IF Impedance and Mixer Gain of Phonon Cooled Hot-Electron Bolometers and the Perrin-Vanneste Two Temperature Model

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Abstract—
We have measured the IF impedance and mixer gain bandwidth of a small area phonon cooled hot-electron bolometer in the 0.05-10 GHz range under a variety of bias and LO pumping level conditions. The device used is a twin slot antenna coupled NbN HEB mixer, with a bridge area of 1 μm x 0.15 μm, and a critical temperature of 8.3 K.

We model the HEB IF impedance following the work of Nebosis, Semenov, Gousev, and Renk, from now on refered to as the "NSGR" model. This model is based on the two-temperature model of Perrin-Vanneste, and is found to accurately describe the measured data at different bias and LO pump levels. Three fitting parameters are used in the NSGR model to describe the frequency dependent modulation of the electron temperature; the electron-phonon interaction time τ_{eph}, the phonon escape time (into the substrate) τ_{es}, and the ratio of the electron and phonon specific heat capacity c_e/c_{ph}. The described time constants and temperature dependent heat capacity ratio obtained from fitting the model to the measured data agrees well with those reported on for thin NbN films in literature.

Using an expression for mixer gain from the NSGR model, but modified to take additional parasitic loss and the complex voltage reflection between mixer and IF load impedance into account, we find when we use the impedance fit parameters τ_{eph}, τ_{es}, c_e/c_{ph}, an excellent agreement between measured and modeled HEB conversion gain.

Keywords— **IF Impedance, phonon cooled hot electron bolometer (pHEB), two-temperature model, "hot" electrons, Perrin-Vanneste, NSGR model, electron temperature modulation, electron-phonon interaction time, phonon escape time, electro-thermal feedback, complex impedance, HEB mixer conversion gain.**

I. Introduction

In the context of history, traditional InSb hot electron bolometers[1] mixers suffer from a small (<100 MHz) IF bandwidth, due to a relatively long electron relaxation time in the film. To enhance the science that may be done with these devices, there has in recent years been a strong push to expand the gain and noise bandwidth of hot electron bolometers. To a large extend success has been achieved with the use of ultra thin ($\approx$ 4-6nm) NbN films with very short phonon escape times. The majority of such films have been supplied by the Moscow Pedagogical State University [2][3]. In fact the band 6 mixers on the HIFI instrument of ESA's Herschel's FIR satellite are all comprised of phonon cooled HEB's with a specified IF bandwidth of 2.4 - 4.8 GHz [4][5]. As the IF bandwidth exceeds several GHz however, a proper knowledge of the IF behavior of thin film hot electron bolometers, and the effect of electro-thermal feedback on the mixer gain is required. Some work on understanding the IF impedance and gain bandwidth has been done by Morales *et.al* [6], however this work is based on large area phonon-cooled hot electron bolometers (1μm x 4μm), and uses a theory developed for diffusion HEB's [7] to fit the IF impedance mixer conversion gain data. Recent trends however have been to reduce the phonon cooled HEB active area by factors of 16 or more, in an effort to accommodate terahertz solid state multipliers (reducing the HEB area reduces the required LO pump power level). These sub-micron area NbN devices are considerably different than those studied by Morales *et.al*, and hence the renewed interest.

In this paper we use a two temperature model introduced by Perrin-Vanneste[8] and expanded upon by Nebosis, Semenov, Gousev, and Renk [9], to describe the phonon cooled HEB IF impedance and mixer gain. The NSGR impedance model includes an electro-thermal feedback mechanism which effects the mixer's non-linear hot spot region by means of complex voltage reflections at the mixer output port. This feedback mechanism is seen to modulate the mixer conversion gain as a function of IF frequency, resulting in some of the observed wiggles and fluctuations in the receiver noise temperature. The inhomogeneous "hot spot", hypothesized to govern the mixing process in hot electron bolometers, is located in the superconducting film that is in contact with the normal metal (Au) contacts pads. This hot spot region is created by the application of bias, LO power, and modulated in a non-linear fashion by incident (RF) radiation. In practice HEB mixers are thus operated at elevated and time dependent electron temperatures, near the critical temperature of the material. We use this information to constrain the fit parameters τ_{eph}, τ_{esc} and c_e/c_{ph} in the NSGR impedance model, as there appears to be not one single unique solution. With the obtained fit values we proceed to calculate the complex IF impedance and mixer gain, inclusive of the discussed electro-thermal feedback, and are able to successfully compared it to actual measurement.

II. Theory

If a hot electron bolometer is exposed to incoming RF radiation (or dc bias) in the form: $P_o[1 + cos(\omega t)]$, then this energy is absorbed by raising electrons in the superconducting film to a higher temperature. The primary cooling mechanism of these "hot" electrons occurs via phonon interaction, with a time constant equal to τ_{eph}. In turn the phonons escape into the substrate with an escape time τ_{esc} , though some may diffuse out of the metal contact pads. In general it has been found that the heat capacities of the electrons and phonons is quite temperature dependent. Following the two temperature analyses of Perrin-Vanneste[8] and the NSGR model[9], where the electron and phonon cooling rates and their respective heat capacities in a superconducting HEB mixer are treated as arbitrary, we find the following heat balanced equations for a linearized ($|T_e - T_0| \ll T_0$) system:

$$c_e \frac{\partial T_e}{\partial t} = P_{dc} + \alpha P_{lo} e^{i\omega t} - \frac{c_e}{\tau_{eph}}(T_e - T_{ph}) \tag{1}$$

$$c_{ph} \frac{\partial T_{ph}}{\partial t} = \frac{c_e}{\tau_{eph}}(T_e - T_p h) - \frac{c_p}{\tau_{esc}}(T_{ph} - T_0) \tag{2}$$

