Twodimensionally distributed Model for HEB based on Random Phase Transitions

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ABSTRACT

According to AFM measurements, ultrathin NbN films are not smooth but exhibit a more or less random distribution of microcrystallites. This gives rise to a superconducting to normal conducting phase transition that follows a first order percolation phase transition. Such a phase transition is easily described by a single critical exponent justifying the assumption of a simple, local R(T) curve used in all distributed HEB models.

Keywords: percolation phase transition, hot electron bolometer, mixing theory, mixing

1. INTRODUCTION

HEB age as a function of film thickness. Due to the random nature of the sputtering process, NbN is deposited in a random manner (c.f. Fig. 1).

Exposing this film to aggressive reactants will remove atom for atom of the film. Keeping in mind, that only close stochiometric $NbN_{0.92...1.08}$ will exhibit superconductivity at elevated temperatures, it becomes clear that any replacement of NbN by NbO locally will change the properties of the NbN film. Any attack will be associated with a reduction of the conduction properties of the film and lead to an increase of the resistivity of the film. For very thin films (as in the case for heterodyne receiver devices with a thickness of merely $3.5\mu m..5.5\mu m$) any individual attack will cause substantial deviations of the conduction pattern. In this paper we will first refer to a model for the random attack of a random thick conducting structure. One arrives at a resistance versus attack curve essentially describing the aging process of a HEB device. This curve is not sufficient to describe the superconducting properties of the film. So therefore the superconducting and super-to-normal conducting properties of the attacked NbN structure are taken into account to calculate the R(T) curve for a specific attack state.

It becomes obvious that the critical current of the film scales differently on the conductivity reduction due to oxididization than the room temperature conductivity. Therefore a local critical current and critical temperature

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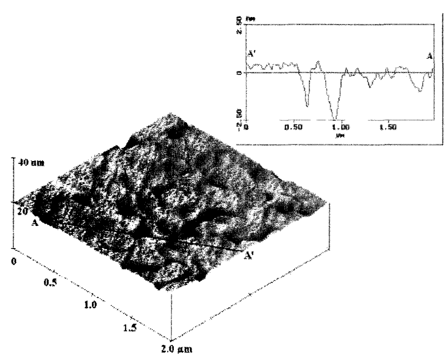

Figure 1. Atomic Force Microscopic cross section of a NbN film used for HEB production The inset shows the height curve along the line A Aín the 2D plot.

distribution is calculated and the two-dimensional current distribution is obtained in a way similar to a Bean model for the superconductor. Superconductivity breaks down as soon as the local distribution of the given bias current exceeds the maximum available critical current density along the most current bearing link in between the antennas. The so-obtained R(T) curve still does not account for local heating phenomena. In a third step the two-dimensional resistance structure is used to calculate a two dimensional temperature distribution by taking the local LO and bias heating powers into account. From this a nonequilibrium resistive transition is obtained that is inherently different from the equilibrium (non-heated) transition used in the R(T) curve. Based on the nonequilibrium heating properties, IV curves are calculated. In a fourth step, these nonequilibrium heating curves are used in a small signal model: The small signal model describes the current and power distribution in the HEB as a function of a (dominant) heating and a (small) beating term at the intermediate frequency. There local hot spots are generated wherever the local heating power density is sufficient to exceed the critical temperature at the (self consistently) obtained local critical current density. Classical expressions as conversion gain, quantum noise, thermal fluctuation noise and thermal noise can be derived from the small signal model. Nevertheless, the journey is not finished here: Creating hot spots in this random structure occurs almost instantaneously (at the time required to expel the magnetic fields from a previously superconducting area). The hot spot model here is no longer limited to a single normal conducting area but normal conducting random islands are created in regions where the heating conditions for local superconducting breakdown are fulfilled. Having created a hot spot, the delivered heating power is used much more efficiently than in the superconducting case due to local Andreev reflection boundaries. Therefore the film will exhibit local hysteresis (clearly seen in underpumped IV curves on HEB). In addition this hysteretic behavior explains part of the 1/f noise seen in HEB receivers. Small spontaneous fluctuations create an isolated hot spot that then takes arbitrarily long time to decay.

