Ultrafast Superconducting Digital Circuits for Analysis and Processing of Microwave Signals

Pascal Febvre, Torsten Reich, Thomas Ortlepp and F. Hermann Uhlmann

***Abstract*—: Rapid-Single-Flux-Quantum (RSFQ) digital circuits based on shunted Josephson junctions have been shown to be able to digitally process signals up to several hundreds GHz. The principle of operation is based on the generation, transport and processing of picosecond voltage pulses of 2.07 mV.ps quantized area, which corresponds to one quantum flux h/2e. Operation of RSFQ circuits can be performed with clock frequencies of the order of 20 to 50 GHz with present technology. We will present current developments and experimental results obtained so far, based on circuits developed in the JeSEF RSFQ process of IPHT Jena[1], using a 1 kA/cm2 shunted Nb/Al-AlOx/Nb junctions. Some particular potential applications for space will be emphasized.**

***Index Terms*—superconductor, RSFQ, Single-Flux-Quantum, superconducting electronics**

I. INTRODUCTION

WITH their high intrinsic speed of several tens of GHz and very low dissipation, superconductive circuits open the way to high-speed digital electronics. In Rapid Single Flux Quantum (RSFQ) digital circuits based on shunted Josephson junctions [2], digital data are transmitted through picosecond voltage pulses with quantized area of 2.07 mV-ps, corresponding to one magnetic flux quantum Φ_0. Hence, the pulse voltage is weak, of the order of 1 mV or less: it corresponds to pulses of about 2 ps duration, depending on the technological process under concern. It is possible to use such a technology to develop different ultrafast processing circuits like Analog-to-Digital Converters (ADCs), autocorrelators, processors or routers. Such circuits can work with clock frequencies above 20 GHz and can reach hundreds of GHz if they are based on the most aggressive technologies. These circuits rely on basic RSFQ cells that can process quanta of magnetic flux under the form of picosecond voltage pulses. The pulses are usually transmitted through RSFQ transmission lines. Flip-flop cells are used to transform the SFQ pulses in Non-Return-to-Zero (NRZ) signals, compatible with classical semiconductor electronics.

II. RSFQ BASICS

The basic element for RSFQ logic is the Resistively-Shunted Josephson junction. For the common RSFQ processes, the resistive shunt is deposited externally. Nevertheless, self-shunted Josephson junctions (naturally occurring for high-T_c materials) present some advantages in terms of integration and ultimate frequencies, though this kind of technology is not yet mastered to make complex circuits. Figure 1 shows the equivalent electrical circuit of a shunted Josephson junction. To keep the following analysis general, we will call R_{shunt} the total shunt resistor of the junction, composed of the junction normal resistance R_N in parallel with an eventual external shunt resistor.

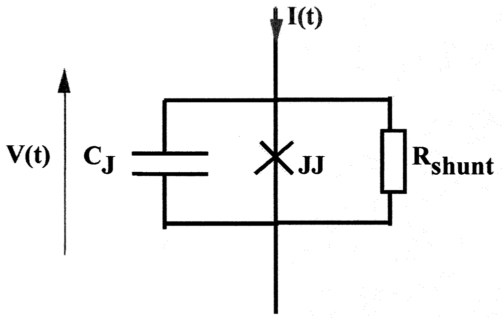

Figure 1: RSJ electrical model of a Josephson junction shunted by an external shunt resistor R_{shunt}. C_J is the Josephson junction capacitance.

The two Josephson equations, controlling the dynamics of the Josephson current are:

$$I_J = I_c \sin\varphi(t) \text{ and } V(t) = \frac{\Phi_0}{2\pi}\frac{\partial\varphi(t)}{\partial t} \tag{1}$$

where $\Phi_0 = \frac{h}{2e}$ is the magnetic flux quantum, φ the difference between the phases of the two macroscopic wavefunctions associated to the two superconductors forming the Josephson junction and I_c its critical current. The total current flowing through the device can be written:

$$I = C_J \frac{dV(t)}{dt} + \frac{1}{R_{shunt}} V(t) + I_c \sin\varphi(t) \tag{2}$$

If one writes L_{J0}, the zero-current Josephson inductance, usually defined by[3]:

$$L_{J0} = \frac{\Phi_0}{2\pi I_c} \tag{3}$$

Manuscript received May 31, 2006.

Pascal Febvre is with the Microwave and Characterization Laboratory (LAHC), University of Savoie, 73376 Le Bourget du Lac, France. (e-mail: pascal.febvre@univ-savoie.fr).

