Characteristics Measurements of Supersensitive Direct Detector Receivers of Submillimeter Waveband Region Using Planck Radiation Source

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Abstract—**A blackbody radiating at the temperatures and frequencies corresponding to the relationship $T \approx \hbar\omega/k$ which is valid when the radiation power spectral density is described by the Planck formula, is considered. This blackbody is used to measure the spectral characteristics and noise equivalent power of low-temperature ($T \approx$ 0.3…0.1 K) direct detectors. The above relationship corresponds to the submillimeter waveband and temperatures ranging from several Kelvins to 40...50 K. It is suggested that the blackbody be placed in the cryostat cold area near a direct detector to prevent external thermal radiation which may overheat a low temperature refrigerator being employed. A measurement technique is developed which involves a Fredholm equation of the first kind and regularization methods.**

***Keywords*—*M*illimeter- and submillimeter-wave direct detectors, super low temperature detectors, super high sensitive radiation detectors, spectral characteristics measurements, noise equivalent power measurements.**

I. Introduction

During the investigation and development of supersensitive low temperature direct detector receivers of submillimeter waveband region a need of measurements of their spectral and sensitivity characteristics is arising. We interpret the receiver as the direct detector, for instance, bolometer, the planar matching antenna into which the direct detector is incorporated (coupled) at submillimeter waves together with input and blocking filters, resonant circuit etc. or without them as well as focusing lens, for instance, immersion lens In the process of choice a measurement method the following circumstances have to be taken into account. The first: said direct detectors have to work at very low temperatures (0.3 – 0.1 K) to achieve super high sensitivity. The submillimeter radiation feeding in from outside to a refrigerator with measurement purposes is complicated problem because it is difficult to filter ambient background thermal radiation which will overheat the refrigerator strongly. The latter could not cope with intensive heat influx. The second: the radiation power value feeding in to the receiver, for instance, from a spectrum analyzer, has to be below of higher level of receiver dynamic range, i.e. before the beginning of non-linear portion of its signal characteristic. For existing now super low temperature submillimeter direct detectors [1] this limit may be two-three orders higher than their noise equivalent power, i.e. $NEP \approx 10^{-18}$ W/Hz$^{1/2}$ or less.

With the purpose to overcome said difficulties we propose to mount the black body source in cold area of receiver cryogenic system, for instance, inside the precooling cryostat at temperature 4.2 K, or to another place at lower temperature. The black body source has to be equipped with the heater and thermometer to be heated from units to ~ 40…50 K as well as with not complicated quasioptical system forming collimated radiation beam directed to the receiver. Using formulae from [2] we obtain the expression for the radiation power spectral density of black body thermal radiation at frequency ω from area S_{rad} in spatial angle $\Delta\Omega$:

$$p(\omega,T) = S_{rad}\cdot\Delta\Omega\cdot\frac{k^3T^3}{4\pi^3c^2\hbar^2}\cdot\frac{x^3}{e^x-1}, \qquad (1)$$

where $x = \hbar\omega/kT$, $\hbar \cong 1.054\cdot10^{-34}$ J·s – Planck constant, $k \cong 1.38\cdot10^{-23}$ J/K – Boltzmann constant, and $c \cong 2.998\cdot10^{8}$ m/s – light speed in vacuum. The maximum of expression $y_\omega = x^3/(e^x-1)$ takes place at $x_m \cong 2.85$ and is $y_{\omega m} \cong 1.415$. The position of this maximum at frequency f_m is connected with temperature as

$$f_m = (kx_m/2\pi h)T \cong 0.594\cdot10^{11}\ T(\mathrm{K})\ \mathrm{Hz}, \qquad (2)$$

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what is one of forms of Wien shift law [2]. In accordance with (2) when the temperature changes from 4.2 to 50 K the frequency f_m moves from $f_m \cong$ 249.5 GHz ($\lambda \cong 1.2$ mm) at $T = 4.2$ K to $f_m \cong 2.97$ GHz ($\lambda \cong 0.1$ mm), i.e. the radiation power density maximum moves trough all submillimeter waveband range. One may estimate the radiation power value of said thermal radiation source in frequency band Δf in vicinity of frequency f_m keeping in mind that $P_{rad} \cong p_{\omega m} \cdot \Delta\omega = p_{\omega m} \cdot 2\pi\Delta f$. Substituting $p_{\omega m}$ from (1) at $y_\omega = y_{\omega m}$ we have

$$P_{rad} \cong 1.415 \cdot S_{rad} \cdot \Delta\Omega \cdot \Delta f \cdot (k^3 T^3 / 2\pi^2 c^2 \hbar^2).$$

