

# A Hot Spot Model for HEB Mixers Including Andreev Reflection

Harald Merkel  
Chalmers University of Technology, SE 412 96 Göteborg, Sweden

## Abstract

A device model for HEB mixers is described that takes two additional effects into account: Andreev reflection at the hot spot boundaries and critical current variations on the bridge. This model is capable to predict IV curves even in the unstable areas with acceptable accuracy. Based on these large signal results a more accurate small signal expansion has been developed: In the framework of this model heating due to a small signal current change acts differently from a small signal voltage change at IF. The small signal model allows accurate predictions of the conversion gain and the mixer noise including thermal fluctuation, Johnson and quantum noise.

## Introduction

Hot spot models for HEB mixers have been proposed in recent years resulting in a substantial improvement in HEB modelling and understanding of the device physics. Such models require the solution of an one-dimensional heat balance. The occurrence of a DC resistance and the device's mixing capabilities are explained by the formation of a hot spot, i.e. a normal conducting zone wherever the quasiparticles exceed the critical temperature. Depending on the applied heating powers a certain temperature profile is obtained on the HEB bridge resulting in a certain hot spot length. Applying a superposition of a strong LO source and a weak RF signal results in a time-averaged RF heating and in a small beating term oscillating at the difference frequency (IF). The latter causes a tiny change of the hot spot length. This yields a small resistance change at IF which creates small signal currents and voltages through the bridge and finally gives rise to conversion gain of the HEB. Unfortunately none of these hot spot models is capable to predict gain and noise simultaneously with acceptable accuracy without introducing additional empirical parameters or by requiring parameter values being in conflict with experimental results. A popular empirical parameter is the local resistive transition width [1] assuming that the film smoothly turns normal around  $T_c$ . This reduction of the resistance slope "helps" to fit the conversion gain but still too much heating power is predicted. Required values for this transition are about 800mK whereas experiments reveal some 50mK. For Nb the case is even worse [2]. There are strong indications that some physical effects are not covered by a simple hot spot based device model. In this paper two additional effects are discussed in order to explain at least part of the discrepancies. These additional effects are due to critical currents and due to Andreev reflection. Throughout this model, **strong localization** is assumed. This applies to quasiparticle and phonon temperatures, the critical current and the quasiparticle bandgap.

## **Critical current effects on the HEB bridge**

In previous models a normal zone is formed wherever the quasiparticle temperature exceeds the critical temperature. This holds only for zero bias current. Otherwise the

normal zone is created wherever the bias current density exceeds the critical current density which is the case at a lower temperature. In the framework of this model, a reduced critical temperature is defined which corresponds to a critical current density equal to the current density caused by the bias current.

### Andreev reflection

In simple hot spot models the HEB bridge is assigned a temperature-depending lateral thermal conductivity of exponential or polynomial form [1],[2]. Due to Andreev reflection at the boundary between the hot spot and the superconducting rest of the HEB bridge, only electrons which energy is large enough to overcome the quasiparticle bandgap participate in heat transport. Andreev reflection provides good thermal insulation of the hot spot. As a direct consequence the electron temperature within the hot spot is now more or less constant. The electron temperature profile and the resulting bandgap distribution is summarized in Figure 1:

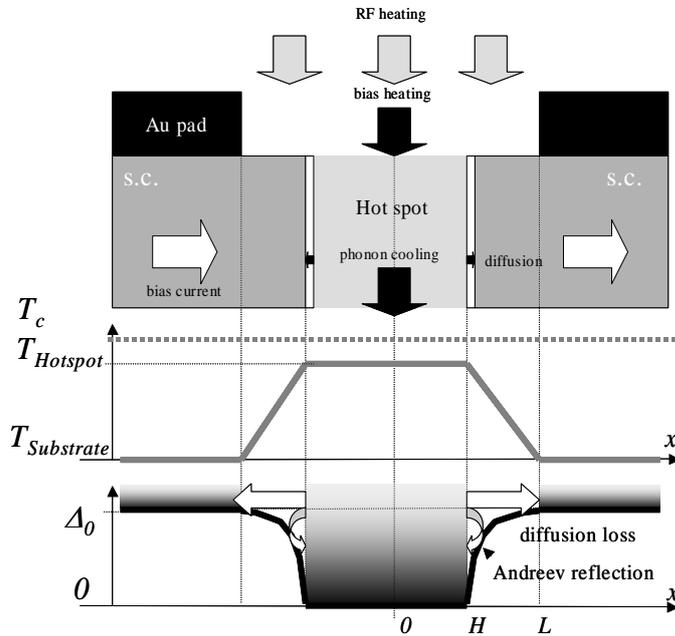


Figure 1: Schematic of a HEB bridge. The whole bridge is heated by RF, the bias heating acts only on the hot spot where superconductivity is suppressed. The electrons are cooled by phonon escape to the substrate and by outdiffusion to the pads. Outdiffusion is reduced by Andreev reflection at the hot spot boundary.

