Graphical Prediction of Trapped Mode Resonances in Sub-mm and THz Networks

Matthew A. Morgan, *Member, IEEE* and Shing-Kuo Pan, *Member, IEEE*

***Abstract*—An analytical method for the visualization and prediction of trapped-mode resonances based on the dimensions of a microwave network is described. The method as explained is intuitive, easy to implement, and has proven itself to be a useful tool in the avoidance of problems associated with trapped modes prior to fabrication, as well as to correct those problems in designs for which the proposed analysis was not carried out in advance.**

***Index Terms*—electromagnetic propagation, orthomode transducer (OMT), radio astronomy, trapped modes**

I. Introduction

WHENEVER the waveguides in a network are overmoded, there exists the possibility of trapped modes. This is especially common with Orthomode Transducers (OMTs) where the required operating bandwidth makes multiple spurious modes difficult to avoid. The most common symptom of trapped modes is the appearance of a suck-out – a sharp, isolated notch in insertion loss – somewhere in the pass-band of the system. For many applications, including some in radio astronomy wherein wideband OMTs are standard components, the presence of a suck-out can be devastating.

Yet, despite a general familiarity with the causes and effects of trapped mode resonances, there appears to be no common practice for their analytical prediction prior to cutting metal. Instead, they are generally regarded as if they were random and unpredictable – perhaps because simulated models, identically lossless and possessed of mathematically-perfect symmetry, do not always reveal them. As such, mode-resonances are most often discovered for the first time during initial testing of the prototype hardware. Then, without a reliable analytical method to guide a revised design, one usually resorts to inserting pieces of absorber to dampen the trapped mode cavity, or placing tighter constraints on manufacturing symmetry to avoid excitation of the unwanted modes. At sub-millimeter wavelengths, the required symmetry is practically impossible to achieve. In addition, field distortion due to losses in the walls near the cutoff of a spurious mode, which is most severe in the sub-mm-wave regime, can also induce mode conversion and provide the linkage between the dominant and trapped modes.

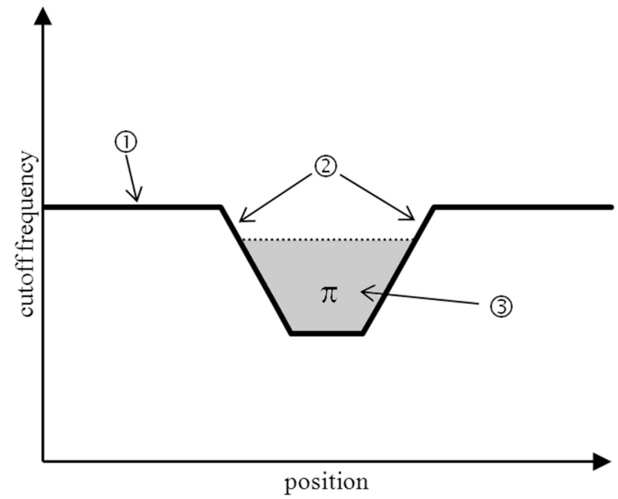

Fig. 1. Graphical representation of a trapped, resonant mode.

In light of these issues, the authors have adopted a graphical approach, which not only helps to visualize potential problems prior to fabrication, but also to suggest easy solutions for relatively mature designs that exhibit resonances. The underlying principles and implementation of this method are described in this article.

II. The Cause of Trapped-Mode Resonances

In order for a suck-out to develop, three conditions must be satisfied:

1. A spurious mode must propagate.
2. The mode must be trapped.
3. The size of the trap must be sufficient for the mode to become resonant.

Notably absent from this list is that most spurious modes also require some kind of asymmetry in the waveguide structure to be coupled with the dominant modes. While strictly true in a theoretical sense, the authors contend that truly perfect symmetry is an impossible ideal that can only be achieved in simulation. In practice, and especially at sub-millimeter-wave

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Matthew A. Morgan and Shing-Kuo Pan are with the National Radio Astronomy Observatory, Charlottesville, VA. 22903 USA (email: matt.morgan@nrao.edu, span@nrao.edu).

