Analysis of subharmonic SIS mixers using SuperMix

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Abstract— **Sub-harmonically pumped SIS mixers operating at very high LO harmonics have been used to measure beam patterns of SIS mixers in the laboratory, and to measure the line-widths and phase-locking performance of flux-flow oscillators. Using the sub-harmonically pumped mode of operation for beam pattern measurements allows the measurement to be carried with a single phase-locked sub-mm source, without a beamsplitter in front of the mixer feed. This allows the near field amplitude and phase to be measured, and allows the far field pattern to be measured at very high angles which would be blocked by the presence of a beamsplitter.**

Kittara, Withington and Yassin[1] have recently described a procedure for modelling the non-linear behaviour of very high harmonic SIS mixers. They use a fully non-linear multitonal mixer theory[2] to analyse the behaviour of a SIS mixer pumped by the 20th harmonic of a 13.5 GHz LO signal, with the mixer down-converting both sidebands around 270 GHz to a 1.4 GHz IF. This analysis shows that sub-harmonic mixers can achieve reasonable dynamic range. The pattern of behaviour seen in the simulations are in remarkable agreement with published experimental results[3].

In this work we describe the small-signal behaviour of subharmonic SIS mixers using the CalTech's SuperMix software[4]. This method has the advantage of faster convergence than the non-linear analysis and hence allows the exploration of the complex behaviour of subharmonic SIS mixers. Our analysis is compared Kittara *et al*'s results. We show that SuperMix can accurately calculate the small-signal behaviour of high harmonic SIS mixers, as well as providing predictions of mixer noise performance.

I. INTRODUCTION

Fundamental mode SIS junction mixers are the most sensitive heterodyne receivers throughout the high mm-wave and sub-mm bands. This sensitivity is due to the very high nonlinearity of the SIS junction IV curve. This same nonlinearity can be used to generate many harmonics of the LO signal, allowing the SIS junction to be used as a subharmonically pumped mixer. Subharmonically pumped mixers using low harmonic number have been reported[5], and higher harmonic numbers have been used to study the line-width and phase-locking of flux flow oscillators[6].

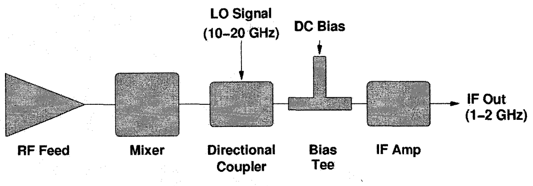

Fig. 1. Schematic of the subharmonic SIS receiver reported by Baryshev *et al*[3].

A particularly effective use of a high harmonic number subharmonic SIS mixer was reported by Baryshev *et al*[3]. They reported the use of a subharmonic pumping of an SIS mixer in measuring the amplitude and phase of the beam pattern of a (fundamental mode) mixer at 640 GHz. The 15 GHz LO signal was injected via directional coupler at the IF output of the mixer (fig. 1), with the mixer being operated at the 42nd harmonic. The RF test signal was provided by a phase-locked Gunn oscillator feeding a Schottky diode doubler and tripler multiplier chain in the far field of the mixer feed. This measurement system has two distinct advantages. Firstly, only one phase-locked mm-wave source is required, and secondly, as the microwave LO signal is injected through the IF output of the mixer, no LO injection optics are required in the optical system and the measured beam pattern is that of the mixer feed alone.

The behaviour of the pumped IV curves and IF output power (and hence mixer conversion loss) reported by Baryshev *et al* is particularly striking. Unlike a fundamental mode mm-wave SIS mixer, no photon steps are visible in the pumped IV curves, as the photon voltages are less than the width of the junction nonlinearity. When the mixer is pumped by a low frequency LO and a mm-wave RF signal, the IF output power against bias curve has several peaks both above and below the junction gap voltage. The widths of these peaks do not seem to be related to either the RF or LO photon voltages, and the widths of the peaks varies strongly with LO pump level.