### Model assumptions

The HEB device model presented here is based on a set of assumptions. All parameters used here are summarized in Table 1 at the end of the paper including typical values for the calculations presented here:

#### 1: Localization and immediate thermalization

The correlation length is of the order of the film thickness. All superconducting parameters are localized. The film properties in vertical direction are homogenous. Besides that one assumes instantaneous thermalization of the heating powers by electron-electron interaction. Then electrons and phonons are described by effective electron and phonon temperatures.

## 2: Heating by superposition

The HEB is heated by LO power being the linear superposition of a local oscillator (LO) signal and a weak RF source. This superposition results in a power deposited in the HEB bridge at the intermediate frequency (IF) of the form:

$$P_{IF} \propto 2\sqrt{P_{LO}P_S}$$

In time average the HEB bridge is heated by the mean value of the LO power and DC power.

## 3: A model for the critical current density on the HEB bridge

Operating a HEB as a mixer requires a substantial bias current to be carried by the HEB bridge. Therefore the superconductivity on the HEB bridge is suppressed wherever the local critical current is exceeded. The theoretical temperature dependence of the local critical current density  $j_c(T,x)$  is given by Ginzburg-Landau theory [3]. Performing a nonlinear best fit a simplified and more convenient relation is obtained (the parameters are explained in Table 1):

$$j_c(T,x) = j_c(0) \cdot \left[ 1 - \frac{T(x)}{T_c} \right]^\gamma \quad (1)$$

Here  $\gamma$  denotes a best fit coefficient set to 0.408. For  $T_c$  ranging from 8.5K to 11.5K this yields a more accurate model than the “usual” setting [3] of  $\gamma=1.5$  for low temperatures and 0.5 for large temperatures. Solving (1) for the quasiparticle temperature, a “reduced” critical temperature is obtained for voltage bias:

$$T_{c,eff,V}(V_0, x_0) = T_c \cdot \left[ \left( \frac{V_0}{j_c(0) \cdot A \cdot R_N \cdot \frac{x_0}{L}} \right)^{\frac{1}{\gamma}} - 1 \right] \leq T_c \quad (2)$$

The results for the reduced critical temperature for voltage bias for a voltage of  $V_0=0.8\text{mV}$  (1.0mV and 1.2mV) are summarized in Figure 2.

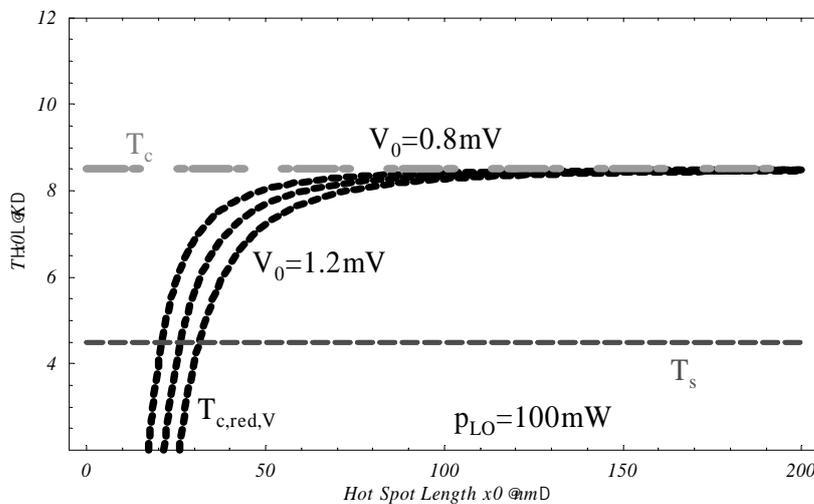


Figure 2: Reduced critical temperature for a bias voltage of  $V_0=0.8\text{mV}$  (1.0mV and 1.2mV , the black dotted curves) with a substrate temperature of 4.2K and a critical temperature of 8.5K.

