizes the time domain theory presented in [1]-[3]. In this first section, only the simple current-to-voltage relationship will be reviewed; no source impedance or junction capacitance is included. To provide further insight, a MATLAB [6] implementation is used to demonstrate junction currents and photon-assisted tunneling.



(a)



(b)

Fig. 1. Unpumped I-V curves that may be used for analysis. (a) Measured I-V curve for a series array of four Nb junctions. This is the typical response of the mixers used within the ALMA Band 3 receivers [7]. (b) I-V curve characteristic mimicking a measured response for a single Nb junction [9]. All MATLAB results contained herein are based on this fitted curve.

Recall that the unique result of the quantum mixer theory is that the unpumped curve provides sufficient information to predict the RF performance of the mixer. Fig. 1a shows a measured I-V curve for a typical mixer used in the ALMA Band 3 receivers [7],[8]. These mixers each contain four junctions arranged in a series array. For simplicity, the I-V curve characteristic used within this paper to demonstrate the time domain theory is shown in Fig. 1b and represents data that might be measured for a single niobium (Nb) junction [9].

In the time domain, the response function is defined in the following manner:

$$\overline{\chi}(t)=\frac{2}{\pi}\int_{0}^{\infty}\left(I_{DC}(\hbar\omega/e)-\frac{\hbar\omega}{eR_n}\right)\sin\omega t\,d\omega \tag{1}$$

where e is the electron charge magnitude and $\hbar$ is Planck's constant scaled by 2π. The function, $I_{DC}(V)$, is the equation that describes the measured unpumped I-V curve and is assumed to be an odd function that tends toward the normal state conductance, $1/R_n$, at large bias voltages. Since $I_{DC}(V)$ is a function of voltage, it is evaluated at the corresponding photon voltages for each ω. The response function characterizes the quantum-mechanical features of the SIS junction; it is based entirely on the I-V curve and is independent of any applied voltage. The response function only needs to be calculated once and, noting the bracketed expression of (1), computation is seen to converge when $I_{DC}(V) \rightarrow V/R_n$.

Fig. 2 depicts the initial time segment of the response function that was calculated for the I-V curve data of Fig. 1b. The response function will oscillate at a frequency proportional to the *gap energy*, 2Δ. Depending on the quality of the junction, i.e., the sharper the nonlinearity of the I-V curve, the longer it will take for the response function to decay [10].

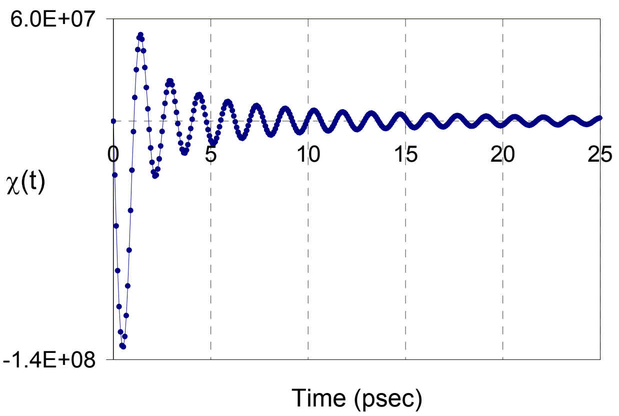


Fig. 2. The response function, χ(t), derived from the I-V curve data shown in Fig. 1b. The response function characterizes the nonlinearities and quantum-mechanical effects of the SIS junction in the time domain.

Consider the full time domain expression for the current across the junction for a given applied voltage, $V(t)$. The expected quasiparticle tunneling current is given as

$$\langle I(t)\rangle=\frac{V(t)}{R_n}+\mathrm{Im}\left(U^{*}(t)\int_{-\infty}^{t}\overline{\chi}(t-t')U(t')dt'\right). \tag{2}$$

where $U(t)$ is the phase factor and is given as

$$U(t)=e^{i\phi(t)} \tag{3}$$

and

$$\phi(t)=\frac{e}{\hbar}\int_{-\infty}^{t}V(t')dt'. \tag{4}$$

As pointed out in [10], the expected current of (2) shows an instantaneous response (the first term) and a delayed component due to the convolution. In this way, the current depends on the past history of the applied voltage. This manifests itself in the frequency domain in the form of a quantum susceptance [3].