c_e and c_{ph} are the electron and phonon heat capacity, α the LO coupling coefficient which is typically 5-10%, and T_e, T_{ph}, T_0 the respective electron, phonon and bath temperatures. Diffusion thru the contact pads is neglected. Following the NSGR analyses, the frequency dependent IF mixer impedance may be solved as:

$$Z = \frac{d}{dI}[I \cdot R(I, T_e)] = R(I, T_e) + I\frac{\partial R}{\partial I} + I\frac{\partial R}{\partial T_e}\frac{\partial T_e}{\partial I} \ , \tag{3}$$

with the film resistance $R(I, T_e)$ obtained from work by Elant'ev [10], and T_c the critical temperature of the superconductor. Here

$$R(I, T_e) \approx \frac{Rn(T_e)}{2}\left(1 + \zeta(T_e) - \frac{[1 - \zeta(T_e)]^3}{[1 + I/I_o - \zeta(T_e)]^2}\right) , \tag{4}$$

with

$$\zeta(T_e) = \frac{1}{1 + e^{\frac{4(T_c - T_e)}{\Delta T_c}}} \ . \tag{5}$$

$Z(\omega)$, the frequency dependent HEB output impedance, may be found by assuming that a small perturbation in the current, $dI = \delta I e^{i\omega t}$, causes a change in the electron temperature $dT_e = \delta T_e e^{(i\omega t + \varphi 1)}$, and phonon temperature $dT_{ph} = \delta T_{ph} e^{(i\omega t + \varphi 2)}$. These partials may then substituted in the linear heat balance Eqn's 1, 2 to give:

$$Z(\omega) = R_o \cdot \frac{\Psi(\omega) + C}{\Psi(\omega) - C} \ , \tag{6}$$

where $\Psi(\omega)$ represents a frequency dependent modulation of the electron temperature, ω the IF frequency, R_o the DC resistance at the bias point of the mixer, and C the self heating parameter[13][14]. The latter is important as it forces the complex part of the impedance (Eqn. 6) to be zero at very low and very high IF frequencies. $\Psi(\omega)$ is defined as

$$\Psi(\omega) = \frac{(1 + i\omega\tau_1)(1 + i\omega\tau_2)}{(1 + i\omega\tau_3)}, \tag{7}$$

and the self heating parameter

$$C = \frac{I^2}{V}\frac{\partial R}{\partial T_e}\left(\frac{\tau_{eph}}{c_e} + \frac{\tau_{esc}}{c_{ph}}\right) = \frac{dV/dI \ - \ R_o}{dV/dI \ + \ R_o} \ . \tag{8}$$

In the transfer function $\Psi(\omega)$; τ_1, τ_2, τ_3 maybe solved as:

$$\tau_1^{-1}, \tau_2^{-1} = \frac{\Omega}{2}\left(1 \mp \sqrt{1 - \frac{4\tau_{eph}{}^{-1}\tau_{esc}{}^{-1}}{\Omega^2}}\right) \ , \tag{9}$$

with

$$\Omega = \left(1 + \frac{c_e}{c_{ph}}\right)\tau_{eph}^{-1} + \tau_{esc}^{-1} \ , \tag{10}$$

and

$$\tau_3{}^{-1} = \frac{c_e}{c_{ph}}\tau_{eph}{}^{-1} + \tau_{esc}{}^{-1} \ . \tag{11}$$

To derive an expression for the conversion gain of the mixer, we use standard formalism to obtain the (frequency selective) responsivity[11][12] of a bolometer. Included in the responsivity is a complex load impedance Z_l, which in any real system is connected across the output port of the bolometer, and the HEB output reflection coefficient Γ_{if}. In this manner electro-thermal feedback, due to (complex) voltage reflections between mixer and IF circuitry, may be taken into the account.

$$S(\omega) = \frac{dV_l}{dP} = \frac{\alpha}{I}\frac{Z_l}{R_o + Z_l}\ \frac{C}{(\Psi + \Gamma_{if}\ C)} \ , \quad \Gamma_{if} = \frac{R_o - Z_l}{R_o + Z_l}. \tag{12}$$

Here α presents the (RF) coupling factor, and I the signal current thru the load (and device). Because the IF load impedance connected to the mixer is in general complex, it is important to use the complex responsivity, and not the absolute responsivity, $|S(\omega)|$, to reflect the true nature of the electro-thermal feedback on the conversion gain, $\eta(\omega)$. To find the (complex) conversion gain of the mixer, we use the standard expression;

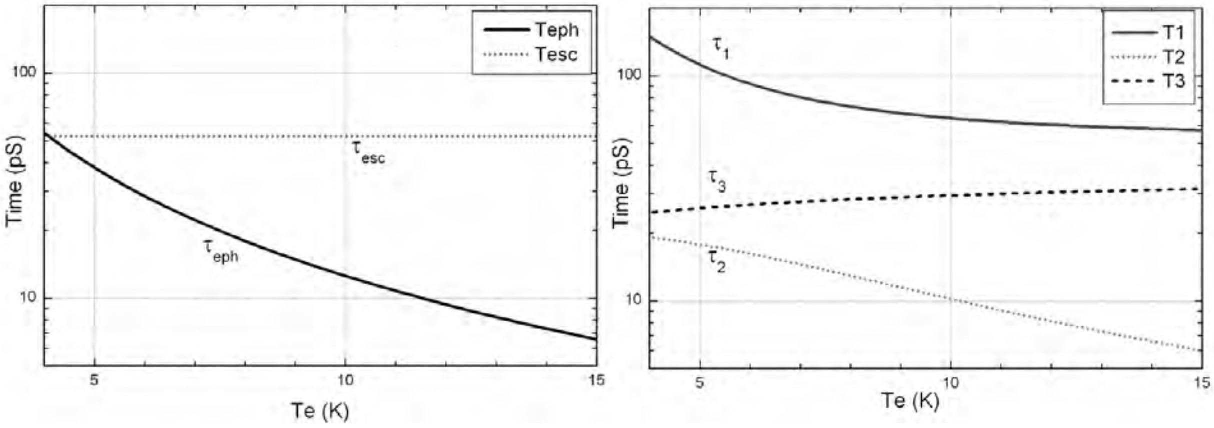

Fig. 1. Time constants and their temperature relationship as described it the text. Values for τ_{eph}, τ_{esc}, and the heat capacity ratio c_e/c_{ph} (not shown) have been obtained from literature. In the plot a 5nm thick NbN film is assumed. Values for τ_1, τ_2, τ_3 are derived from Eqn's [9-11], and serve to constrain the impedance model (Secion VI). Fit values obtained for τ_{eph}, τ_{esc}, and c_e/c_{ph}, for different HEB bias and LO pump conditions, are presented in Table I.