For instance at S_{rad} = 0.1 cm^2, $\Delta\Omega \cong 10^{-2}$ sr and $\Delta f = 0.05 f_m$ Hz the calculation gives $P_{rad} \cong$ 1.75.10^{-14} and $P_{rad} \cong$ 2.44.10^{-10} W for 4.2 and 50 K respectively. These values, especially near to 50 K, could be higher than indicated above possible radiation power level. It can be reduced making S_{rad} and $\Delta\Omega$ smaller. Keeping in mind the variation of radiation power along whole submillimeter waveband region it is necessary to provide for several, at least two, black body source apertures.

The described circumstances give good possibility to measure the characteristics of supersensitive low temperature direct detector receivers of submillimeter waveband region using black body thermal radiation source with variable temperature from ~ 1 to ~ 50 K ($T \ll \hbar\omega / k$) and several apertures. We have named it by ***the Planck radiation source*** (PRS) in conformity with the characteristics measurements of said above receivers. This possibility and initial relations including the equation (3) (see below) were formulated in [3].

Below the methods of measurement of the spectral characteristics and the noise equivalent power (*NEP*) of said above receivers using the PRS are considered.

II. Spectral characteristics measurements.

Not losing the common formulation of the problem we consider the method of spectral characteristics measurements of submillimeter receivers using the PRS applying it for the direct detector based on the electron heating in transition edge sensor (TES bolometer) with Andreev electron reflection [1]. The output current (detected signal) $|\Delta I|$ of such direct detector when the radiation power P_{abs} absorbed is: $|\Delta I| = S_I \cdot P_{abs}$, where S_I is current responsivity of direct detector. For a direct detector receiver with spectral characteristic (transfer function) $k(\omega)$ one may obtain the receiver current response:

$$\left|\Delta I(T)\right| = S_I \cdot P_{abs}(T) = S_I \int_{\omega_{\min}}^{\omega_{\max}} k(\omega) \cdot p(\omega,T)\, d\omega, \quad (4)$$

where $\omega_{\min}$ and $\omega_{\max}$ are lower and higher limits of chosen frequency range for spectral characteristic measurement and $p(\omega,T)$ is determined by (1). We rewrite (4) in the next form

$$Ak(\omega) = \int_{\omega_{\min}}^{\omega_{\max}} p(\omega,T) \cdot k(\omega)\, d\omega = I(T),\ T_{\min} \le T \le T_{\max}, \quad (5)$$

where $T_{\min}$ and $T_{\max}$ are lower and higher limits of chosen temperature range of the PRS, for instance 4.2 K and 50 K, and $I(T) = \left|\Delta I(T)\right| / S_I$. The equation (5) is the Fredholm integral equation of first kind with non-precisely given right side (measurement errors, noise) [4, 5]. The solving methods of such equations for various applied problems using regularizing methods, for instance Tikhonov method, are well developed [4-6]. We have followed by well known approach [6] when the problem is reduced to the boundary problem for Euler equation:

$$\left.\begin{aligned}
&\int_{\omega_{\min}}^{\omega_{\max}} k(\omega) \cdot \overline{p}(\omega,T)\, d\omega + \alpha\{k(\omega) - qk''(\omega)\} = g(\omega),\\
&k(\omega_{\min}) = 0, \quad k(\omega_{\max}) = 0,\\
&\overline{p}(\omega,T) = \int_{T_{\min}}^{T_{\max}} p(t,\omega) \cdot p(t,T)\, dt,\\
&g(\omega) = \int_{T_{\min}}^{T_{\max}} p(t,\omega) \cdot I(t)\, dt.
\end{aligned}\right\} \quad (6)$$

In this case we have replaced the initial equation (5) with the equation with stabilizer $\alpha\left\{k(\omega) - qk''(\omega)\right\}$ which permits to find approximate solution but stable for small variations of measured data at some positive stabilizer parameters α and q. For calculations the equation (6) is replacing with its difference equivalent at the even grid with increment i besides the middles of segments are accepting as grid nodes, i.e.:

$$i = (\omega_{\max} - \omega_{\min}) / n,\ t_z = 0{,}5 \cdot i + (z-1) \cdot i, \quad (7)$$
$$z = 1, 2, ..., n.$$

As the result we obtain the difference equation:

$$\sum_{j=1}^{n} \overline{p}(\omega_j, T_z) \cdot i \cdot k(\omega_j) + \alpha \cdot k(\omega_z) +$$
$$+ \alpha \cdot q \cdot \frac{2 \cdot k(\omega_z) - k(\omega_{z-1}) - k(\omega_{z+1})}{i^2} = g(\omega_z). \quad (8)$$

The solution of this equation is reducing to the solution of linear equation set with amount of frequency and temperature sampling points usually from 32 to 512. Main difficulties during the solving of this problem are the large range of data variation and the large amount of variational parameters using for the stability providing and the solution precision improvement.