In order to optimise the effectiveness of subharmonic mixers used in these applications, it is vital that the behaviour of the mixer can predicted. In this work we compare two numerical models of subharmonic SIS mixers. The first of these is CalTech's SuperMix software library,which has been widely used to simulate and design fundamental mode mixers. The second is the MultiTone software package, based on a recently published fully nonlinear model of quantum mixing[2]. This second package is of particular interest to non-astronomical uses of subharmonically pumped SIS mixers, as it allows the dynamic range of the mixer to be predicted. However, the SuperMix library has other advantages, as it can incorporate complex superconducting circuits, and several SIS junctions.

II. SUPERMIX AND MULTITONE SIMULATIONS OF SUBHARMONIC MIXERS

SuperMix[4] is a software library developed at the California Institute of Technology to allow the simulation of

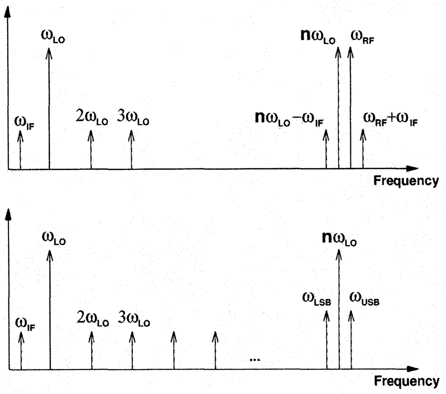

Fig. 2. Harmonics included in (top) MultiTone and (bottom) SuperMix simulations of the subharmonic mixer.

fundamental mode SIS mixers and their associated superconducting circuits and IF systems. The portion of the library that simulates the behaviour of SIS mixers is based on a generalisation of Tucker's theory of quantum mixing in SIS junctions[7]. SuperMix allows any number of harmonics of the LO frequency and any number of SIS junctions to be simultaneously solved for large signals, while the conversion between the RF and IF sidebands is calculated by perturbing the harmonic balance over the LO harmonics, in the limit of small sideband signals.

A further generalisation of Tucker theory has recently been presented by Withington, Kittara and Yassin[2]. Although currently limited to a single tunnel junction, this completely quantum model of SIS mixing allows the tunnel junction to be excited by any number non-harmonically related frequencies of arbitrary amplitude. This model allows signals at all sidebands and LO harmonics to be arbitrarily large, therefore allowing the behaviour of saturating SIS mixers to be rigorously calculated. We have previously shown that the MultiTone software, based on this model gives identical results to SuperMix, in the limit of small sideband signals[8].

Kittara *et al*[1] have recently carried out simulations of subharmonic SIS mixers using the MultiTone software. The MultiTone model of subharmonic mixers is somewhat complicated by a limitation of the software (but not the overall model), due to which only the first three harmonics of a signal can be included. The MultiTone model of a subharmonic mixer therefore uses separate signals at the LO frequency ω_{LO}, the nth harmonic of the LO $n\omega_{LO}$, the upper sideband of the nth harmonic $\omega_{RF} = n\omega_{LO} + \omega_{IF}$ and the IF signal ω_{IF}. In order to get accurate results, undriven signals must also be included at the lower sideband of the nth harmonic $\omega_{LO} - \omega_{IF}$, and $\omega_{RF} + \omega_{IF}$. This scheme is outlined in fig. 2. The effect of the mixer circuits is included by setting an embedding impedance for each harmonic included in the harmonic balance.

In the SuperMix model of the subharmonic mixer, the first n harmonics of the LO frequency are included in the harmonic balance, while the mixer conversion between the upper and lower sidebands and the IF are calculated in the small-signal limit. SuperMix divides the embedding circuits of the mixer into three distinct circuits; the DC bias circuit, the IF output circuit and the RF circuit, to which the LO must be connected. In order to simulate a subharmonic mixer with different LO and RF embedding impedances, the RF circuit of the subharmonic mixer consists of two branches, with the signals in each branch selected by idealised bandpass filters between the embedding impedance and the junction (fig. 3). The RF bandpass filter has perfect transmission between $n\omega_{LO} - \omega_{IF}$ and $n\omega_{LO} + \omega_{IF}$, while the LO bandpass filter has perfect transmission in a narrow band about ω_{LO}. Outside of these bands, both filters are perfectly reflecting.