Even though the time domain implementation can accept an arbitrary applied voltage, in order to provide continuity with the frequency domain approach, consider the large signal voltage defined as

$$V(t)=V_{DC}+V_{LO}\cos(\omega_{LO}t). \tag{5}$$

This waveform shows a DC bias voltage component combined with a local oscillator voltage signal of magnitude V_{LO}. It is assumed that $V(t)$ is the voltage that falls across the junction and, therefore, no LO source impedance has been taken into account.

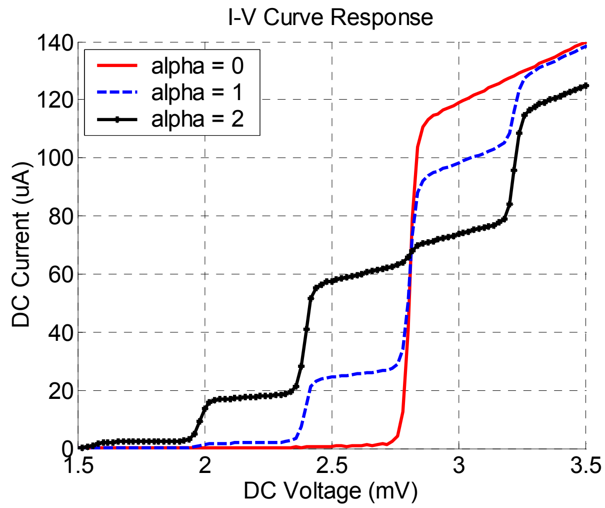
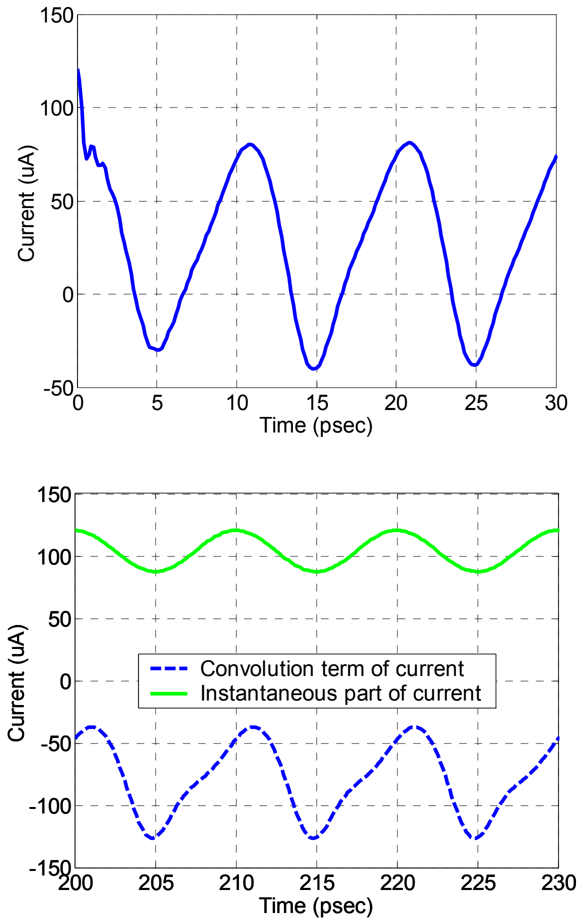


Fig. 3. Photon-assisted tunneling demonstrated with pumped I-V curves. Each curve was generated by extracting the DC component from the time domain response. No source impedance has been taken into account.