We have computer simulated the solution of problem of spectral characteristic $k(\omega)$ measurement of mentioned above direct detector receivers for various forms of $k(\omega)$. The set of transfer function was given analytically. They were substituted to (5) and $I(T)$ for T = 4.2 – 50 K were calculated for each of them. After that using calculated

$I(T)$ the reverse problem was solved by described above method using the equation (8). The obtained transfer functions were compared with initial ones as the result of what the conclusion on the method effectiveness was made. At Fig. 1 the example of results of simulation with the transfer function $k(f)$ in form of resonant circuit

$$k(f) = Q^2 / [Q^2(1 - f^2 / f_0^2)^2 + f^2 / f_0^2] \tag{9}$$

with resonance frequentcies f_0 = 150 GHz and f_0 = 2000 GHz and the quality Q = 20 are given. At first the search of a position and form of the direct detector response in wide frequency band was made (Fig.1,a,b) during the reverse problem solving. Then the frequency band was narrowed in accordance with determined approximate position and form and the procedure was repeated. This permitted to determine the position and the form.of response more precisely. The result of such repeated procedure for resonant transfer function for f = 2000 GHz is given at Fig. 1,c. The peak of reconstructed curve is ~ 20% less and width is ~ 20% larger in comparison with initial curve. Similar differences take place for another Q's: for instance ~ 10% for Q = 10 and ~ 5% for Q = 5. At the same time it was found that the relation $A[k(f)] \cong 1/Q$, where $A[k(f)]$ is the area under curve $k(f)$, takes place as for initial so for reconstructed transfer characteristics. Knowing the value of $A[k(f)]$ we may correct with sufficient precision the curve width and the peak height in necessary direction. The similar results in respect of peak heights and curve widths were obtained for the transfer functions in form of Gaussian, parabola and Π-shape curves what gives possibility to conclude that form of the transfer function is not so important. It was possible to reconstruct without difficulties the transfer functions in presence of up to five percent noise in the measured current as well as for double-

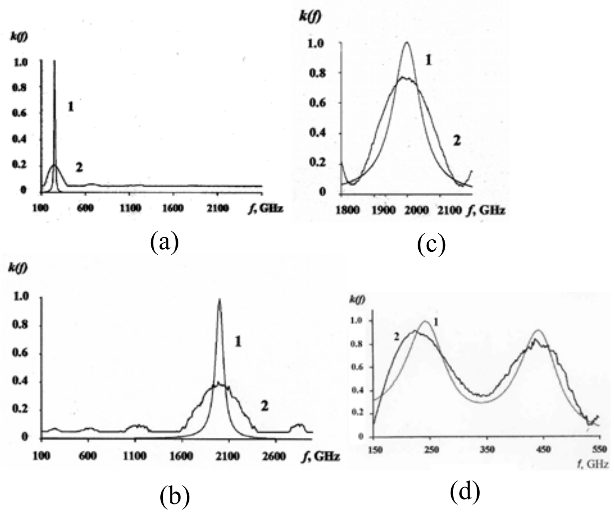

Fig. 1. The results of computer simulation with the spectral characteristic in form of resonant circuit with Q = 20: (a) and (b) – the search of position and form for f_0 = 150 GHz and f_0 = 2000 GHz respectively, (c) – the refining of the position and the form for f_0 = 2000 GHz; the same for double-resonance characteristic (d); **1** – initial and **2** – resulting spectral characteristic.

peak curve of two coupled resonant circuits. All these approve the normal work of measurements method using the PRS up to $Q \approx 100$. Measurement simulation result for double-resonance spectral characteristic is given at Fig. 2. One may see that PRS method has acceptable frequency resolution.