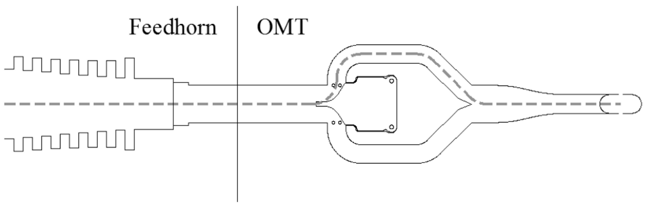

Fig. 2. Section drawing of the ALMA Band 6 (211-275 GHz) feedhorn and OMT. Mode analysis is performed along the dotted line path. Trapped modes are present in the shaded areas.

frequencies, there is always some asymmetry present, if only as a consequence of imperfect manufacturing or part-to-part alignment. Further, the development of a suck-out is intrinsically non-graceful – the onset of bad effects is practically instantaneous on either side of perfect symmetry. It is far better to assume that if a trapped mode can propagate, it will propagate. Fine-tuning the symmetry can only reduce the amount of coupling into the trapped mode, effectively increasing the loaded Q of the resonator. For these reasons, the authors believe that attempts to improve the symmetry may reduce the level of a suck-out, even to the extent that it can no longer be detected in a laboratory setup, but if the above three conditions are met, then it will always be present to some degree.

III. Graphical Representation of Trapped Mode Resonances

The three conditions for mode resonance introduced in the previous section can be visualized graphically as shown in Fig. 1, wherein the cutoff frequency of a spurious mode is plotted as a function of position along the signal path in a waveguide network. Since waves will propagate in this mode at any frequency above this curve, the first condition is met simply if the line at any point passes below the upper frequency limit of the operating band for the system in question. The second condition is then met if at any point the curve reaches a local minimum – a *well*, where the mode can propagate over certain frequencies that are cutoff at both ends. Finally, the third condition is met if the well is deep enough and/or long enough that the cavity is half of a guided-wavelength long at a frequency where the mode is still trapped. (To be rigorous, the exact resonant frequency will depend on the reactive impedances presented by the discontinuities at both ends, as well as intermediate reflections from internal impedance-changes, but the half-wave condition is a good approximation in most cases.) This may be estimated by integrating the propagation constant across the cavity to find the total phase

$$\theta(f) \approx \int_{x_1}^{x_2} \beta(x)dx = \int_{x_1}^{x_2} \sqrt{k^2 - k_c^2(x)}dx = \frac{2\pi}{c}\int_{x_1}^{x_2} \sqrt{f^2 - f_c^2(x)}dx \quad (1)$$

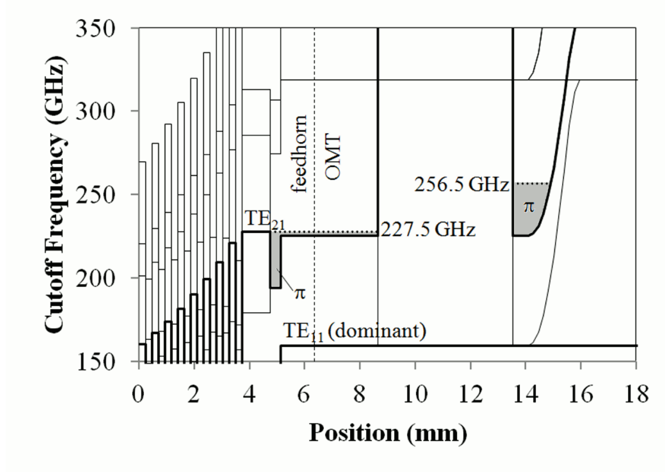

Fig. 3. Plot of the ALMA Band 6 mode analysis predicting two in-band resonances at 227.5 GHz and 256.5 GHz.

and represented graphically by shading in the bottom of the well. The resonance condition is satisfied when $\theta=\pi$. The method thus consists of plotting the cutoff frequencies of all the lowest order modes that can propagate in the structure through each branch of the waveguide network, examining these plots to identify local minima, and then evaluating the integral (1) for each of the minima to determine if the trapped mode will resonate. No assumption is made about how the trapped modes are excited. As stated above, it is simply assumed that if a mode can propagate, then it will be present at some level.