An interesting verification of the time domain equations is to demonstrate the quantum-mechanical effect of photon-assisted tunneling, as shown most clearly in the pumped I-V curve. The time-varying current was calculated at incremental DC bias voltages, from which the DC component of the current was then extracted through a Fourier transform. Fig. 3 shows the pumped curves that were computed for different pumping strengths, as defined in (9). Each step has an approximate width of 0.4 mV, which correctly corresponds to the photon voltage of the local oscillator (f_{LO} = 100 GHz).

Since, in practice, the junction is most often biased near the midpoint of the first photon step below the gap voltage, it serves well to look at the transient current for this condition. Fig. 4a shows the initial transient behavior and how, after several local oscillator periods, the signal will reach a steady state. The next figure displays a steady state sample of this same waveform separated into the individual components of (2): the instantaneous current and the current arising from the convolution term.



(a)

(b)

Fig. 4. (a) Transient response of the junction current when the DC bias voltage is centered on the first photon step and the LO is pumped at $\alpha = 1$. (b) Comparison of the two contributions of current once it has reached a steady state.

II. Accounting for the Source Impedance

To solve for the large signal waveform across the junction within an embedding network, the time domain equations have been used in the *voltage update method* (VUM) [4],[5]. Even using this method, however, the nonlinear diode current (computed using the time domain equations) is related to the rest of the circuit in the Fourier domain. A short review of this technique is given and then a variation is presented, allowing full computation of the current in the time domain.

A. Voltage Update Method (VUM)

During the large signal analysis of the quantum mixer theory, approximations are made with respect to the LO and its harmonics. For example, when using a three-port approximation, all harmonics are considered to be shorted out, and the LO is treated as a pure sine wave. As higher-order harmonics are included, the resulting LO waveform across the mixer becomes increasingly complex. A further complication is due to the dispersive nature of the LO source impedance so that higher-order harmonics see a different impedance than the fundamental.

Fig. 5 demonstrates how the VUM allows the user to specify the source impedance, $Z_{source}(\omega)$, at each harmonic frequency. The generator voltage, V_{gen}, is represented by a DC bias and a pure sinusoidal source at the LO frequency. The dotted line divides the nonlinear and linear parts of the circuit.

The correct voltage falling across the junction is determined through an iterative process designed to match the terminal voltage of the linear network, $V^{LIN}(t)$, to the nonlinear part, $V^{NL}(t)$. The procedure starts with an initial guess on the voltage, $V^{NL}(t)$, falling across the junction and the corresponding diode current is found. Using circuit theory, the linear network voltages at the interface are found by

$$V_n^{LIN} = Z_n^e I_n^{LIN} + V_{gen,n} \tag{6}$$

where $n = 0, 1, 2, \ldots, N$ denotes the index of the Fourier amplitude corresponding to the n$^{\text{th}}$ harmonic of the local oscillator, and $Z_n^e = \left(Z_{source,n} \| Z_{C,n}\right)$ is the equivalent source impedance that includes the junction capacitance. The Fourier amplitudes of the current are found through transformation.

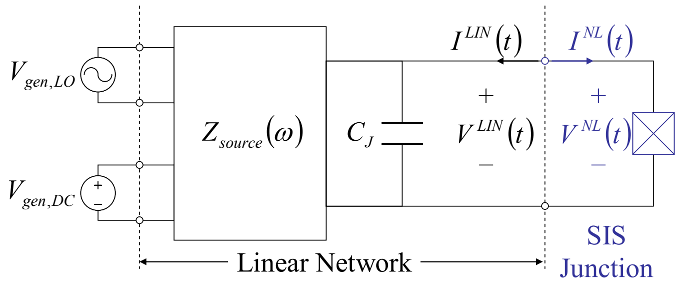


Fig. 5. Circuit describing the relation of the nonlinear diode current to the rest of the circuit in the voltage update algorithm [5]. The nonlinear current is transformed into the frequency domain and then linearly related to each harmonic of the LO.