#### 4: Almost perfect Andreev reflection at the hot-spot boundaries

A hot spot is formed wherever the bias current exceeds the critical current. The remaining parts of the bridge are in perfect Meissner state. The hot spot is heated by the absorbed bias power and the uniformly absorbed LO power. The quasiparticles in the hot spot are cooled by electron-phonon interaction and the heat is removed from the film by phonon escape. At the NS interfaces Andreev reflection [4] determines the amount of heat being able to leave the hot spot by diffusion. In the ideal case (perfect Andreev reflection) no heat transfer will occur across the interface and the whole cooling power is carried by the phonon path. In reality only the fraction of “normal” electrons with the energy of the quasiparticle levels in the superconductor will be able to carry heat across the NS interface. As a first order approximation we neglect the fact that the quasiparticle bandgap opens slowly on a length given by the thermal healing length and assume the bandgap to be its coldest value reached at substrate temperature immediately. This is an acceptable assumption since we are only interested in the net heat loss of the hot spot to the antenna pads – at some point in the superconductor all electrons with energies smaller than the local bandgap will be reflected and only those being able to overcome the highest bandgap will remove heat laterally from the hot spot. The fraction of electrons transporting heat  $\alpha$  across the hot spot boundary is estimated using a Fermi-Dirac distribution function for the electron density  $n_E(E)$  [5]:

$$\alpha = \frac{\int_{\Delta}^{\infty} n_E(E) dE}{\int_0^{\infty} n_E(E) dE} = \frac{kT \ln \left( 1 + e^{\frac{\Delta(T)}{kT}} \right) - \Delta(T)}{kT \ln 2}, \quad \Delta(T) = \Delta_0 \left[ 1 - \frac{T}{T_c} \right]^{\gamma} \quad (3)$$

Calculations show that typical values of the Andreev transmission for critical temperatures around 10K and pad temperatures of the order 5K are in the range 1% to 10% providing good thermal isolation of the hot spot. It is important to note, that this model assumes the antenna pads of the HEB to be in perfect Meissner state. Using this model for HEB configurations with normal conducting antenna pads, the maximum value of the quasiparticle bandgap needs to be changed appropriately. In the next section, the hot spot size is calculated based on the previous model assumption by approximating the solution of an one-dimensional heat transport equation.

#### Solving for the temperature and the size of the hot spot

The quasiparticle temperature on a hot spot of given (but yet unknown) length  $2x_0$  is determined by an equilibrium between electron-phonon cooling  $P_P$ , RF and bias heating and cooling due to net outdiffusion through the NS interfaces  $P_D$ . One obtains then [6]:

$$P_{LO} + \frac{V_0^2}{R_N \frac{x_0}{L}} = P_P + P_D = \frac{2 \cdot x_0 \cdot D \cdot W}{L} \quad (4)$$

The net heat loss due to heat conduction from a hot spot with temperature  $T_{c,eff,V}$  is determined by the gradient of the quasiparticle temperature between the hot spot and the antenna pads:

$$P_D = 2\alpha \cdot \lambda \cdot \frac{A}{V} \cdot \frac{T_s + T_{c,eff,V}}{(x_0 - L)} = 2\alpha \cdot \lambda \cdot \frac{T_s + T_c \left[ \left( \frac{L \cdot V_0}{j_c AR_N x_0} \right)^{\frac{1}{\gamma}} - 1 \right]}{x_0 \cdot (x_0 - L)} \quad (5)$$

For the electron-phonon interaction one is left with the “usual” expression [6]:

$$P_P = \sigma_E (T^n - T_p^n) \quad (6)$$

The power being transferred to the phonons heats the film phonons that is cooled by phonon escape to the substrate. The heat transport by phonons in direction of the film is neglected. Then a heat balance for the phonons becomes:

$$\sigma_E (T^n - T_p^n) = \sigma_P (T_p^m - T_s^m) \approx \delta \cdot \sigma_E (T_p^n - T_s^n) \quad (7)$$

Inserting (7) in (6) and subsequently in (5) and (4) the temperature of a hot spot  $T$  for a given length  $x_0$  is obtained by:

$$\frac{\delta \cdot \sigma_E}{1 + \delta} (T^n - T_s^n) + 2\alpha \cdot \frac{T_s + T_c \left[ \left( \frac{L \cdot V_0}{j_c AR_N x_0} \right)^{\frac{1}{\gamma}} - 1 \right]}{x_0 \cdot (x_0 - L)} = \frac{P_{LO} + \frac{V_0^2}{R_N \frac{x_0}{L}}}{2 \cdot x_0 \cdot D \cdot W} \quad (8)$$

For (8) a closed form analytical solution is available. A typical result for the quasiparticle temperature is shown in Figure 3. Note that with perfect Andreev reflection (i.e. no diffusion losses) the hot spot will violate the boundary condition under the antenna pads.