Acttual measurements of the receiver spectral characteristic should involve measurements of dependence $|\Delta I(T)|$ instead of $I(T)$ in (5). Then, spectral characteristic $k(\omega)$ in (5) must be replaced by the function $K(\omega) = S_I Lk(\omega)$. Here, factor L characterizes the receiver (including detector and matching elements) losses. It is assumed that these losses slightly depend on frequency only within the frequency range of the measured spectral characteristic of a receiver. Function $K(\omega)$ is a solution of (5) and can be represented in the form

$$AK(\omega) = \int_{\omega_{\min}}^{\omega_{\max}} p(\omega, T) K(\omega) d\omega = |\Delta I(T)|, \tag{10}$$

$$T_{\min} \le T \le T_{\max},$$

The amplitude of obtained function $K(\omega)$ is equal to $K(\omega)_{\max} = S_I L = S_{Iopt}$. This value corresponds to the optical current response of a direct detector. The dividing of $K(\omega)$ by $S_I L$ yields dimensionless spectral characteristic $k(\omega)$; the dividing of $K(\omega)_{\max}$ by S_I yields microwave (optical) loss L by a direct detection receiver.

III. *NEP* MEASUREMENTS

The standard method of receivers *NEP* measurements at microwave frequencies is the well known method of cold/warm loads (sources). This method is working at frequencies and temperatures when $x << 1$ ($\hbar\omega \ll kT$), i.e. when the relation (1) becomes

$$p(\omega, T) \cong S_{rad} \cdot \Delta\Omega \cdot (\omega^2 / 4\pi^3 c^2) \cdot kT$$

(Rayleigh-Jeans formula) what means that $p(\omega, T) \propto T$. At frequencies and temperatures when $\hbar\omega \sim kT$ there is no proportionality between $p(\omega, T)$ and T and the method of cold/warm loads is not valid. Instead of the using two temperatures of the radiation source, in our case of the PRS, we propose to use two apertures of this radiation source. Then the absorbed powers by the receiver will be proportional to S_{rad}. It is convenient to set such PRS temperature when the radiation frequency corresponding to the maximum of its spectral radiation density will coincide with the middle of of receiver frequency bandwidth. These data are known from the foregoing measurements of the receiver spectral characteristic. In this case the *NEP* measurement will consist of measurement of two output signals at two apertures and hash width at the receiver output. Then the hash width has to be expressed in thermal power units of the PRS.

When realizing the described measurement method using the PRS in conformity with definite direct detector receiver design the additional examination is needed. This is important, for instance, when the receiver design comprises planar antenna/antennas matching an incident radiation with the detector/detectors [1]. We consider, as an example, the radiation beam passing in optical camera of a possible imaging radiometer (Fig. 2) from the telescope output through lenses to antenna array with direct detectors (for

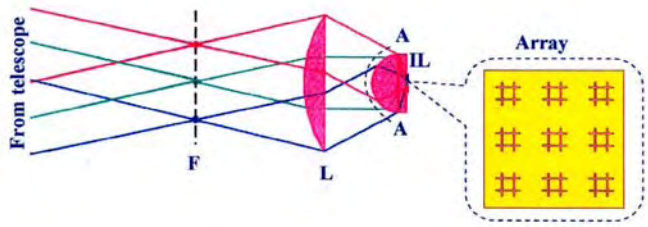

Fig. 2. Simplified scheme of a possible submillimeter imaging radiometer optical camera with 3×3 detector array: F is the telescope focal plane, L is the intermediate lens, IL is the immersion lens focusing radiation beam onto the antennas-detectors array.

instance, bolometers) coupled into antennas. The beam passing in vicinity of antenna array, moreover, from antennas to detectors of course can not be described in conception of geometrical optics: the examination in conception of wave optics is needed. It has to be done actually with the purpose to construct the optical camera with maximum possible radiation power transmission factor from the immersion lens input to detectors. Setting this problem apart we may formulate the problem of direct detector receiver characteristics measurement in conformity with considered case choosing the surface AA (Fig. 2) as an input of the receiver. Exactly in this place the spectral characteristic and noise equivalent power of receiver have practical importance. If so, whole presented above consideration has to be applied on the basis of conception that the surface AA (Fig. 2) is the receiver input. It is necessary to emphasize that spectral characteristics of receiver pixels are determined first of all by frequency characteristics of separate matching antennas with detectors. Losses in each pixel will worsen the receiver *NEP*. Both characteristics are determined in the measurement procedure as described above. We suppose that characteristics of each pixel have to be measured. Keeping in mind the optical camera scheme shown at Fig. 2 it is necessary to repeat that we have to create a precisely collimated radiation beam in the optical measurement scheme corresponding to the receiver array structure.

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