2. LUMPED CIRCUIT EQUIVALENT OF A NBN FILM

The HEB and its adjacent antenna pads are discretized using resistors in a 2D cartesian mesh. The local conductivity averaged over a discretization area is directly translated in a resistance value for this discrete element. For the antenna pads, the resistance of the film plus the Au cover resistance is taken into account. For the HEB film, the resistivity depends on the number of intact NbN molecules per area element. Discretizing the number of available NbN molecular layers, one ends up with a two dimensional resistance mesh containing all local properties of the element. Therefore, for each node point $[i : j]$ on the mesh, one has to solve Kirchhoff's law requiring the continuity of currents:

$$I_{source,i,j} - I_{ground,i-1,j} = I_{X,i,j} + I_{Y,i,j} - I_{Y,i,j-1} - I_{X,i-1,j} \tag{1}$$

There $I_{source,i,j}$ is the current injected at the node $[i, j]$ (mostly zero expect at the outer terminals), $I_{ground,i,j}$ is a current flowing from the node directly to ground (set zero here) and I_X (I_Y)denotes the current in X(Y) coordinate direction through the resistor "to the right of" (respectively "above") the node. The resistors are grouped in R_X (R_Y) being "horizontally" ("vertically") oriented and the indices i (j) runs in X(Y) coordinate direction (c.f. 2). In this part of the analysis $Y_{G,i;j} = 0$.

Relating the potentials of the neighboring nodes, one has to set up Ohm's Law for all resistors in the net allowing to eliminate the unknown currents from the above Equation system. One arrives in a linear set of Equations with the nodal potentials $\phi_{i,j}$ as unknown variables.

$$I_{X,i,j} = Y_{X,i,j} \cdot (\phi_{i,j} - \phi_{i+1,j}) \tag{2}$$

$$I_{Y,i,j} = Y_{Y,i,j} \cdot (\phi_{i,j+1} - \phi_{i,j}) \tag{3}$$

$$I_{ground,i,j} = Y_{G,i,j} \cdot (\phi_{i,j}) \tag{4}$$

This complete lumped element equivalent has no unique solution. It is nevertheless uniquely specified up to an arbitrary additive constant to all potentials. Therefore setting one arbitrary potential to zero, there is always exactly an unique solution (supposed the resistors are nonzero and finite).

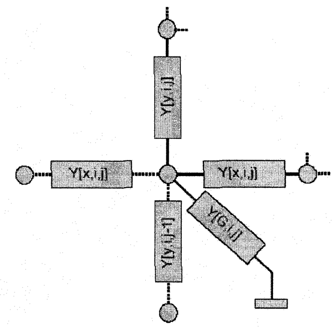

Figure 2. Lumped Element Mesh used to model the HEB circuit.

3. HEB AGING AS A RANDOM DESTRUCTION PROCESS

Aging is modeled starting with a film with a given thickness distribution: One sets all pad resistors to their (constant) values and the conductors on the NbN film are obtained using

$$Y_{X,i,j} = \frac{n[1,i,j]}{N_0} \cdot Y_{0X,i,j} \tag{5}$$

$$Y_{Y,i,j} = \frac{n[2,i,j]}{N_0} \cdot Y_{0Y,i,j} \tag{6}$$

where the "zero values" $Y_{0...}$ are taken from sheet resistance measurements. Assuming a certain number of atomic layers to be present in the film (e.g. $N = 5$ for a $35\AA$ thick film), the initial state matrix is set $n[..., i, j] = N$. Now random coordinates are calculated and to each random coordinate pair, the amount of intact layers is reduced by 1 until 0 is reached. During this Monte-Carlo like process ([1]), the currents and voltages through the HEB structure can be calculated at any aging state. Instead of a time axis the number of successful oxidizing attacks is used. Obviously, providing water free environments and applying protection layers on the film will affect the scaling law relating the absolute number of attacks to the number of attacks per time unit and provides therefore a time axis in "storage time under given conditions" to the aging analysis. The scaling law to be used contains the temperature and moisture of the environment in the easiest possible way and models the applied protection layers simply as linear reduction factors of the oxidization attacks. One arrives at a relation where $\alpha \approx 0.1ln2\frac{1}{K}$ according to the thermodynamics of a binary process, a fit parameter $\beta \approx 1.2\frac{1}{h}$ and the values for the protection layers are found in Table 1 :

$$\eta = \frac{\#attacks}{time} = \beta \cdot r_{protection} \cdot c_{H2O} \cdot e^{\alpha T} \tag{7}$$

From aging experiments comparing a set of protection layer technologies with each other, one finds delay factors summarized in the following Table 1.

Typical developments of the resistance versus number of attacks are shown in Figure 3. It is necessary to repeat the Monte Carlo process several times to arrive at a statistically relevant base for the development of the resistance with time. It is obvious, that this statistic nature of aging is also reflected in a spread in the measured resistance values in aging experiments. The following data refers therefore to mean values obtained by running a vast number of independent Monte Carlo simulations on the same geometry.