IV. Analysis Example and Measurements

To illustrate this method, we consider an example, shown in Fig. 2. This diagram consists of a section of a conical, corrugated feedhorn and Bøifot-style OMT made for Band 6 (211-275 GHz) of the Atacama Large Millimeter Array (ALMA) [2]. The transition from circular-to-square waveguide occurs inside the feedhorn and the flange interface between the horn and the OMT is in square waveguide. A thin, metal septum and vertical wires complete the Bøifot junction. Strictly speaking, the forward tip of the septum creates a very short section of quasi-coaxial waveguide, but this detail is omitted from the analysis for simplicity. The feed-OMT combination is modeled instead as a simple series of circular-, square-, and rectangular-waveguide sections following the side-arm branch of the OMT. The cutoff frequencies are plotted in Fig. 3.

Two trapped-mode resonances are predicted from this plot. The first, at 227.5 GHz, arises from the $TE_{21,a}$ mode trapped between the circular-to-square junction in the feedhorn and the Bøifot junction in the OMT. This resonance is particularly insidious because it requires the two components to be mated to manifest itself, and would be sensitive to (among other things) the alignment of the flanges at the interface. Measurements of either component alone may not even show a resonance, and if they did, it could be at an entirely different

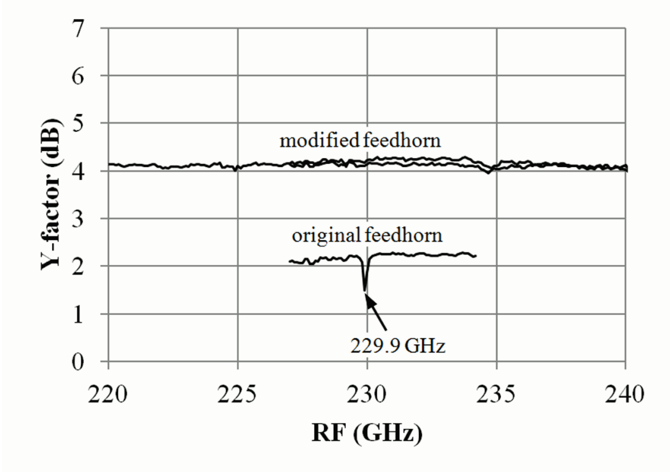

Fig. 4. Measured 230 GHz resonance in an ALMA Band 6 receiver with original and modified feedhorns. Vertical offset added for clarity.

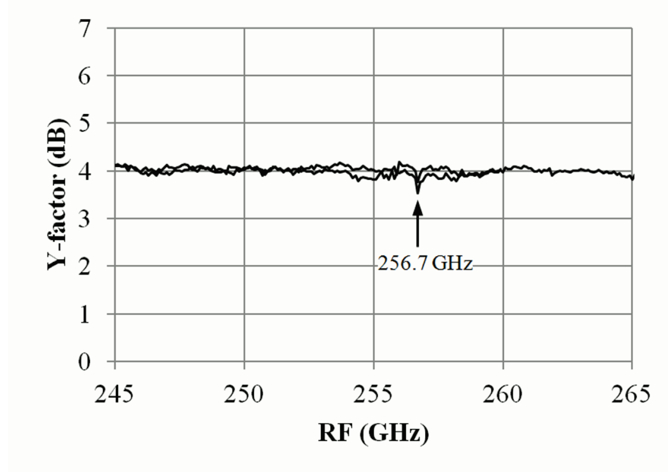

Fig. 5. Measured 256 GHz resonance in an ALMA band 6 receiver.

frequency. This fact highlights the need to consider whole systems, not just sub-components, and to be especially careful of interfaces that occur in overmoded waveguide – or better yet, to avoid such interfaces altogether.

The second resonance, predicted at 256.5 GHz, occurs in the passive combiner between the two side-arms. The two rectangular waveguides are first joined to form a square waveguide, then the square waveguide is tapered back down to rectangular again. This causes the modes in the combiner to drop in cutoff frequency, then rise again, forming a cutoff well. As we have now seen, such a feature is a good recipe for trapped-mode resonances to occur. Both the $TE_{21,a}$ and TM_{01} modes, which are degenerate in square waveguide, exhibit this resonance.