$$\eta(\omega) = \frac{2S(\omega)^2}{Z_l} P_{lo} \; . \qquad (13)$$

After substitution of Eqn. 12, and making the assumption that most of the signal current thru the device is in fact DC bias current, i.e. P_{dc}=$I^2 \cdot R_o$ we find after some algebraic manipulation the magnitude of the conversion gain as

$$\eta(\omega) = \frac{2\alpha^2 P_{lo}}{P_{dc}} \left| \frac{R_o Z_l}{(R_o + Z_L)^2} \frac{C^2}{[\Psi(\omega) + \Gamma_{if}\ C]^2} \right| . \qquad (14)$$

To obtain a better understanding of the range of plausible values for τ_{eph}, τ_{esc}, and c_e/c_{ph}, and to constrain the fit parameters to our data set, we resort to values used in literature. For the electron-phonon interaction time, we used an empirical relation $\tau_{eph} \approx 500T^{-1.6}$ [12]. Similarly, the phonon-escape time has been noted [19] [20] [5] to follow the relationship $\tau_{esc} \approx 10.5\ d$ (ps/nm), where d equals the NbN film thickness. Finally, taken from [20] [5], the ratio of the electron to phonon heat capacity is seen to be approximately 18.77 $T_e/T_{ph}\ ^3$. When the electron temperature is similar to the phonon temperature, i.e. $T_e \sim T_{ph}$, as is ordinarily the case under optimal bias conditions, then the ratio of c_e/c_{ph} follows a T^{-2} dependence. These quantities are plotted in Fig. 1 for a 5 nm NbN superconducting film.

III. Experiment and Calibration

In Fig. 2 we describe the setup and calibration of the experiment. A twin-slot NbN HEB mixer chip (M12T-F2) with a bridge area of 1 μm x 0.15 μm is glued to the back of a silicon lens. The twin-slot antenna is positioned at the second foci of the ellipse, and produces an essentially diffraction limited beam with an f/D ratio of $\approx$ 20. The co-planar wave HEB IF output connects, via a number of parallel wire bonds, to a broad bandwidth (Baryshev, Kooi $et.al$) grounded CPW-to-microstrip transition and then via a 50 Ohm transmission line to a SMA bulkhead output connector. Details on the device's noise temperature, mixer gain as a function of bias, and R-T curve maybe found in a separate paper by Yang $et.al$ [21].

To calculate the LO pumped HEB IF impedance we use the following procedure: First we measure the reflection coefficient at the mixer block SMA connector reference plane with a vector network analyzer. Included in this measurement is the bias Tee (not shown in Fig. 2. Next we use HFSS[26], a full 3D finite element electromagnetic field simulator, to obtain a 2 port S-parameter model of the IF circuitry, including wire bonds, via holes, and air space. And finally we use a linear circuit simulator [27] to de-embed the IF circuit from the measured complex input reflection coefficient, and use it to fit the two-electron temperature NSGR IF impedance model (Eqn. 6). This procedure provides the fit parameters that determine the HEB frequency selective IF impedance and mixer gain: τ_{eph}, τ_{es}, and c_e/c_{ph}.

Actual network analyzer calibration was done at room temperature. To correct for thermal contraction and increased conductivity of the coax cable intern to the cryostat upon cooling, we did a reflection measurement at 77K and at LHe temperature with the HEB biased at 20mV. At this bias voltage the device impedance maybe expected to be purely real. We did attempt to bias, and calibrate at 0mV, however instability in the HEB prevented a proper measurement. The resultant calibrations at room temperature, 77K and 4.5 Kelvin are shown in Fig.'s 3 and 4. Modeled vs. measured calibration is very good up to about 8 GHz, after which some discrepancy develops. This is most likely due to the way the SMA connector is mounted to the pc board/mixer unit. In the fits, the frequency range below 8 GHz has been weighted extra heavily for this reason.

IV. IF Impedance

In Fig.'s 5-9 we present the de-embedded and modeled IF impedance of the HEB. The IF impedance of the device has been obtained at a number of carefully chosen bias and LO pumping levels, as demonstrated in Fig. 5a.

To model the IF impedance we use equations 6-11. Parameters used to fit the impedance model are the electron-