Torsten Reich, Thomas Ortlepp and F. Hermann Uhlmann are with the Department of Electrical Engineering and Electromagnetic Fields, University of Technology Ilmenau, PO Box 10 05 65, D-98684 Ilmenau, Germany.

and $i = \frac{I}{I_c}$ the reduced current flowing through the device, equation (2) can be rewritten under the usual form, using equations (1):

$$i = L_{J0} C_J \frac{\partial^2 \varphi(t)}{\partial t^2} + \frac{L_{J0}}{R_{shunt}} \frac{\partial \varphi(t)}{\partial t} + \sin \varphi(t) \qquad (4)$$

For low values of the phases, the sinus is about linear and equation (4) is simply the one of a parallel RLC circuit, whose dynamics is well known. Three natural frequencies appear:

1- the plasma angular frequency of the parallel LC circuit $\omega_{pl} = \frac{1}{\sqrt{L_{J0} C_J}}$ associated to the period: $\tau_{pl} = 2\pi \sqrt{L_{J0} C_J}$;

2- the cutoff angular frequency of the parallel LR circuit $\omega_{LR} = \frac{R_{shunt}}{L_{J0}}$ associated to the relaxation time $\tau_{LR} = \frac{L_{J0}}{R_{shunt}}$;

3- the cutoff frequency of the parallel RC circuit associated to the relaxation time $\tau_{RC} = R_{shunt} C_J$.

Equation (4) can be rewritten:

$$i = \frac{\partial^2 \varphi(t)}{\partial \tau^2} + \frac{1}{\sqrt{\beta_c}} \frac{\partial \varphi(t)}{\partial \tau} + \sin \varphi(t) \qquad (5)$$

where $\tau = \omega_{pl} t$ is the time normalized to the Josephson junction plasma period and β_c is the McCumber parameter defined by:

$$\beta_c = \left(\frac{\omega_{LR}}{\omega_{pl}} \right)^2 = \frac{\tau_{RC}}{\tau_{LR}} = \frac{R_{shunt}{}^2 C_J}{L_{J0}} = \frac{2\pi R_{shunt}{}^2 C_J I_c}{\Phi_0} \qquad (6)$$

Normalized equation (5) is identical for all Josephson junctions and can be considered as a law of corresponding states for Josephson devices. The time constants defined above can be rewritten as follows:

$$\tau_{RC} = \frac{\sqrt{\beta_c}}{2\pi} \tau_{pl}; \tau_{LR} = \frac{1}{2\pi \sqrt{\beta_c}} \tau_{pl}; \tau_{pl} = \sqrt{\frac{2\pi \Phi_0 C_S}{j_C}}$$

where C_s is the specific capacitance of the Josephson junction and j_C the Josephson junction critical current density.

It is well known that, for a linear parallel RLC circuit ($\sin \varphi \approx \varphi$) the damping coefficient is:

$$\xi = \frac{1}{2\sqrt{\beta_c}} \qquad (7)$$

while the envelope of the signal decreases as $\exp(-\xi \omega_{pl} t)$ in the damped oscillatory and critical regimes. This means, from the above expressions of ξ and ω_{pl}, that the relaxation time constant of a shunted Josephson junction excited by a short electrical signal, is 2 τ_{RC}. For junctions without external shunts, the parallel resistor of figure 1 is simply the normal state resistance R_N. For such hysteretic junctions, the McCumber parameter is usually high, above 10, which corresponds to quite high values of the relaxation time constant 2 τ_{RC}. For instance, for a typical 4 μm^2 junction with a current density of about 10 kA/cm^2, 2 τ_{RC} = 4.6 ps. If one defines the ultimate frequency by $1/(12\tau_{RC})$, corresponding to the time needed by the junction to switch to a resistive mode (3 times $2\tau_{RC}$) and switch back to initial state (3 times $2\tau_{RC}$ again), one finds an ultimate frequency of operation of about 40 GHz with an aggressive high-current density technology. This partly explains why the initial Josephson latching logic developed in the early eighties never met the expectations and has been given up.

In order to increase the speed of the circuits, the RC constant has to be lowered, by either reducing R or C_J. Since the capacitance is proportional to the size of the junction, a reduction of the junction size will not lower the RC constant since the normal resistance R_N will increase proportionnally. Even for the case of a resistor R made out of an external shunt resistor, independent of the junction size, a reduction of the junction size will decrease the critical current, increase the Josephson inductance, hence the LR constant. Moreover, some noise considerations have to be taken into account if one uses very small Josephson junctions. A better solution is to externally shunt the Josephson junction to reduce the RC constant while keeping it high enough not to increase too much the LR constant.