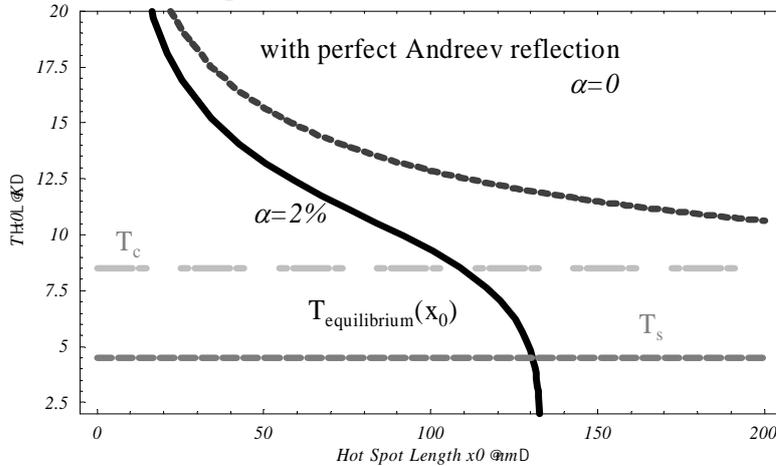


Figure 3: Equilibrium temperature of a hotspot with variable length (for a device length of  $2 \times 200\text{nm}$ ) with 2% Andreev transmission (solid black line) and perfect Andreev reflection (dotted dark gray line) together with the substrate temperature  $T_s$  and the critical temperature  $T_c$ .

Obviously the temperature in the hot spot (and therefore also at the end of the hot spot) must be equal to the quasiparticle temperature required to break superconductivity for the given bias current. From this the hot spot length is calculated. A graphical solution for a single operating point is shown in the following Figure:

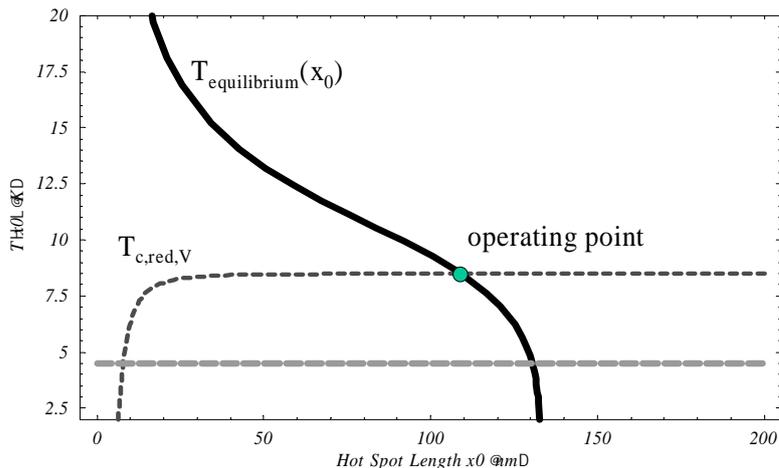


Figure 4: Solving for a hot spot length at 0.3mV bias voltage and 100nW LO power.  $T_c$  is 8.5K and the substrate is at 4.2K. The device length is 400nm, its width is 4 $\mu$ m and the thickness is 50 $\text{\AA}$ .

Figure 5 shows a comparison between theory (thin and partially dotted curves) and experiment (thick curves) for various LO powers. The topmost curve is obtained for no RF heating at all and the lowest one for 300nW. The curves in between are obtained in steps of 25nW. For the measured curves, the topmost is obtained at about 25nW LO power and the lowest at about 300nW.