Table 1. Protection factors for a set of protection layer technologies

Protection Layer	r [1]	Comments
Photoresist	1.00	Reference case
SiO_2	1.90	
Si	3.1	
$Si + Si$	4	Worst Case
$Si + Si$	11	10 devices
$SiO_2 + SiN$	13	

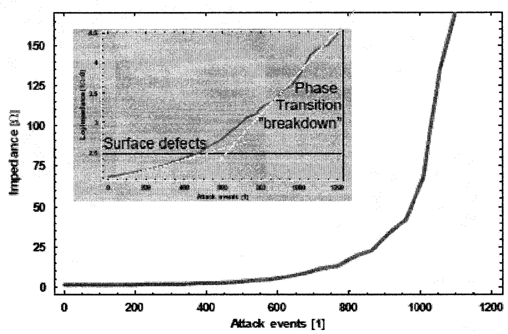

Figure 3. Average room temperature resistance as a function of successful attacks for 100 individual random model runs.

4. A TWO DIMENSIONAL HEATING MODEL

Using a similar approach to model the RF performance of the HEB, we note that, the HEB chip is electrically small compared to a THz wavelength. Therefore the purely resistive electrical equivalent described in the above is still valid.

Knowing the evolution of the room temperature resistance, it is a straightforard extension to include superconducting effects in the model: One assigns a critical temperature and a critical current density to the film that is at first hand linear in the amount of still available molecular layers. Then the local resistance is either zero (when neither critical current and critical temperature are exceeded locally) or it is a normal resistor with the value given by the remaining thickness.

Obviously we have to calculate the temperature in each resistor of the equivalent lumped element circuit. For the first point, we know the current and voltages in each node and in each resistor allowing us directly to calculate the dc heating power locally. In addition, we know the absorbed THz signal power (being either uniformly in space or restricted to the normal conducting areas when not exceeding the quasiparticle bandgap). The only missing link is to relate the electron temperature in the resistor to the applied heating powers. Discretizing the heat balance equation ([2]) in two dimensions one arrives at a thermal equivalent mesh - the thermal resistors "to ground" contain the phonon cooling process and the resistors linking the neighboring sections of the film contain the diffusion cooling process.For the phonon cooling one obtains:

$$T_(P) = Y_0 + Y_{phonon} \cdot P + ... = \left(T_0{}^{\mu} + \frac{P_{lo}}{\sigma}\right)^{\frac{1}{\mu}} + \frac{\left(T_0{}^{\mu} + \frac{P_{LO}}{\sigma}\right)^{-1+\frac{1}{\mu}} P}{\mu\,\sigma} + \left(\frac{\left(T_0{}^{\mu} + \frac{P_{LO}}{\sigma}\right)^{-2+\frac{1}{\mu}}}{2\,\mu^2\,\sigma^2} - \frac{\left(T_0{}^{\mu} + \frac{P_{LO}}{\sigma}\right)^{-2+\frac{1}{\mu}}}{2\,\mu\,\sigma^2}\right) P^2 \tag{8}$$

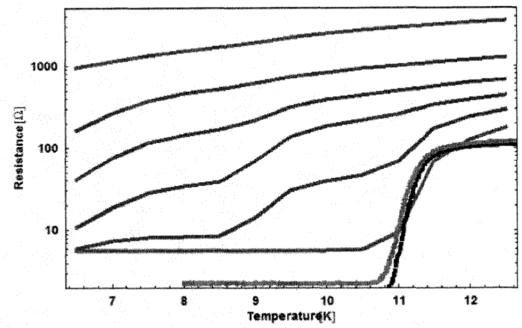

Figure 4. R(T) Curve for the TERASEC mixer chip (pixels 17 and 15 selected)

Please keep in mind that the node indices $[\tilde{i}, \tilde{j}]$ are indices referring to each resistor and are related to the voltage node indices by $[\tilde{i}, \tilde{j}] = [i + \frac{1}{2}, j]$ for a horizontal (X) thermal resistor and $[\tilde{i}, \tilde{j}] = [i, j + \frac{1}{2}]$ for a vertical (Y) thermal resistor. Now the heat flow from and to each node $\psi_{...}$ are related to the electron temperature T by:

$$\psi_{X,\tilde{i},\tilde{j}} = Y_{thermal,X,\tilde{i},\tilde{j}}(T) \cdot (T_{\tilde{i},\tilde{j}} - T_{\tilde{i}+1,\tilde{j}}) \tag{9}$$

$$\psi_{Y,\tilde{i},\tilde{j}} = Y_{thermal,Y,\tilde{i},\tilde{j}}(T) \cdot (T_{\tilde{i},\tilde{j}+1} - T_{\tilde{i},\tilde{j}}) \tag{10}$$

$$\psi_{ground,\tilde{i},\tilde{j}} = Y_{phonon,\tilde{i},\tilde{j}}(T) \cdot (T_{\tilde{i},\tilde{j}}) \tag{11}$$

Using this equivalent, the temperature distribution in the HEB is obtained by solving the above linear equation system.