The optimum trade-off is clearly obtained when the RC constant is lowered to reach the LR constant. From equation (6), this case is reached when the McCumber parameter β_c is equal to unity. Consequently, the LR and RC time constants become directly connected to the Josephson plasma time constant. The new characteristic time constant is $\tau_0 = \sqrt{\frac{\Phi_0 C_S}{2\pi j_C}} = \frac{1}{2\pi} \tau_{pl}$. The associated maximum clock frequency of operation can be defined by $f_{0,\max} = \frac{1}{2\pi \tau_0} = \frac{1}{\tau_{pl}} = \sqrt{\frac{j_C}{2\pi \Phi_0 C_S}}$. It is shown in figure 2.

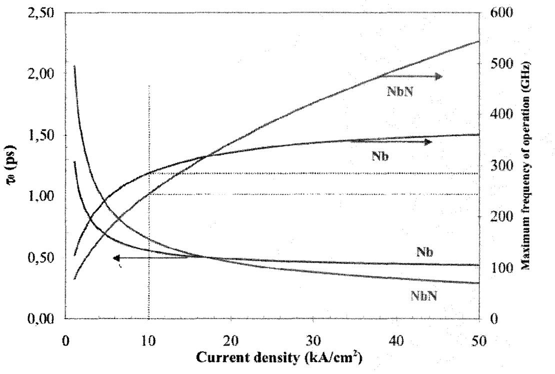

Figure 2: Expected maximum frequency of operation of RSFQ circuits for Nb-based and NbN-based Josephson junctions.

By appropriately shunting the Josephson junctions so that the McCumber parameter is one, one can then perform dynamic switching at high speed [4]. Depending on how the junctions are arranged within superconducting loops, quantized picosecond pulses, associated to the dynamic switching of the Josephson junctions, can be generated, propagated and stored in some cells under the form of quanta of magnetic flux. To account for such behaviour, the non-linear response of Josephson junctions need to be put back in equation (2). Though the solution of this general equation is much less trivial than its linear counterpart, the global behaviour, in terms of relaxation times, remains unchanged.

III. APPLICATION TO DIGITAL SQUIDS

A. Introduction

It is possible to use the RSFQ technology to develop digital Superconducting Quantum Interferometer Devices (SQUIDs). These devices can count quanta of magnetic flux under the form of picosecond voltage pulses. The pulses are processed by RSFQ transmission lines and flip-flop cells which transform the SFQ pulses in Non-Return-to-Zero (NRZ) signals, compatible with classical semiconductor electronics. The main advantage of digital SQUIDs, compared to their analog counterparts, is their theoretically infinite dynamics, which should permit to measure absolute magnetic fields with a very high accuracy. The other advantage is connected to their intrinsic very high slew-rate, of the order of 10^9 Φ_0/sec, corresponding to the digital clock rate of the SFQ circuits.

B. Principle of operation and results

The principle of operation of the digital SQUID is presented in [5-6]. Figure 3 shows the circuit diagram of the SQUID.

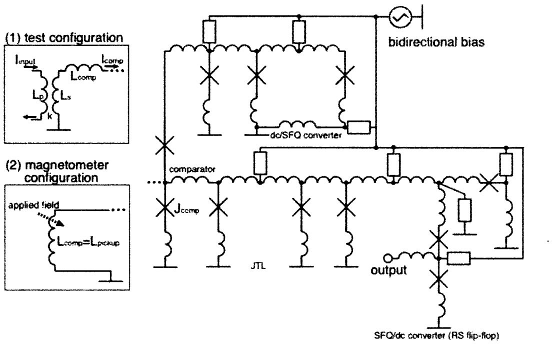

Figure 3: Lumped circuit diagram of the digital SQUID device. Test configuration (1) is used to test the SFQ part of the SQUID device, while the magnetometer configuration (2) is the configuration to be used for regular operation.

A new interesting feature about such SQUIDs deals with its bidirectional bias which consists of a train of alternate negative and positive SFQ pulses, generated by the clocked dc/SFQ converter. It allows to "follow" the signal to measure with only one digital output. Indeed, the output signal consists of negative or positive SFQ pulses, depending respectively whether the signal to measure increases or decreases. The key part of this device is the ac SQUID defined by the comparator junction J_{comp} in parallel with inductance L_{comp} (see figure 3). This junction switches, generating an SFQ pulse which is processed by the JTL and the SFQ/dc converter, in two cases: a) when the additional current produced by an increase of the magnetic field to detect adds up in the comparator junction with the current associated to a positive SFQ pulse: a positive SFQ pulse is getting out of the ac SQUID loop; b) when the current negative variation produced by a decrease of the magnetic field to detect adds up with the current associated to a negative SFQ pulse: a negative SFQ pulse is getting out of the ac SQUID loop. In other cases, for each negative of positive clock pulse coming on J_{comp} junction, there is no generated output pulse. A picture of such a digital SQUID is shown in Figure 4. It has been fabricated in a certified ISO9001 foundry at IPHT Jena [1]. The shunted Josephson junctions, based on a Nb/Al-Al_2O_3/Nb trilayer, have a current density of 1 kA/cm^2 and an $R_N I_C$ product of 256 µV.