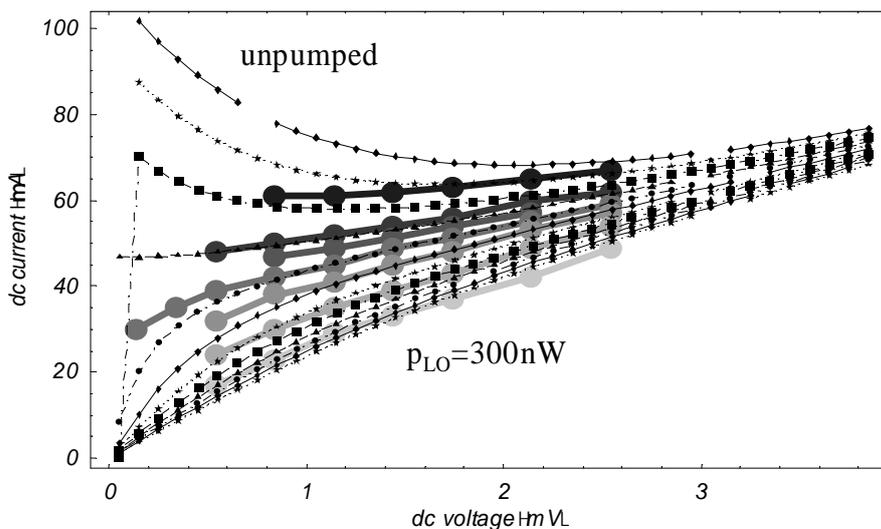


Figure 5: Measurement and theoretical results for the IV curve for a NbN HEB on MgO with the dimensions 400nm x 4 $\mu$ m x 55 $\text{\AA}$  measured at 2.5THz. The measured points are connected by thick gray lines. The black curve is an unpumped curve, the light gray is pumped with about 300nW LO power according to a standard isothermal method applied far away from the optimum point for large bias voltages. The calculated values are obtained for 0nW up to 300nW LO power with a step size of 25nW.

### Small signal model

The HEB mixer topology is shown in Fig. 6. It is similar to the topology from [7] where the biasing resistor has been replaced by a large inductance serving as RF coil:

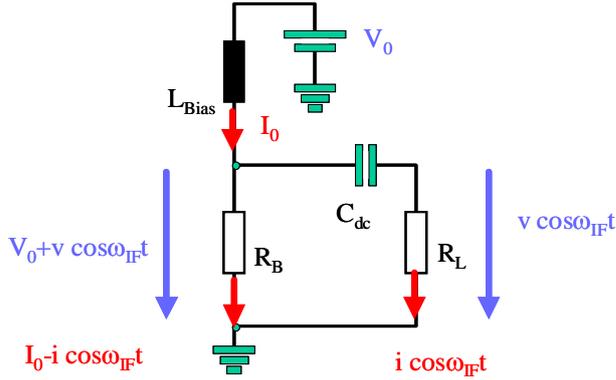


Figure 6: Mixer topology for a typical HEB application – the inductance ensures proper voltage biasing for choosing a proper DC operating point. For IF signals, the inductance poses an open circuit and the IF signal is coupled to the load resistance by a DC block capacitor.  $\omega_{IF}$  denotes the intermediate frequency i.e. the difference frequency between the LO and the RF signal.

In this model, the large signal relations behave differently in current and voltage. This is contrasted by older models [1],[7],[8] where only heating powers are considered. Let us assume that the bolometer resistance given by the hot spot length depends on LO heating power, bias current and bias voltage. Then the small signal resistance change in the bolometer  $r_B$  is modelled by:

$$r_B = C_{rf} \sqrt{P_{LO} \cdot P_S} + C_V v \cdot I_0 - C_I i \cdot V_0 \quad (9)$$

From this, the power in the load resistance can be calculated and one obtains for the conversion gain:

$$G = \frac{P_L}{P_S} = \frac{2 \cdot I_0^2}{(R_B + R_L)^2} \cdot \frac{C_{rf}^2 \cdot P_{LO}}{\left[ 1 - I_0^2 \cdot \frac{C_I R_B - C_V R_L}{R_B + R_L} \right]^2} \quad (10)$$

Values for the conversion loss of a HEB and comparison with measurements are indicated in Figure 7.

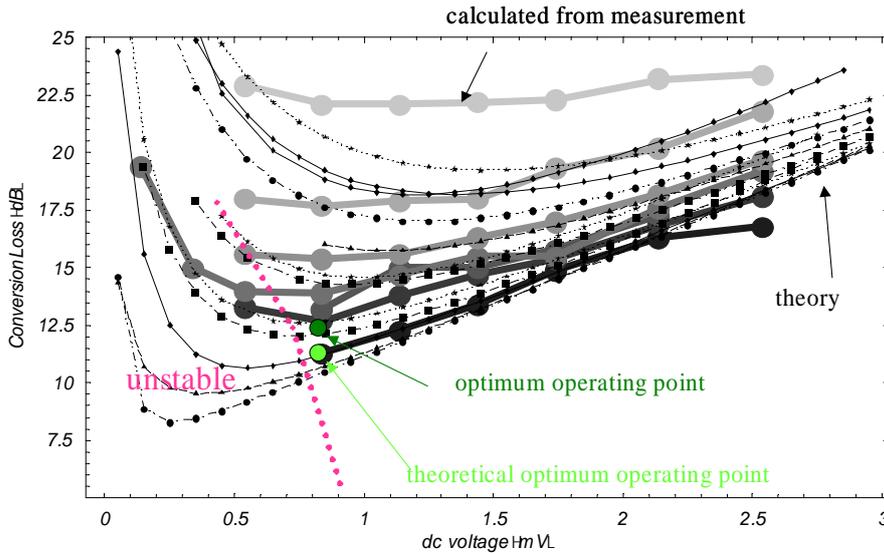


Figure 7: Calculated intrinsic conversion gain based on measurements (thick gray curves) and theoretical results (dashed and dotted curves) for the intrinsic conversion loss for a NbN HEB on MgO with the dimensions  $400\text{nm} \times 4\mu\text{m} \times 55\text{\AA}$  measured at  $0.6\text{THz}$ . The measurement data have been obtained at an IF frequency of  $1.5\text{GHz}$ .