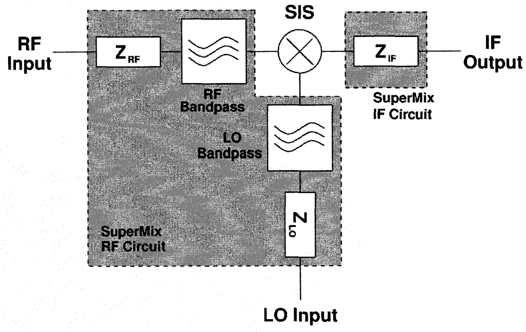

Fig. 3. Schematic of the circuits used within the SuperMix simulations of the subharmonic mixer.

We have found that SuperMix's harmonic balance routine will often fail to converge when simulating a subharmonic mixer, particularly when the LO pump level is close to the optimum. This problem occurs because the accuracy of the calculation of the RF currents through the junction is hard coded in the SuperMix SIS junction model. Although the preset tolerances are adequate for most fundamental mode mixers, these tolerances cause the balancing of very high harmonics to fail. We have worked around this problem by including many higher LO harmonics in the calculation, e.g. 90 harmonics are required to simulate the 20th harmonic mixer presented here at all pump levels. This comes at a cost of greatly increased execution time, as the harmonic balance must now be carried out over many more harmonics than strictly necessary. A better alternative would be to alter the SuperMix library, so that the user can set the tolerances in the RF current calculation.

III. SIMULATION RESULTS

In this section we compare the results of SuperMix based simulations with those obtained from the MultiTone software for a specific idealised mixer. The mixer is pumped by an LO at a normalised frequency of 0.02, corresponding to 13.5 GHz for a niobium junction. The mixer is operated at the 20th harmonic of the LO (normalised frequency 0.40 $\sim$ 270 GHz) with an IF of 0.002 $\sim$ 1.35 GHz. The embedding impedances of the IF, LO, the 20th harmonic of the LO, and the RF sidebands are set to unity, while all other intermediate (and higher) LO harmonics have an embedding impedance equal to zero. The response function of the junction is given by the polynomial quotient approximation

$$I(V) = \frac{V^n}{1 + V^{n-1}}, \tag{1}$$

with $n = 50$. The sharpness of this IV curve roughly corresponds to that of a high quality niobium/aluminium oxide junction.

Fig. 4 compares the pumped IV curves from SuperMix and MultiTone at various LO drive level $\alpha = \omega_g V_{LO}/\omega_{LO}$.

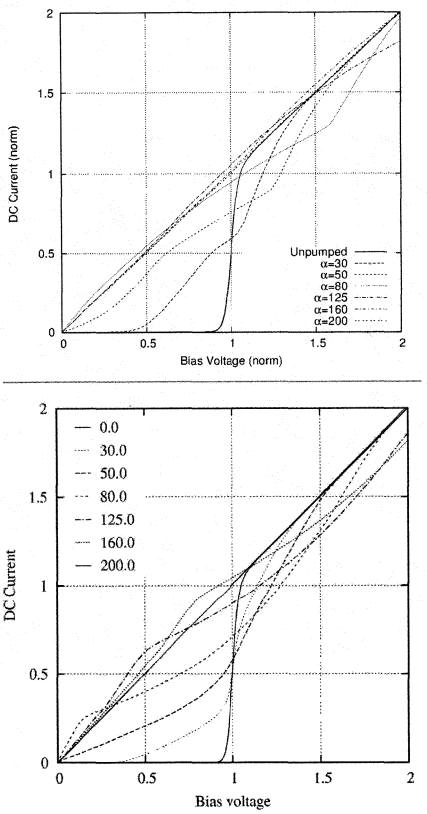

Fig. 4. (top) SuperMix and (bottom) MultiTone simulated pumped IV curves for a 20^{th} harmonic mixer pumped at $\omega_{LO} = 0.02 \sim 13.4\,\mathrm{GHz}$ for a Nb junction. In all of the following results, the MultiTone results have a gap voltage of 2.8 mV, while the SuperMix bias voltages are normalised to a gap voltage of 1.0.