Once the Fourier amplitudes, V_n^{LIN}, of the linear network voltages have been found, $V^{LIN}(t)$ is calculated through the inverse Fourier transform. If $V^{LIN}(t) = V^{NL}(t)$, then the junction voltage has been found. Otherwise, the voltage may be updated according to

$$V^{NL}(t) = p\,V^{LIN}(t) + (1-p)V^{NL}(t) \tag{7}$$

where p is a convergence parameter valid over the range of $0 < p < 1$. Convergence is better for the VUM when the terminating impedances approach a short-circuit, but may become a problem when the source impedance is high compared with the input impedance of the junction [5].

B. Variation: Time Domain Voltage Update Method (TDVUM)

Since the VUM computes the linear network voltages in the frequency domain, it is assumed that the voltage waveforms only contain harmonics of the LO and are in a steady state. If the VUM is modified so that all calculations are performed in the time domain, denoted as the *time domain voltage update method* (TDVUM) for comparison purposes, true transient behavior and arbitrary time signals can be analyzed [11]. To be compatible with time domain circuit and field solvers, the new algorithm must be able to evolve with each additional time step, yet still be accurate to the complexities of (2).

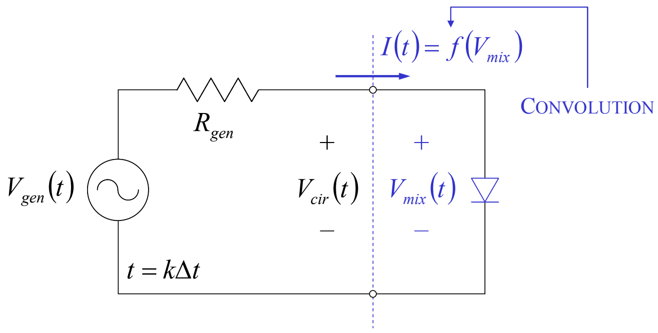


Fig. 6. Circuit describing the time domain voltage update method (TDVUM). The source impedance is represented here as a simple resistor, but may be a more complicated network. For each time step, Δt, the mixer voltage is iteratively matched to the circuit voltage at the terminal interface. However, since the quantum mixer current requires a convolution of previous voltage values, each iteration also requires a convolution. Note that V_{gen} includes the DC voltage bias.

Fig. 6 depicts a simplified circuit representation for the TDVUM such that the source impedance is given by a simple resistor. The generator voltage is assumed to include the DC bias. As with the VUM, the terminal voltages are matched through an iterative process. Given a specified generator voltage, an initial guess on the mixer voltage, V_{mix}, is made. The nonlinear diode current is calculated, and the circuit voltage is found by

$$V_{cir}(t)=V_{gen}(t)-I(t)\,R_{gen} \tag{8}$$

where $t=k\,\Delta t$ and is evaluated for the k^{th} time step. If the terminal voltages match, then the algorithm is complete, otherwise, V_{mix} is updated according to (7). However, this is not as simple as it appears, because the current, found by (2), requires a convolution operation that depends on the past history of the voltage across the junction. Therefore, not only is the convolution operation called during each time step, but also for numerous times within each iteration of that time step – creating a very demanding computational algorithm.

One simplification to the convolution procedure is that the storage of the entire voltage time sample is not required; only a time sample equal to the length of the response function, calculated by (1), is necessary. This, of course, assumes that the time sample of the response function is long enough so that it has converged (e.g., longer than what is shown in Fig. 2). Furthermore, the convolution within each k^{th} time step may be broken up into two parts: a term depending on the most recent k^{th} voltage guess and a term that makes use of the previous time samples already determined, denoted as the *k – 1 convolution term*. By re-using this latter term during the iteration process, the computational effort is greatly diminished.