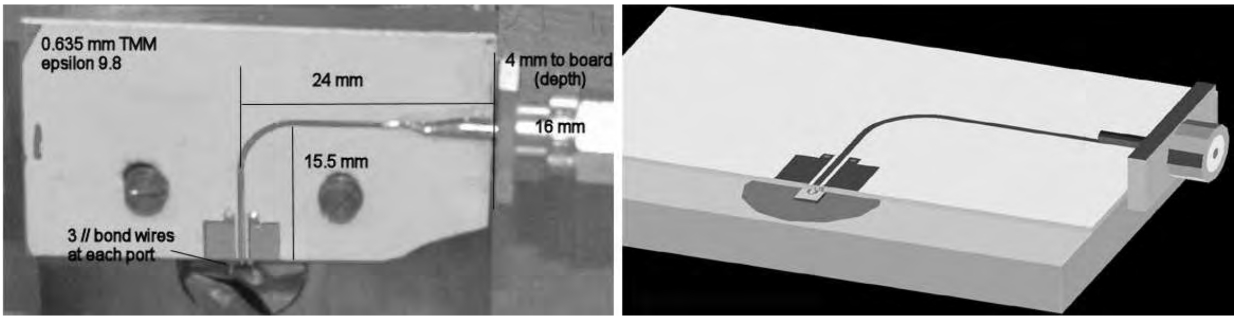

Fig. 2. Left: Quasi optical mixer block with a broad bandwidth grounded cpw to microstrip transition. The mixer block was designed to measure HEB gain and noise bandwidth to $\sim$ 10 GHz. Right: HFSS 3D model. Dielectric material was Rogers, Tmm 10i (Er=9.8) with a board thickness of 635 μm. Wire bond, air space, via holes, electrical conductivity have all been taken into account.

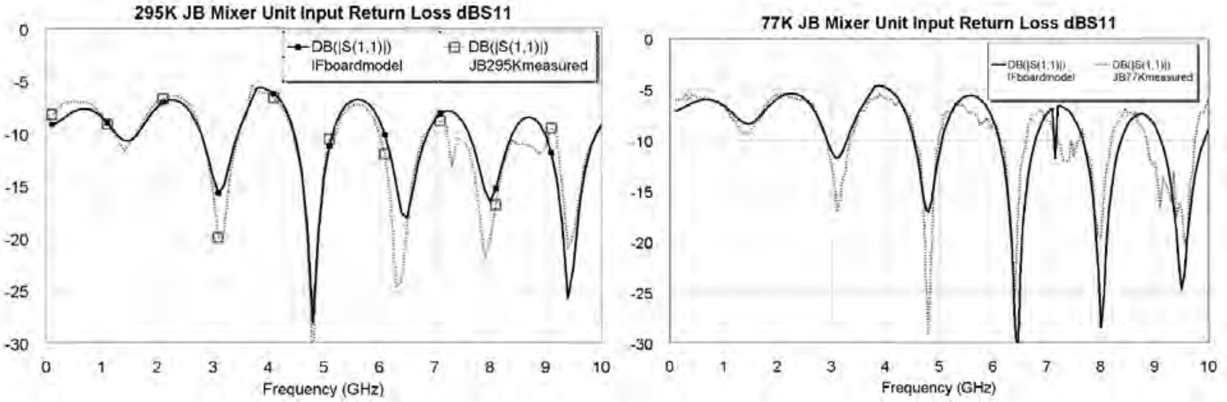

Fig. 3. Left: Calibration at room temperature. Shown is the measured (dotted), and modeled (black) input return loss at the SMA mixer block output flange. Data in all cases was taken at a power level of -65 dBm, with no effect on the pumped I/V curve. 64 averaged traces were obtained to integrate down the noise. Right: Quality of the calibration correction at 77K. Here we used the measured DC impedance (R_o) of the HEB to calibrate for thermal contraction (4.37mm) and reduction in (frequency dependent) loss of the coaxial cable connected to the mixer unit. At 77K the HEB impedance was reduced by $\approx$ 3-4% from room temperature which is attributed to the reduction in Ohmic loss in the Au wiring of the HEB mixing chip. Vertical scale is in units of dB.

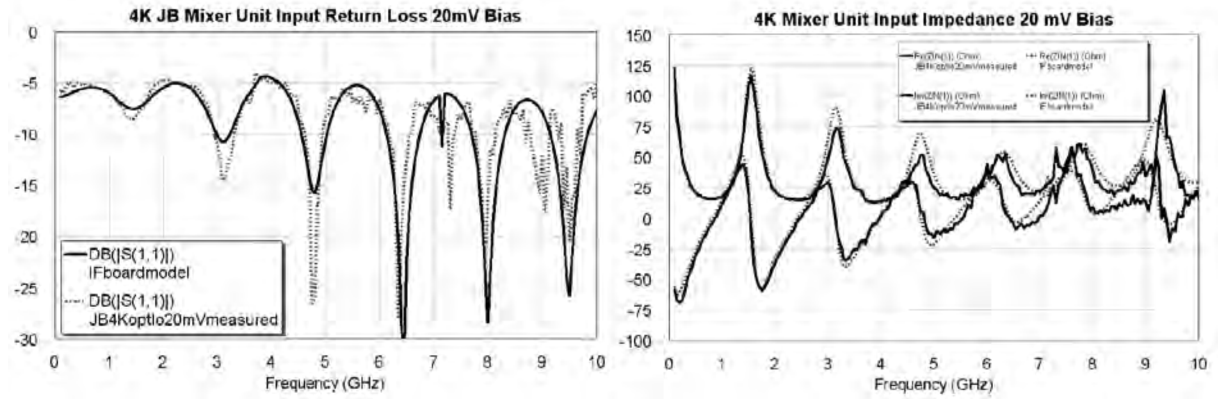

Fig. 4. Left: Measured (dotted) vs modeled (black) input return loss at the SMA flange. Right: Measured (black) vs modeled (dotted) input impedance at the SMA flange. In both cases, the mixer unit was at 4.5 Kelvin, with the HEB biased at 20 mV. Vertical scale is in units of dB.

phonon interaction time τ_{eph}, the phonon escape time τ_{es}, and the ratio of the electron and phonon specific heat capacity c_e/c_{ph}. Particularly in the underpumped LO case, the HEB IF impedance demonstrates quite a large real and reactive component. For bias voltages > 2mV, the situation reverses and the reactive part $\rightarrow$ zero.