For each temperature calculation we have to run a set of steps: 1: calculate the random reduced amount of superconducting layers for the given HEB assuming a electron temperature to be equal to the bath temperature (e.g. 4.2K) to all resistors, 2: Find out, which resistors are superconducting and which resistors are normal conductors by comparing temperature and currents with the critical temperature and critical currents. 3: Calculate the voltages and currents on this structure. 4: Calculate the DC and LO powers present in each resistor in the structure. 5: Calculate the electron temperature distribution on the "dual grid". 6: Return to point 2 and repeat the loop until a fixed point is reached.

Performing this process, R(T)- curves (c.f. Fig 4)are readily obtained by setting a very low measurement current (e.g. $1\mu A$) and varying the bath temperature within the desired area.

Specifying a set of bath temperatures, IV curves are shown in Fig. 5.

5. A SMALL SIGNAL MODEL FOR HEB

For a small signal equivalent, the electric model is largely unchanged with the exception of a modified bias circuitry coupling the current sources to an inductor and coupling a 50Ω load (the IF amplifier) to the bias supply using a dc block capacitor. All this is easily accomplished within the 2D lumped element approach described below.

A more fundamental difference is found in the thermal equivalent. For the thermal equivalent, we have to take into account that the phonon cooling to the substrate is a reaction speed limiting bottleneck. Therefore the conductor to ground $Y_{phonon,\tilde{i},\tilde{j}}$ takes the form of a first order low pass RC circuit.

$$T_{\tilde{i},\tilde{j}} = Y_0 + Y_{phonon} \cdot P = \left(T_0{}^{\mu} + \frac{P_{lo}}{\sigma}\right)^{\frac{1}{\mu}} + \mu\,\sigma \cdot \frac{\left(T_0{}^{\mu} + \frac{P_{LO}}{\sigma}\right)^{-1+\frac{1}{\mu}}}{1 + i \cdot \tau_{phonon}\omega_{IF}} \cdot P \tag{12}$$

From the power dissipated in the load resistor, the conversion gain of the HEB is readily obtained.

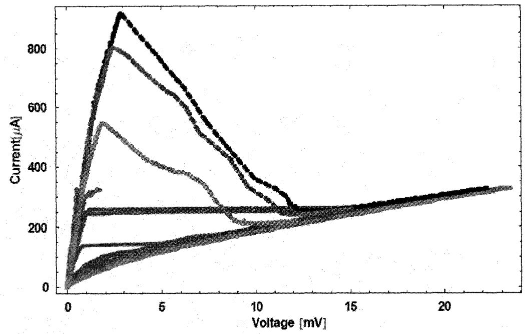

Figure 5. IV Curve for the TERASEC mixer chip (pixels 15 and 17 selected) at 4K, 6K and 8K measured and calculated using LO powers from $0nW$ to $1500nW$

6. 1/F NOISE AND SWITCHING BEHAVIOR IN HEB

For low heating powers, one observes that the conversion gain diverges. This is caused by instability of the HEB and to the failure of any small signal equivalent there. It is found by performing the iterative solution steps from Section 4, that for low LO powers, the obtained point on the IV curve does not converge to a single value but "jumps" between a set of values in a more or less periodic manner. This instability is caused by the fact that a newly formed hot spot (in one iteration) can be easily bypassed as long as there is still a more conductive superconducting link on the device eventually removing heating power from the normal conductor. In the next iteration step, the region is again superconducting and the process starts anew. The real time dependence of this process is determined by the inductive load on the bias supply and on cable lengths and cable position in the cryostat. Frequencies between $30kHz$ and $2MHz$ have been observed experimentally by moving one and the same device between two measurement setups. The switching regime begins as soon as the first resistor in the HEB sees superconductivity suppressed. This switching regime ends upon more pumping or more heating when the heating power loss in the normal conducting areas (due to bypassing) is equalled by the more efficient usage of heating power (due to the upcoming Andreev barrier around the hot spot). For hard pumping, the possibility and efficiency of bypassing shrinks (due to an overall warming of the HEB) therefore reducing the instable area and there is a pumping level where instability is suppressed completely.

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