Although the SuperMix and MultiTone pumped curves are different, both are reasonably consistent with the measured IV curves reported by Baryshev *et al.* The differences between the two simulation methods are probably due to both the different harmonics used in each calculation, and differences between the embedding circuits used. More work, including alterations to both software packages, will be required to get good agreement between these results.

Figures 5 and 6 compare the mixer conversion losses predicted by the two software packages at two different LO drive levels. Due to differences between the operation of the two software packages, the actual values of the conversion loss cannot be directly compared. Instead the IF output power predicted by MultiTone for a small fixed RF input power is compared with the small-signal conversion loss from SuperMix. Both sets of results produce the same number of peaks in the IF output, although heights and widths of these peaks differ, with the MultiTone results looking closer to the experimental results of Baryshev *et al.* The uneven nature of the peaks in both the experimental and MultiTone results is due to the high RF signal level used in both cases. This situation cannot be simulated by SuperMix.

In both the experimental data and the MultiTone simulations, the RF signal can be larger than the signal generated at the nth harmonic of the LO, for reasonably small RF signals. In this case the RF and LO nth harmonic swap roles, with the mixer being partially pumped by the RF signal. Despite

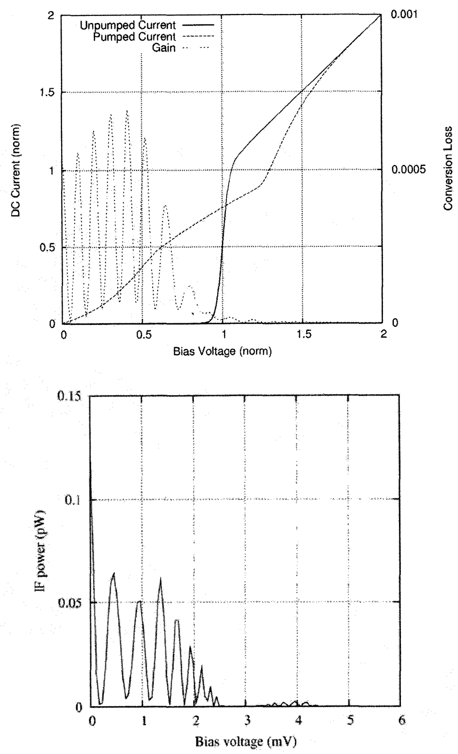

Fig. 5. (top) SuperMix calculated conversion gain and (bottom) MultiTone simulated IF output power for the 20^{th} harmonic mixer. The LO drive level is $\alpha = 50$, and the RF signal power in the MultiTone simulation is 472 pW.

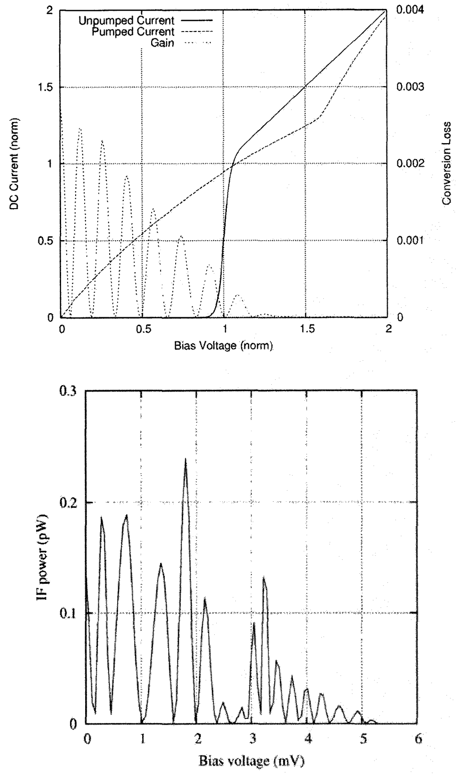

Fig. 6. (top) SuperMix calculated conversion gain and (bottom) MultiTone simulated IF output power for the 20^{th} harmonic mixer. The LO drive level is $\alpha = 80$, and the RF signal power in the MultiTone simulation is 472 pW.