Because there is no uniqe solution to the best fit values of τ_{eph}, τ_{esc}, and c_e/c_{ph}, we use literature to provide appropriate boundary conditions (Fig. 1, Section II). The results in Table I provide for some interesting statistics on the material properties of the NbN film, and assumptions of the temperature dependence of τ_{eph}, and c_e/c_{ph} used in literature. For example, the mean escape time for the phonon's into the substrate is 62.2 $\pm$ 4.8 pS. Using the empirical re-

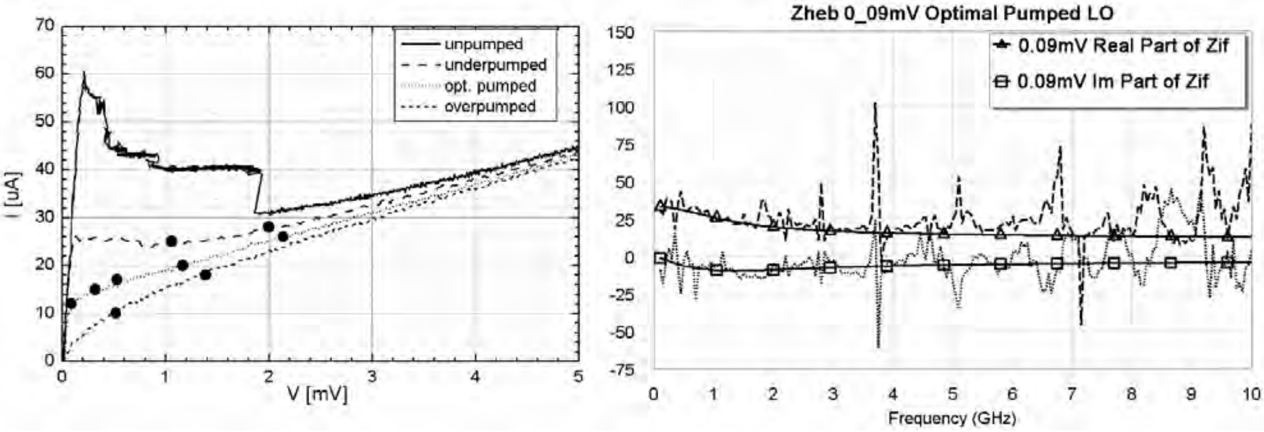

Fig. 5. Left: Unpumped, Under, Optimal, and Over pumped I/V curves. Circles indicate HEB reflection measurements bias points. Right: 0.09 mV (de-embed and modeled) HEB IF impedance at optimal pumped LO. τ_{eph}=12.1 pS, c_e/c_{ph}=0.166, and $< T_e >$= 10.45 K. Vertical scale is in Ohm.

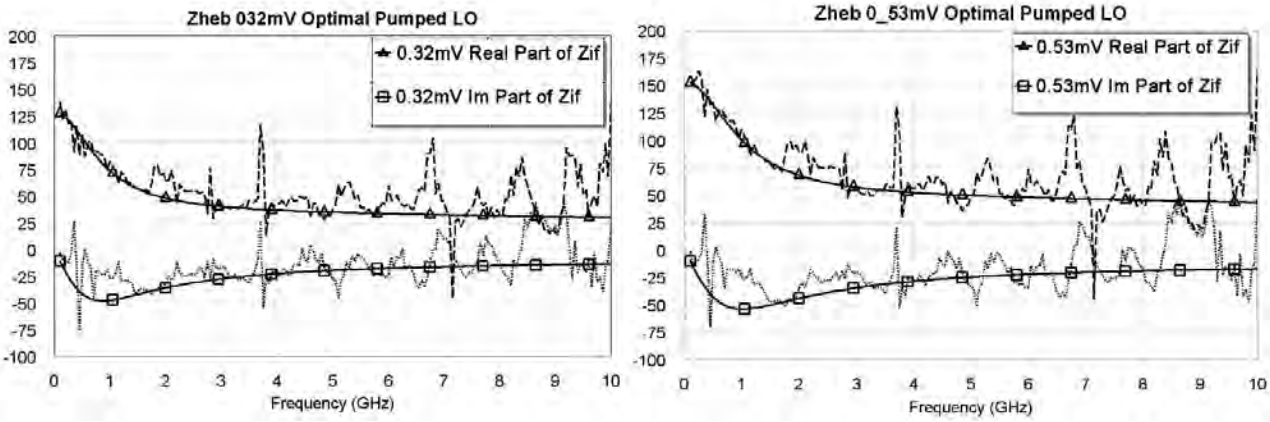

Fig. 6. Left: 0.32 mV (de-embed and modeled) HEB IF impedance, optimal pumped LO. τ_{eph}=11.4 pS, c_e/c_{ph}=0.161, and $< T_e >$= 10.73 K. Right: 0.53 mV HEB IF impedance, optimal pumped LO, τ_{eph}=10.4 pS, c_e/c_{ph}=0.167, and $< T_e >$= 10.95 K.