Implementing these concepts, a MATLAB algorithm was programmed in the following steps:

- *Initialize parameters* – the I-V curve parameters and time step are determined and the response function is calculated. Then for each k^{th} time step:
- *Guess initial voltage* $V_{mix}=V_{gen}$.
- *Compute current* – the phase factor and current are calculated for the initial voltage guess and the $k-1$ convolution term is stored.
- *Perform iteration and voltage update* – the circuit of Fig. 6 is solved for each new test voltage. Each voltage guess requires re-calculating the k^{th} term of the phase factor. The current is calculated for the new test voltage and the $k-1$ convolution term is re-used each time. If $V_{cir}=V_{mix}$, then the correct voltage has been found, otherwise, the new test voltage is modified by (7).

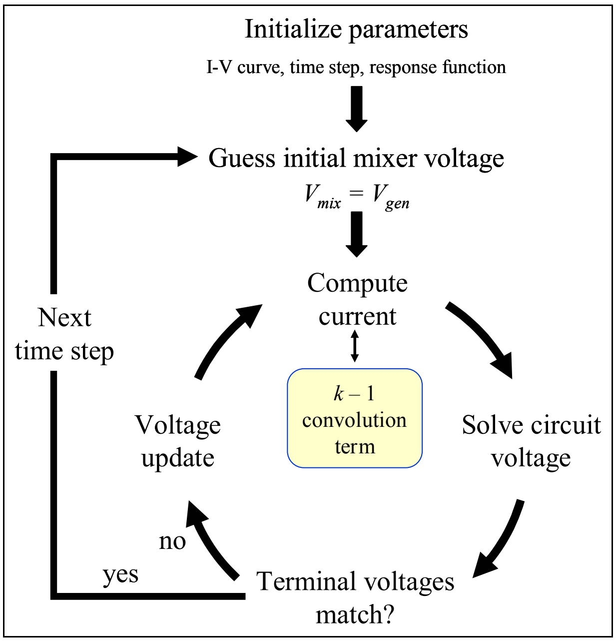


Fig. 7. Algorithm of the time domain variation of the Voltage Update Method.

As a consequence of working within the time domain, the source impedance of Fig. 6 must be implemented using real elements. The implication is that if a resistor is used, it is considered ideal and present for all frequencies including DC. In reality, mixer designs have DC bias networks, dispersive transmission line impedances and junction capacitance. More complex source networks, including DC bias elements, can be modeled, but the intention of this paper is to keep the demonstration simple.