## HEB Noise

Noise in the HEB is caused by Johnson noise since the hot spot forms a resistor with a certain temperature  $T_{Hotspot}$ . The noise contribution at the mixer output  $T_J^{out}$  is given by [9],[10]:

$$T_J^{out} = \frac{4 \cdot R_L \cdot R_B \cdot T_{Hotspot}}{(R_B + R_L)^2} \cdot \frac{1}{1 - I_0^2 \frac{C_V R_L - C_I R_B}{R_B + R_L}} \quad (11)$$

Any system of a given temperature with a given thermal coupling to a cold reservoir and a certain volume exhibits thermal fluctuations [9] resulting in noise. For a hot spot this results in [10]:

$$T_{TF}^{out} = \frac{I_0^2 R_L}{(R_B + R_L)^2} \cdot \frac{1}{1 - I_0^2 \frac{C_V R_L - C_I R_B}{R_B + R_L}} \cdot \left[ \frac{\partial R}{\partial T} \Big|_{T=T_{hotspot}} \right]^2 \cdot \frac{4T_{hotspot}^2 \tau_{e,relax}}{C_e \cdot V \cdot \frac{x_0}{L}} \quad (12)$$

A third noise contribution is caused by quantum noise [11],[12]:

$$T_Q^{out} = 2G \cdot \left( L_{optics} L_{optics4K} \left[ \frac{R_B}{R_S} + 1 \right] - 1 \right) \cdot \frac{h\nu}{2k} \quad (13)$$

Adding up all the contributions and transferring them to the mixer input, the DSB input noise temperature  $T_{in}$  is obtained [11]:

$$T_{in} = L_{optics} \frac{T_{TF}^{out} + T_J^{out} + T_{IF} + T_Q^{out}}{2G} + (L_{optics} - 1) \frac{h\nu \cdot B}{e^{\frac{h\nu}{kT_{optics}}} - 1} \quad (14)$$

Results for the DSB receiver noise temperature are summarized below:

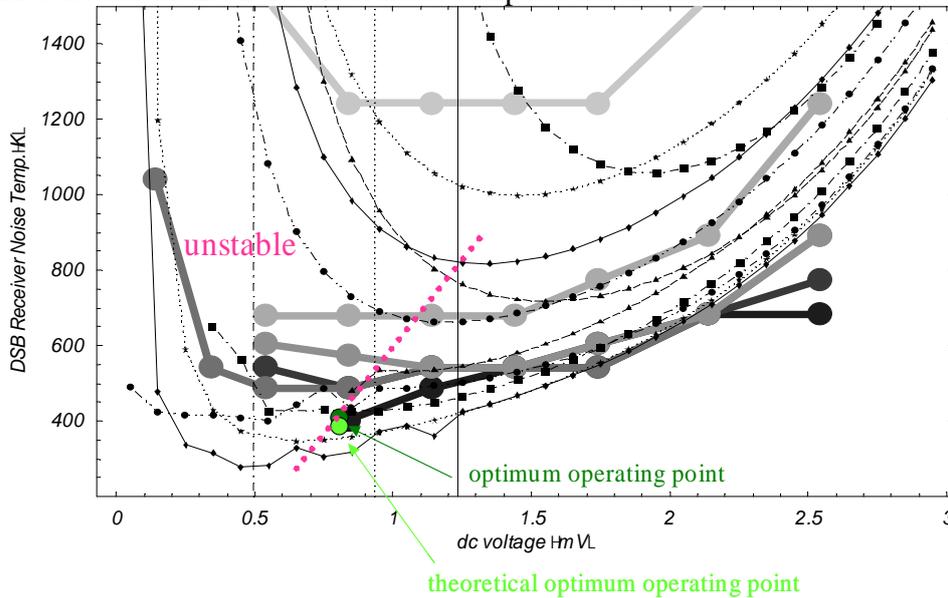


Figure 8: Measured receiver noise (thick gray curves) and theoretical results (dashed and dotted lines) for the DSB receiver noise temperature for a NbN HEB on MgO with the dimensions 400nm x 4µm x 55Å measured at 0.6THz.