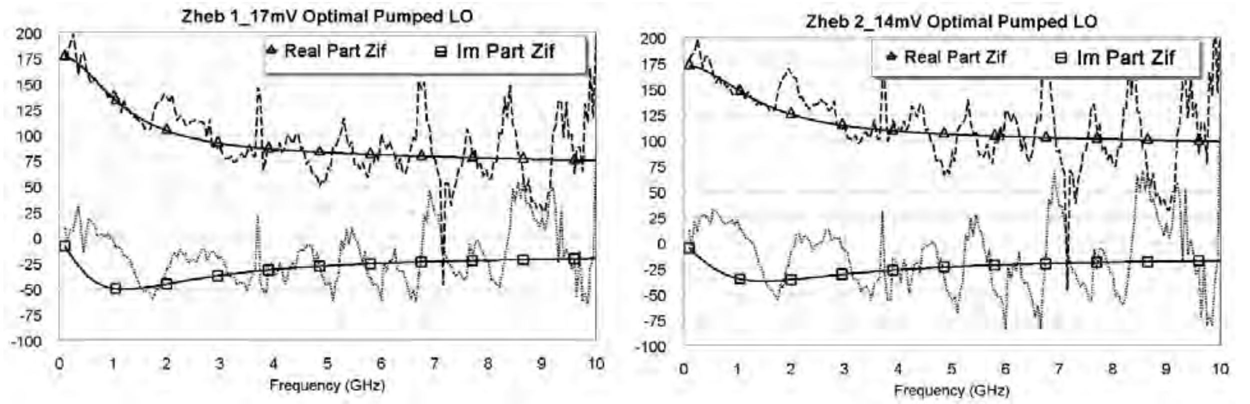

Fig. 7. Left: 1.17 mV HEB IF impedance, optimal pumped LO. τ_{eph}=10.0 pS, c_e/c_{ph}=0.163, and $< T_e >$= 11.15 K. Right: 2.14 mV HEB IF impedance, optimal pumped LO. τ_{eph}=8.82 pS, c_e/c_{ph}=0.167, and $< T_e >$= 11.55 K.

lationship that $\tau_{esc} \approx 10.5\, d$ (pS/nm), we find a suggestive NbN film thickness of 5.9 ± 0.44 nm. This is supported by a recent study of the film by Transmission Electron Microscopy (TEM), in which the thickness seems to be around 5 nm instead of the intended thickness of 3.5 nm [22]. Secondly, the temperature relationship of the electron-phonon interaction time, and the ratio of the electron-phonon heat capacities may, to a first order, be verified. Using the empirical relationships (Section II) that for thin NbN films, $\tau_{eph} \approx 500\, T^{-1.6}$ (pS·K) and $c_e/c_{ph} \approx 18.77\, T^{-2}$, we obtain an estimate for the mean (or effective) electron temperature in the NbN bridge. The last two colums in Table I show the calculated results. The mean electron temperature, $< T_e > \; = \; < T_e(eph) + T_e(c_e/c_{ph}) >$ is reported

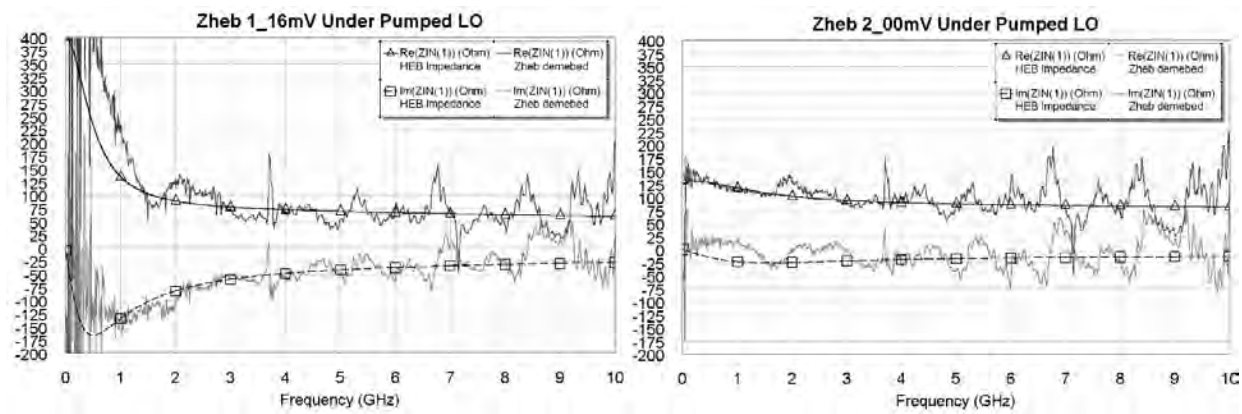

Fig. 8. Left: 1.06 mV HEB IF impedance, under pumped LO. τ_{eph}=14.9 pS, c_e/c_{ph}=0.213, and $< T_e >$= 9.21 K. Right: 2.00 mV HEB IF impedance, under pumped LO. τ_{eph}=12.9 pS, c_e/c_{ph}=0.210, and $< T_e >$= 9.67 K.

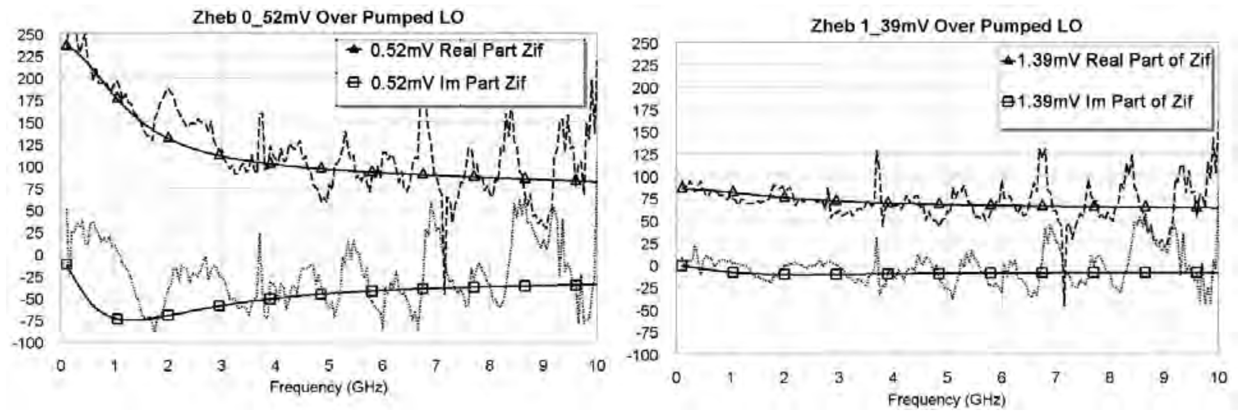

Fig. 9. Left: 0.52 mV HEB IF impedance, over pumped LO. τ_{eph}=7.99 pS, c_e/c_{ph}=0.094, and $< T_e >$= 13.7 K. Right: 1.29 mV HEB IF impedance, over pumped LO. τ_{eph}=7.04 pS, c_e/c_{ph}=0.088, and $< T_e >$= 14.46 K. Details on Fig.'s 5 - 9 may be found in Table I below. Vertical scales are in Ohm.