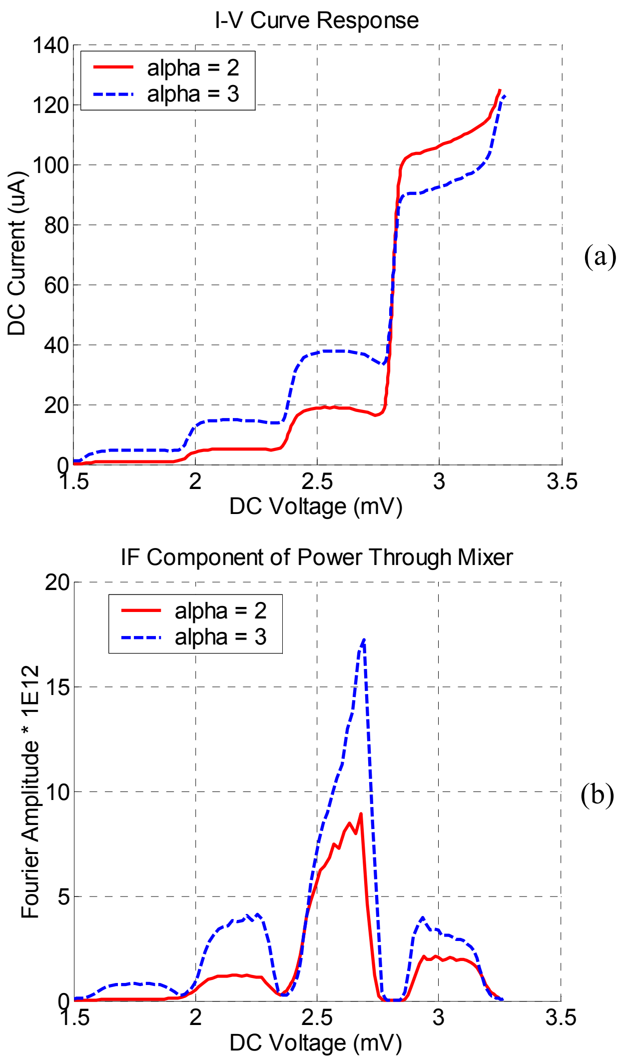


Fig. 8. (a) Pumped I-V curve response for two different source pumping strengths. The source impedance has been implemented as an ideal resistor of 10 Ω. (b) Two small signal tones, spaced at ±10 GHz from the LO and with amplitude of $hf_{LO} / 20e$, were combined with the LO across the junction. The Fourier amplitude of the IF component of the power through the junction was extracted and plotted with respect to the DC voltage bias.

Using the I-V curve of Fig. 1b, the optimum source resistance is found to be $R_{S,opt} \approx 10\ \Omega$ [12]. Fig. 8 demonstrates the responses for the simple source resistance shown by the circuit of Fig. 6. It is appropriate to define a *source* pumping parameter

$$\alpha_{source} = eV_{gen,AC} / \hbar\omega_{gen} \tag{9}$$

where the distinction is made from [3] in that the voltage, $V_{gen,AC}$, is the amplitude of the AC *generator* voltage, not the voltage established across the junction. By adding two small signal tones to the source, each of amplitude $hf_{LO} / 20e$ and set at ±10 GHz from the LO, the full spectrum of mixing products are available. Fig. 8b displays the IF component of the power through the mixer and shows the familiar peaks corresponding to each photon step, with the greatest amplitude at the first photon step below the gap voltage.

III. Implementation of TDVUM into 3D EM Field Solver (MEFiSTo)

After demonstrating the algorithm of the TDVUM, the groundwork has been laid for a full embedding into a circuit solver or field solver. MEFiSTo-3D Pro [13] is a full wave 3D electromagnetic field solver that is based on the *transmission line matrix* (TLM) method [14],[15]. An interconnection between SPICE [16] circuit models and the field has already been developed in MEFiSTo by means of representing the TLM network by an equivalent Thévenin or Norton source and impedance [17]. The transmission line impedance is found through an equivalent combination of input link lines. A similar technique was used to implement the SIS quantum theory via a TLM-MATLAB connection, such that the full time domain SIS mixer algorithm was implemented within the MATLAB element containing the TDVUM scripts written to calculate the SIS junction current.

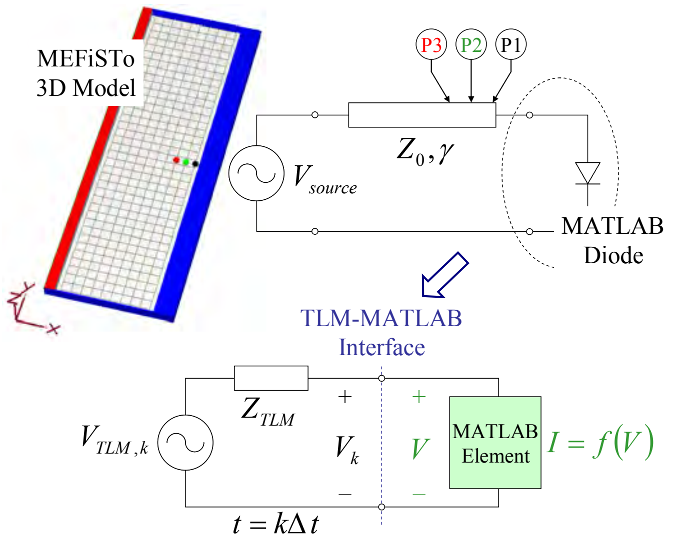


Fig. 9. MEFiSTo implementation of a parallel plate waveguide terminated with a MATLAB diode. The 3D model shows a distributed source connected to a short section of transmission line with three probes placed to monitor the voltage and current at each location. The lower figure describes the TLM-MATLAB interface where the distributed TLM mesh has been reduced to a Thévenin source and resistance .