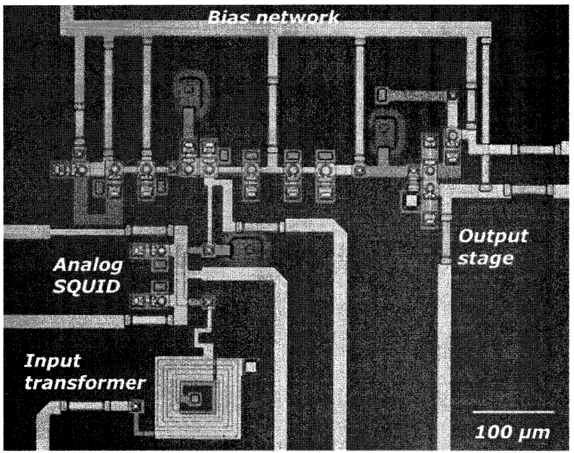

Figure 4: Picture of the digital SQUID made of 11 shunted Nb/Al-Al2O3/Nb Josephson junctions. Junctions have a current density of 1 kA/cm2. Fabrication has been performed at IPHT Jena in Germany. The analog SQUID on the left is not part of the overall circuit described in figure 1. It has been added there for additional functionalities, not described in this article.

Figure 5 shows the output signal measured with a low-frequency 1 kHz digital clock. The signal to be measured is a 5 Hz sinusoïdal pattern.

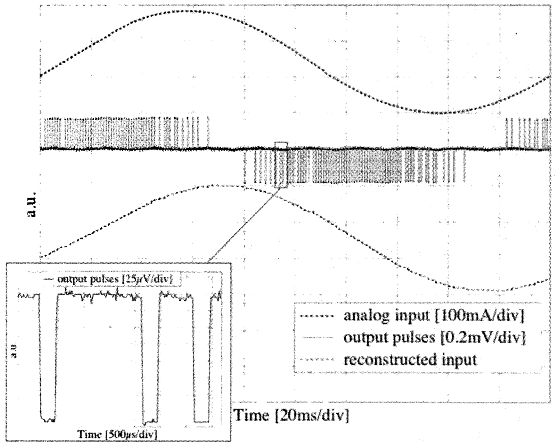

Figure 5: Measurement of a sinusoïdal input signal corresponding to a magnetic field variation (top curve). The central curve shows the measured signal at the digital SQUID output. The bottom curve shows the input signal reconstructed from the digital measurements.

IV. CONCLUSION

The first successful measurements of RSFQ circuits for digital SQUIDs have been performed. This technology presents some interest for very high-speed processing of ultrafast signals, that can be fed from and transmitted to the "room-temperature world", and sensitive detection of magnetic and electro-magnetic signals. To that regards, space scientific applications have always been a powerful engine to pull such technologies. Connected to that particular point, we foresee different fields for which such technologies are of high interest, like low-consumption, high-speed and high resolution broadband autocorrelators, or on-chip multiplexing for future submillimeter heterodyne imaging systems.

ACKNOWLEDGMENT

We want to strongly acknowledge the contribution of Juergen Kunert and Hans-Georg Meyer from IPHT Jena for the device fabrication and helpful advice. One of us (T Reich) would like to thank SCENET—European Network for Superconductivity for financial support during his scientific exchange stay that allowed him to perform the experimental measurements presented in this paper.

REFERENCES

[1] http://www.ipht-jena.de/BEREICH_1/abt13_cryo_electronics/index.html

[2] K. K. Likharev and V. K. Semenov, "RSFQ logic/memory family: A new Josephson-junction digital technology for sub-terahertz-clock-frequency digital systems," *IEEE Trans. Appl. Supercond.*, vol. 1, pp. 3-28, 1991.

[3] A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect*, John Wiley & Sons Inc., 1982.

[4] W. Chen, A. V. Rylyakov, Vijay Patel, J. E. Lukens, and K. K. Likharev, " Rapid Single Flux Quantum T-Flip Flop Operating up to 770 GHz," *IEEE Trans. on Appl. Supercond.* vol. 9, pp. 3212-3215, June 1999.

[5] T. Reich, T. Ortlepp and H.F. Uhlmann, Digital SQUID sensor based on SFQ technique, *IEEE Trans. Appl. Supercond.*, vol. 15, no. 2, pp. 304-307, 2005.

[6] T. Reich, T. Ortlepp, H.F. Uhlmann and P. Febvre, Experimental analysis of a digital SQUID device at 4.2 K, *Superconductor, Science and Technology*, vol. 18, no. 8, pp. 1077-1081, August 2005.