TABLE I
HEB Parameters for different bias Conditions.

Units of dV/dI, R_o, R_o^* are in Ω, τ_{esc} and τ_{eph} in pS, $T_e(eph)$ and $T_e(c_e/c_{ph})$ in Kelvin, and ν_{-3dB} in GHz.

Vbias	dV/dI	R_o	R_o^*	C	C^*	τ_{esc}	τ_{eph}	c_e/c_{ph}	$T_e(eph)$	$T_e(c_e/c_{ph})$	ν_{-3dB}
0.09mV Opt	42	7.5	9.5	0.697	0.551	56.5	12.1	0.166	10.24	10.67	1.842
0.32mV Opt	110	21.3	21.3	0.675	0.715	58.8	11.4	0.161	10.62	10.84	2.122
0.53mV Opt	167	31.2	31.2	0.685	0.662	57.5	10.4	0.167	11.25	10.65	2.551
1.17mV Opt	168	58.5	58.5	0.483	0.504	67.6	10.0	0.163	11.53	10.77	2.981
2.14mV Opt	169	82.3	82.3	0.345	0.355	67.6	8.82	0.167	12.47	10.64	3.020
20.0mV Opt	150	140	140	0.034	0.033	—	—	—	—	—	—-
1.06mV Under	600	42.4	47.4	0.868	0.788	68.2	14.9	0.213	8.99	9.42	2.426
2.00mV Under	230	71.4	71.4	0.526	0.300	65.2	12.9	0.210	9.83	9.51	2.797
0.52mV Over	80	52.0	52.0	0.212	0.639	58.2	7.99	0.094	13.27	14.18	4.081
1.39mV Over	127	77.2	52.2	0.243	0.240	60.4	7.04	0.088	14.36	14.66	3.727

$T_e(eph)$ is the effective electron temperature based on the fitted electron-phonon interaction time, τ_{eph}, and temperature relationship obtained from literature. $T_e(c_e/c_{ph})$ is electron temperature based on the, from literature obtained temperature, relationship of electron and phonon heat capacity. R_o^* and C^* area actual values used in the model fit. Note that for the Under and Over pumped I/V curves there appears some discrepancy between model and measurement. ν_{-3dB} is the calculated -3dB HEB mixer gain roll-off frequency based on $\Psi(\omega)$ and τ_p.

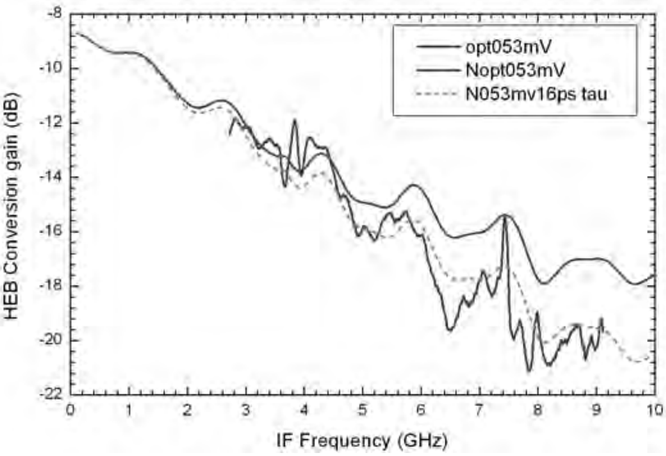

Fig. 10. Measured and modeled HEB mixer conversion gain as a function of IF frequency for optimal LO, and 0.53 mV bias. Input parameters to the model are: τ_{eph}=10.4 pS, τ_{esc}=57.5 pS, c_e/c_{ph}=0.167, and T_e = 10.95 K. The effect of electro-thermal feedback is taken into account by means of the (modeled) complex IF load impedance. The dotted curve indicates the mixer gain when Eqn. 14 is modified with an additional at 10 GHz pole ($\tau = 15.8\ pS$), see text for details.

in figures 5-9, and shows a consistent trend with bias and LO pump level [23].

V. Mixer Conversion Gain and the Effect of Electro-thermal Feedback

To properly model the HEB mixer conversion gain, the effect of voltage reflection modulations of the hot electron temperature, and subsequent mixing efficiency ($\partial R/\partial T$) needs to be taken into account. Because, as part of the de-embedding exercise, an accurate 3D EM model[26] of the IF embedding circuitry was developed (Fig. 2),

we can use this model to accurately predict the IF impedance presented to the HEB mixer IF output port. The model includes discontinuities such as the wire bonds that typically connect the IF board to the HEB output port. With this information we are able to calculate Γ_{if} and $[R_o \cdot Z_l/(R_o + R_l)^2]$ in Eqn. 14. It has been observed that the addition of a 10 GHz ($\tau = 15.8\ pS$) pole to Eqn. 14 helps to improve the accuracy of the modeled conversion gain. At low IF frequencies where the vast majority, if not all, of the HEB's operate the addition of an added pole to $\eta(\omega)$ is of little consequence. As to the cause of the additional high frequency pole, it is most likely the result of unaccounted for parasitic reactances in the HEB mixer stripline circuitry, Ohmic contact pads, and wirebonds that contact the HEB chip. Since parasitic reactances have not been taken into account in the "idealized" responsivity formulism of Eqn. 12, it may be advisable to include them. The HEB mixer gain modified for device parasitics may thus be rewritten as

$$\eta(\omega) = \frac{2\alpha^2 P_{lo}}{P_{dc}} \left| \frac{R_o Z_l}{(1 + i\omega\tau_p)^2 (R_o + Z_L)^2} \frac{C^2}{[\Psi(\omega) + \Gamma_{if}\ C]^2} \right|, \tag{15}$$

where $\tau_p \approx 15.8$ pS. Note that τ_p will be device and application dependent.