The noise measured at the IF output of the HEB is a collection of the warm load at the input  $T_{lab}$  of the HEB collected in both sidebands, the fluctuation, Johnson and quantum

noise contributions and the contribution of the optics losses at a given temperature of the optics:

$$T_{out} = T_{lab} \cdot 2G + T_{TF}^{out} + T_J^{out} + T_Q^{out} + (L_{optics} - 1) \frac{h\nu \cdot B}{e^{kT_{optics}} - 1} \cdot 2G \quad (15)$$

Results for the output noise temperature is summarized below for the same device as in Fig. 5.

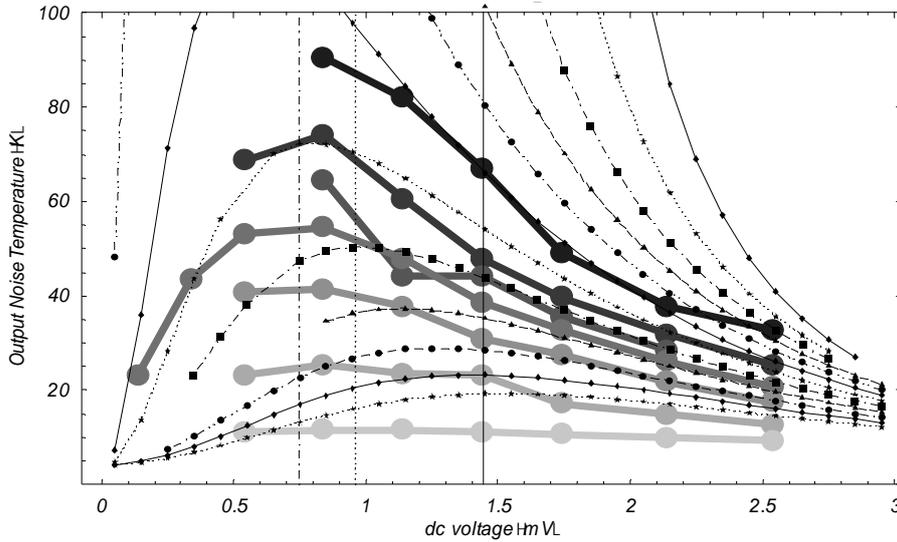


Figure 9: Measurement (thick, gray curves) and theoretical results (dashed and dotted lines) for the output noise temperature for a NbN HEB on MgO with the dimensions  $400\text{nm} \times 4\mu\text{m} \times 55\text{\AA}$  measured at  $0.6\text{THz}$ .

## Conclusion

A HEB model including critical current effects and Andreev reflection at the hot spot ends together with a small signal model where heating due to RF, IF currents and voltages is treated differently improves the quality of performance predictions by HEB device models substantially. In addition complete relations for the quantum noise in HEB receivers are now available (c.f. the contribution of K.S.Yngvesson and E.L.Kollberg in this issue) and have already been introduced in the model presented here. The model describes IV curves with satisfying accuracy and yields reasonable conversion gain and noise figures within the accuracy of the measurements. The device model has been successfully tested on various NbN HEBs with different geometries ranging from  $120\text{nm} \times 1\mu\text{m}$  to  $400\text{nm} \times 4\mu\text{m}$ . More tests on NbN on a larger geometry range and HEBs based on other materials have to be done to yield conclusive results about the overall model quality.

## Model Parameters

Parameter	Description	Value used for model calculation
$A$	Film cross section	$A = D \cdot W$
$\alpha$	Transmissivity of the s-n interface due to leaky Andreev reflection	$\alpha \approx 0.02$ typical
$B$	RF antenna bandwidth	$B = 500\text{GHz}$
$C_E$	Electron thermal capacity	$C_E = 1600 \frac{W_s}{m^3 K}$ at $T_c$