Fig. 9 illustrates this concept whereby a short section of parallel plate waveguide is terminated by a shunt diode. The distributed field components at the TLM-MATLAB connection are reduced to a simple Thévenin source and resistance. Since the signals are within the time domain, the source impedance is real and the phase information is preserved within the signal itself. The result is that even though complex source and dispersive tuning networks surround the junction, they may be reduced at the junction terminal to the simple circuit shown in Fig. 6.

In cases where either the voltage or current tends to zero at the interface, extrapolation is necessary to obtain the true response. According to [17], the actual TLM-MATLAB connection is made halfway between the MATLAB cell boundary and the MATLAB node. Another note is that since the time step within MEFiSTo is dependent on the mesh resolution, it is important that the mesh be discretized uniformly in all directions and that the time step is kept the same within the MATLAB code.

A great advantage of embedding the SIS model into a time domain field solver is the ability to visualize the fields, including the transient and standing waves. Fig. 10 shows the field animation along a parallel plate waveguide designed to have a characteristic impedance of 10 Ω. The transmission line is terminated by a SIS MATLAB element and excited using a DC and LO voltage generator (which may be defined independently in MEFiSTo).

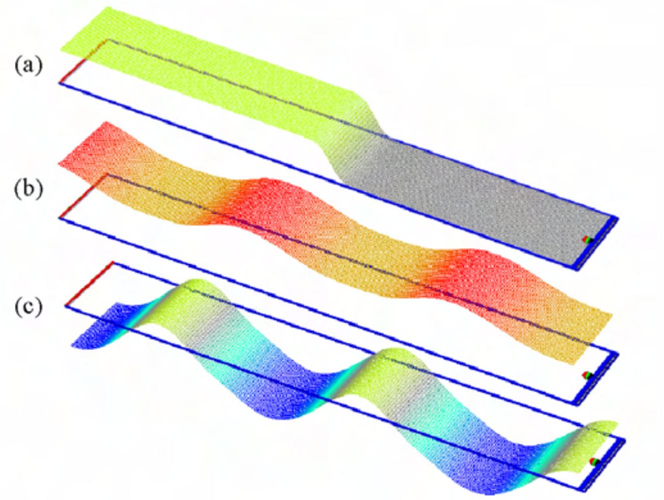


Fig. 10. MEFiSTo model of a parallel plate waveguide terminated with a SIS mixer diode. (a) Transient view capturing the DC bias voltage applied from the generator (left) toward the SIS junction (right). (b)-(c) Steady state view of the resulting voltage and current field components, respectively, due to an applied local oscillator.

IV. Conclusion and Future Work

The time domain theory of the quasiparticle tunnel junction has been reviewed and a new time-stepping algorithm, based on the voltage update method, has been demonstrated using MATLAB. The new time domain variation of the voltage update method (TDVUM), has immediate application in determining large signal voltage developed across the junction while accounting for the source impedance.

It has also been shown that the TDVUM algorithm is well suited for embedding within a time domain field solver where the fields may be reduced to a Thévenin or Norton equivalent at the junction terminals.

Future work would include implementing a 3-terminal, 2-port MATLAB element. In the above results, a 2-terminal, 1-port element was used to terminate a section of transmission line. Using the 2-port element, an output IF section can be modeled as it appears on the mixer chip. MEFiSTo already has this feature for SPICE elements. Further experimentation with complex geometries, including biasing networks, junction capacitance, and tuning elements must also be completed.

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