Several additional observations may be made. First, to minimize receiver noise temperature modulation across the IF operating bandwidth, one has to carefully consider ways to 1) minimize the complex part of Z_l such that Γ_{if} is frequency independent and 2) have $Z_l \approx R_o$ such that $\Gamma_{if} \rightarrow 0$. Secondly, to extend the HEB mixer IF bandwidth, the time dependent response of the electron temperature, $\Psi(\omega)$, needs to be increased. As discussed by Gao *et.al* [24], the most effective way to do so is to use films with higher T_c. The physical reason for the increase in IF bandwidth with higher T_c films is that at a higher T_c, or higher electron and phonon temperature, the phonon specific heat (c_{ph}) increases much faster than the electron cpecific heat (c_e). In this way the phonons can act as an important intermediate heat bath between the electron gas and the substrate. Of course, if we can reduce the thickness and thereby τ_{esc}, it too will enlarge the IF bandwidth.

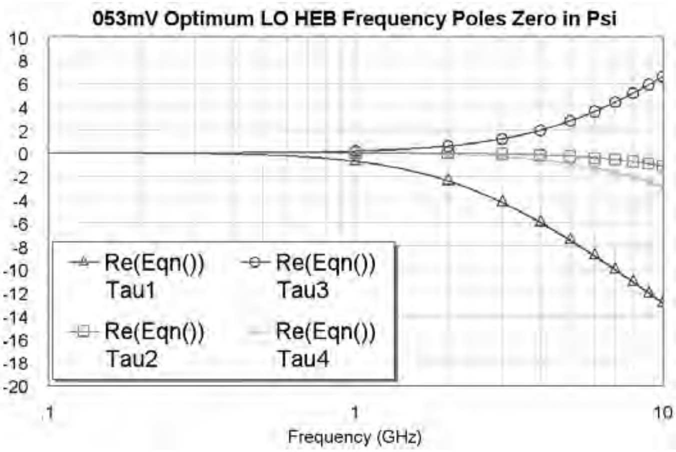

Fig. 11. $\Psi(\omega)$, the frequency dependent transfer function of the electron temperature at 0.53mV bias and optimal LO pumping level. τ_1 = 68.79 pS, which results in a pole at 2.313 GHz. τ_2 = 8.69 pS with a pole at 18.3 GHz, and τ_3 = 29.9 pS with a zero at 5.32 GHz. Included in the plot is τ_p due to unaccounted for device parasitics (see text). The three poles and zero effectively synthesize a "single pole" with ν_{-3dB} listed in Table I. To increase the frequency response of the system ν_{-3dB} will need to be increased. Refer to text for details.

This concept is illustrated in Fig. 11, where we plot the time dependent transfer function, $\Psi(\omega)$, of the electron temperature at 0.53mV bias and optimal LO pump level. τ_3 (5.32 GHz) is seen to slightly compensate τ_1 (2.31 GHz), whereas τ_2 (18.3 GHz) enhances the effect of τ_1, though to a very small extend. Adding a third pole (τ_p) to take into account residual device parasitics, we effectively synthesize a "single pole" mixer gain transfer function governed by $\Psi(\omega)$ and tau_p. This is depicted by ν_{-3db} in Table I. At 0.53 mV the synthesized "single pole" corresponds to a -3 dB roll-off frequency of 2.551 GHz, in a good agreement with measurements by Baselmans *et.al* [25].

VI. Conclusion

A novel de-embedding technique has been demonstrated to obtain the IF impedance of a small area (0.15 μm^2) phonon cooled HEB under a variety of bias and LO pump level conditions. In the same setup the HEB mixer conversion gain has, at an LO frequency of 1.3 THz, been measured in a 2.5-9 GHz IF bandwidth. To understand the observations, we have successfully modeled the HEB IF impedance and conversion gain based on a two-temperature electron cooling model first introduced by Perrin-Vanneste, and expanded upon by Nebosis, Semenov, Gousev, and Renk *et.al*. Good agreement between model and theory is obtained, and we are able to extract from the NSGR model (using published temperature and thickness relationships) values for the electron-phonon interaction time τ_{eph}, the phonon escape time τ_{esc}, the ratio of the electron and phonon specific heat capacity c_e/c_{ph}, and effective electron temperature of the NbN bridge as a function of bias and LO pump level. From these relationships, the NbN film thickness may be infered to be 5.9 $\pm$ 0.44 nm.

The mean electron temperature is indirectly calculated to vary, in the case of an optimal LO pumped HEB from 10.5-11.6 K, in the case of an under pumped HEB from 9.2-9.6 K, and in the case of an over pumped HEB mixer from 13.7-14.5 K. In addition, the presented analyses allows us to calculate the effective -3 dB mixer gain as a function of bias and LO pumping level. As an example, at 0.53 mV the -3dB mixer roll-off is observed to be 2.551 GHz, in good agreement with measurement. To extend the HEB IF response to higher frequencies, it can be seen from Eqn. 9-11 that either τ_{eph} and/or τ_{esc} ought to be reduced. This may in practice be accomplished with the use of a higher T_c superconducting material, thinner film, or both.

By using the complex IF impedance presented to the HEB chip we are able to demonstrate, for the first time, the effect of electro-thermal feedback on the mixer gain. Flat mixer gain (receiver noise temperature) within IF band may only be achieved if the variance of the complex load impedance presented to the HEB is small compared to the HEB DC resistance at it's operating point. Finally, using the NSGR model with the presented values for τ_{eph}, τ_{esc}, c_e/c_{ph} and a measured (LO pumped) I/V curve, expressions for $Z(\omega)$ and $\eta(\omega)$ for thin NbN films may now be derived.

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