$C_I$	HEB resistance change due to a small current change at IF	$C_I = \frac{R_N}{2L \cdot V_0} \cdot \frac{\partial x_0}{\partial i} \Big _{P_{LO}=\text{const}, v=\text{const}}$
$C_{rf}$	HEB resistance change due to a small change of the RF heating power at IF	$C_{rf} = \frac{R_N}{2L} \cdot \frac{\partial x_0}{\partial P_{LO}} \Big _{v=\text{const}, i=\text{const}}$
$C_V$	HEB resistance change due to a small voltage change at IF	$C_V = \frac{R_N}{2L \cdot I_0} \cdot \frac{\partial x_0}{\partial v} \Big _{P_{LO}=\text{const}, i=\text{const}}$
$D$	Film thickness	$D = 35 \text{ \AA}$
$\delta$	Phonon to electron efficiency ratio	$\delta \approx 0.2$
$\Delta(T)$	Quasiparticle bandgap as a function of quasiparticle temperature	
$\Delta_0$	Quasiparticle bandgap extrapolated to 0K	$\Delta_0 = 800 \text{ GHz}$
$G$	Conversion gain of the HEB	
$\gamma$	Exponent in the temperature dependence of the critical current, obtained by best fit to Ginzburg-Landau expression	$\gamma = 0.408$
$h$	Planck's constant	
$I_0$	Bias current (DC) across the HEB bridge	$I_0 = \frac{V_0}{R_B}$
$i$	Small signal current (IF) across the HEB bridge	
$j_c(T, x)$	Local critical current density, function of quasiparticle temperature	
$j_c(0)$	Maximum critical current density at 0K, related to maximum critical current by division by bridge cross section	$j_c = \frac{I_c}{D \cdot W} \quad I_c = 160 \text{ \mu A}$
$k$	Boltzmann's constant	
$\lambda$	Lateral thermal conductivity	$\lambda = 1 \frac{W}{Km}$
$\lambda_{eff}$	Lateral effective thermal conductivity across the s-n boundary	
$2 \cdot L$	HEB bridge length (Length between the pads, contact zone under the pads not taken into account)	$L = 200 \text{ nm}$
$L_{optics}$	Loss of the optics at room temperature	$L_{optics} = 1.3$
$L_{optics4K}$	Loss of the optics at cryogenic (substrate) temperature	$L_{optics4K} = 1.7$
$m$	Exponent for the temperature dependence of phonon escape	$m = 4.0$
$n$	Exponent for the temperature dependence of electron-phonon interaction	$n = 3.6$
$\nu$	RF frequency	$\nu = 1600 \text{ GHz}$
$P_D$	Power leaving the hot spot by electron diffusion	
$P_{IF}$	Power absorbed by the HEB at the intermediate frequency	
$P_L$	Power delivered to the load at IF	
$P_{LO}$	Local oscillator power absorbed by the HEB	variable
$P_P$	Power leaving the hot spot by phonon cooling	
$P_S$	RF signal power absorbed by the HEB	

$r_B$	Small signal HEB resistance change due to IF beating	
$R_B$	Resistance of the HEB in the operating point	
$R_L$	Load resistance	$R_L = 50\Omega$
$R_N$	Normal resistance of the HEB bridge (at 20K)	$R_N = 65\Omega$
$R_S$	Antenna impedance (Real part)	$R_S = 100\Omega$
$\sigma_E$	Electron-phonon cooling efficiency	$\sigma_E = \frac{C_e}{3.6T^{2.6}\tau_{e \rightarrow p}}$
$\sigma_P$	Phonon escape efficiency	$\sigma_P \approx \delta\sigma_E$
$\tau_{e \rightarrow p}$	Electron- phonon interaction time constant	$\frac{1}{\tau_{e \rightarrow p}} = 8GHz$
$\tau_{e,relax}$	Electron energy relaxation time constant	$\frac{1}{\tau_{e,relax}} = 5GHz$
$T(x), T$	Local quasiparticle temperature	
$T_{equilibrium}$	Hypothetic hot spot temperature, where heating and cooling powers are equal	
$T_c$	Critical temperature of the HEB bridge	$T_c = 8.5K$
$T_{c,effI}$	Reduced critical temperature due to critical current effects under current bias conditions	
$T_{c,effV}$	Reduced critical temperature due to critical current effects under voltage bias conditions	
$T_{hotspot}$	Hot spot temperature in a given operating point	
$T_{IF}$	Noise temperature contribution of the IF amplifier	$T_{IF} = 7K$
$T_{in}$	Noise temperature at the input of the receiver, DSB receiver noise temperature	
$T_J^{out}$	Noise temperature at the output of the mixer due to thermal (Johnson/ Nyquist) noise	
$T_{lab}$	Room temperature in the surrounding laboratory	$T_{lab} = 292K$
$T_p$	Temperature of the phonons in the hot spot	
$T_Q^{out}$	Noise temperature at the output of the mixer due to Quantum noise	
$T_s$	Substrate temperature under the HEB bridge	$T_s = 4.5K$
$T_{TF}^{out}$	Noise temperature at the output of the mixer due to Thermal Fluctuation noise	
$V$	HEB bridge volume	$V = D \cdot W \cdot L$
$v$	Small signal voltage (IF) across the HEB bridge	
$V_0$	Bias voltage (DC) across the HEB bridge	variable
$2 \cdot x_0$	Length of the hot spot	
$W$	Film width	$W = 4\mu m$

Table 1: List of used parameters with their abbreviations and model values for the calculation presented in the paper

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