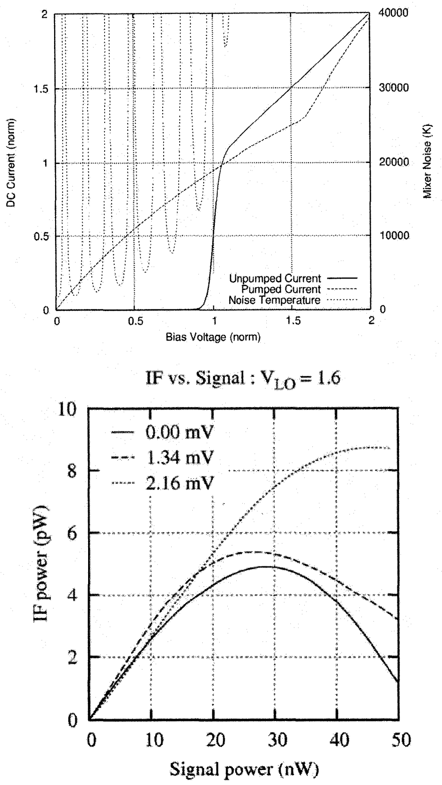

Fig. 7. (top) SuperMix calculated noise temperatures of the 20th harmonic mixer at a LO drive level of $\alpha = 80$. (bottom) MultiTone results for the saturation of the IF signal at increasing RF signal levels at three bias voltages.

this, the IF output of the mixer is still linear in the RF power up to a few nanowatts. The relatively large RF signal is the cause of the differences between the SuperMix small-signal conversion gain and the IF output power in the MultiTone and experimental results, which use a moderate RF signal level.

Figure 7 illustrates one of the main differences between SuperMix simulations and the MultiTone software package. MultiTone cannot produce simulated noise temperature data, while SuperMix can only analyse mixers in the limit of small signals at frequencies other than the LO and its harmonics. The mixer noise temperature is dominated by the high conversion loss (more than 25 dB at all bias points) rather than high noise in the IF band, and this could be considerably improved with better choice of the various embedding impedances.

Finally, in figure 8 we plot the mixer conversion loss from SuperMix against the LO drive voltage at three bias points. At non-zero bias voltages the conversion loss against LO voltage curve is strongly peaked. The best conversion loss occurs at zero bias, and at this point the mixer is least sensitive to variations in the LO power.

IV. CONCLUSIONS

Both SuperMix and MultiTone based simulations can analyse subharmonically pumped SIS mixers, although some contortions are required to carry out these calculations. SuperMix simulations are particularly useful when designing, and finding the basic operating state of subharmonically

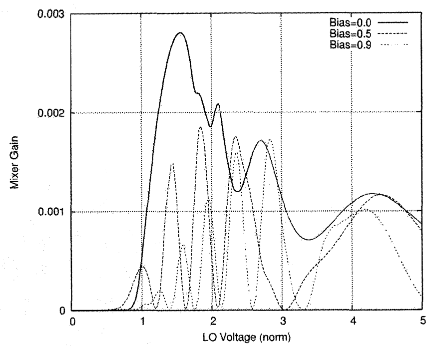

Fig. 8. SuperMix calculated conversion gain of the 20th harmonic mixer as a function of LO power at three bias voltages.

pumped mixers. However in many applications, the subharmonic mixer will be operated with a moderate RF signal power. MultiTone simulations of the mixer will be essential in ensuring the linearity of the mixer over the dynamic range of the measurement in these applications, and when comparing simulations with the performance of mixers fed by a moderate RF signal.

Both software packages require further work to carry out these simulations in a simple and transparent way. In particular, the accuracy of SuperMix's RF current calculation routine should be adjusted to allow the accurate calculation of high LO harmonic currents, and the MultiTone software should be altered to include all the harmonics of the LO signal up to the harmonic number of the mixer. It should then be simple to simulate more realistic subharmonically pumped SIS mixers, and to compare measured performance with both SuperMix and MultiTone simulations.

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