To be fair, the potential for trapped modes were well understood by the designer of this OMT [2] who warned against carelessness at the interface to the feed and documented the tradeoffs associated with the sidearm combiner. Nevertheless, despite these warnings the issues were not fully appreciated by the broader engineering community, highlighting the lack of attention to trapped modes that is common in the design phase of a project.

Although it is unfortunate for the Band 6 project that these issues were not better understood until after most of the production feedhorns and OMTs had been fabricated, the situation does provide us a unique opportunity to verify the theory through measurement. The lower (and, it turns out, more prominent) resonance at 227.5 GHz was not observed during qualification of the prototype components because, as described earlier, it is only present at that frequency when the horn and OMT are joined together. Resonances which were measured at the component-level were dismissed as artifacts of the tapered square-to-rectangular transitions used in the test set – an assumption seemingly confirmed when those resonances disappeared after inserting resistive vanes into the tapers. Of course, since those resistive vanes absorb energy from the higher-order modes as well as the orthogonal dominant mode for which the vanes were intended, the basis for dismissing these resonances was not entirely justified.

Y-factor and noise temperature measurements proved to be the most effective way of observing these resonances in actual hardware. Fig. 4 and Fig. 5 show measurements of these resonances in an actual ALMA Band 6 receiver. Multiple curves on each plot correspond to the different LO tunings and/or upper- and lower-sidebands. The first resonance is evident in Fig. 4 at 229.9 GHz, slightly higher than predicted. A small shift upward in frequency, about 0.4%, was expected due to shrinkage of the metal (brass) which is cooled in this application to 4 Kelvin. The discrepancy here is much larger than that, however, and is probably due to the geometric approximations that were made around the septum area to simplify analysis. The cutoff plane for the $TE_{21,a}$ mode must be closer to the tip of the septum than was estimated, effectively shortening the trapped-mode cavity. Nonetheless, an error of only 1.1% relative to prediction (0.7% after accounting for thermal contraction) is an encouraging validation of the theory. The higher predicted resonance at 256.5 GHz was also detected (Fig. 5), although in fewer production units and at a weaker level. In this case, the full geometric detail of the cavity was included in the model, and the predicted location of the resonance was much more accurate.

Because the lower suck-out appears near the bottom of the Band 6 operating range, it was initially suggested that the leading square waveguide in the OMT could be lengthened, thus lowering the resonant frequency and tuning it out of band. However, the mode in question is very near cutoff in this region, and thus has an extremely long guided wavelength, approaching infinity as the resonant frequency is lowered. Large changes in length have only a small effect on resonant frequency, and no matter how long this section was made, the resonant frequency would never get lower than the cutoff of the upper step in the well at 225.5 GHz, which is still in band. Close examination of Fig. 8 suggests a better solution. With the TE_{21} mode cutting off very near to 230 GHz in the circular throat of the feedhorn, the well is just barely deep enough as it is to support the first resonance. This analysis confirmed that a mere 0.8% increase in the diameter of this one section would be sufficient to eliminate the resonance with a comfortable margin. Interestingly, the suck-out in this way is not tuned out of band, but entirely out of existence, vanishing from the spectrum in place, and with little observable effect on the behavior of the rest of the system. This modification was made in a small number of later feedhorns and the resulting clean spectrum for a typical unit is shown in Fig. 4. The curve exhibiting the resonance was offset vertically for clarity.

V. CONCLUSIONS

The causes and effects of trapped-mode resonances or suck-outs in overmoded waveguide systems have been reviewed in detail. A graphical technique for visualizing the conditions that lead to trapped-mode resonances has been described, and used to analyze a number of real-world examples in the sub-millimeter-wave regime. The technique was then verified by comparing measured evidence of trapped modes in fabricated hardware against the predictions. Despite a number of simplifying assumptions, the suck-outs were observed within 1% of their predicted frequencies, and disappeared from the data when the corrective modifications suggested by the technique were implemented. These examples highlight the power of this kind of analysis to visualize the problems as well as the simplest solutions to trapped-modes in complex waveguide networks.

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