Metamaterial-Like Properties of the Distributed SIS Tunnel Junction

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Abstract— **The propagation factor of the Niobium-based distributed Superconductor-Insulator-Superconductor (SIS) tunnel junction has been modelled for frequencies between 70-200 GHz. In the modelling, the distributed SIS junction was simulated as a microstrip distributed component. Furthermore, in our studies, the reactive part of the SIS tunnel junction quantum admittance was included in the model circuit diagram. It was shown that the SIS quantum reactance has significant effects on both the loss factor, Re (γ), and phase factor, Im (γ), where γ is the complex propagation constant.**

I. Introduction

The Superconducting-Insulator-Superconductor (SIS) mixers have been extensively studied both theoretically and experimentally since the 1970s [1],[2]. The outcome of the research allowed fundamental understanding in the SIS physics as well as receiver design topologies using SIS junction as the mixer element. Currently the receivers based on SIS mixers are the most sensitive heterodyne receivers available from approximately 100 GHz -1000 GHz [3],[4],[5],[6]. These receivers are mainly used for radio astronomy applications. However, much of the the previous studies of the SIS mixers considered lumped SIS elements.

In 1996, V. Belitsky published an extensive theoretical paper treating the Nb-AlOx-Nb SIS junction as a distributed microstrip line tunnel junction [7]. However, the theory did not include the reactive component of the SIS quantum admittance, which originates from the real part of the response function, I_{KK}, as a result of the Kramers-Kronig transform of the DC tunnel current [1]. Moreover, it can be found from [1] that the reactive part of the quantum admittance is proportional to I_{KK}.

The reactive admittance was not included in [7] because it was assumed to be negligible compared to the geometrical capacitance of the SIS junction, which in normal operation of the SIS mixers is often the case. However, for some bias points, the reactive part of quantum admittance has significant effect. Since the quantum admittance is "connected" in parallel to the geometric capacitance of the SIS microstrip tunnel junction, this will have further effects on the behavior of distributed SIS junctions. Furthermore, the reactive part of the quantum admittance can take inductive as well capacitive values depending on the DC bias voltage [8]. For these reasons, the reactive part needs to be included in the SIS microstrip line tunnel junction model, especially when considered as a non-mixing element.

The aim of this paper is to study how the inclusion of the reactive part of the SIS quantum admittance changes the behavior of the SIS distributed tunnel junction, specifically the propagation factor.

II. Theory

In order to calculate the reactive part of the SIS quantum impedance, we need to calculate the Kramers-Kronig transform of the DC tunnel current, $I_{KK.}$ The reactive part of the SIS quantum admittance can then be calculated according by the following expression [1]:

$$B_{11} = \frac{e}{2\hbar\omega}\sum_{n=-\infty}^{\infty}[J_{n-1}^2(\alpha) - 2J_n^2(\alpha)+J_{n+1}^2] \times I_{KK}\left(V_0 + \frac{n\hbar\omega}{e}\right) \qquad (1)$$

where $J_n(\alpha)$ is the Bessel function of the order n.

The current response function *r(V)* is defined as following:

$$r(V) = I_{KK}(V) + jI_{dc}(V) \qquad (2),$$

where $I_{dc}(V)$ DC tunnel current i.e., the un-pumped I-V characteristic and $I_{KK}(V)$ is the Kramers-Kronig transform of $I_{dc}(V)$ according to [1]

$$I_{KK} = \int_{-\infty}^{\infty}\frac{d\bar{V}}{\pi}\frac{I_{dc}(\bar{V})-\bar{V}/R_N}{\bar{V}-V} \qquad (3)$$

The real part of the current response functions (3) can be calculated using the measured I-V of the tunnel junction. Here, we used a model similar to [9].

A. Reactive part of the SIS quantum admittance

Equation (1) is used to calculate the reactive part of SIS quantum admittance, B_{11} at 100 GHz. The susceptance in Fig.2 is plotted as function of the DC bias for three different pumping levels, α ($\alpha = eV_{LO}/\hbar\omega$). The pumped IVC are shown in Fig.3. It is evident from Fig.2 that the susceptance,

B_{11} can either be inductive or capacitive depending on bias voltage and pumping factor α.

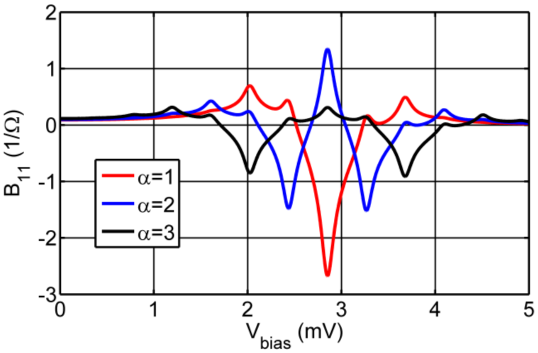

Fig.2: Reactive part of SIS quantum impedance, B_{11} for 100 GHz. The reactance is plotted as function of DC bias for three different pumping levels, α ($\alpha = eV_{LO}/\hbar\omega$).

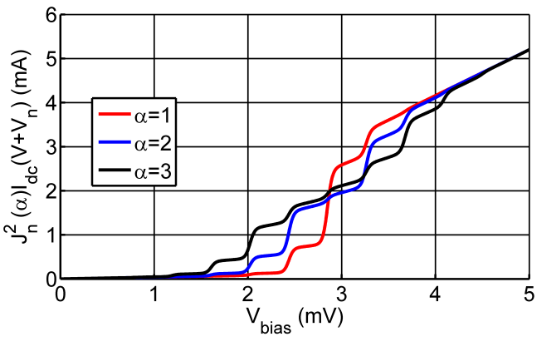

Fig.3 The curves represent the pumped I-V characteristic, $\mathbf{J_n^2(\alpha)I_{dc}(V+V_n)}$, for different pumping factor, α. $\mathbf{V_n} = \boldsymbol{n}\frac{\hbar\omega}{e}$ is quantized photon voltage.

B. Properties of SIS microstrip line propagation factor

For the modelling, we follow the approach suggested in [7]. However, in this case, the reactive part of the quantum impedance, B_{11} was not neglected in the circuit diagram suggested in [7].

From the circuit diagram, the propagation factor, γ, for a microstrip line with series impedance Z and admittance Y of unit length can be expressed as

$$\gamma = \sqrt{\left(j\omega\mu_0 s + \left(Z_{ss} + Z_{sg}\right)\right) \cdot \left(G_{RF} + j\omega C_s + jB_{RF}\right)} \quad (4)$$

where Zss and Zsg are the surface impedance of the superconducting counter and ground electrodes respectively, C_S is the specific capacitance in pF/μm^2 [10], G_{RF} $(\Omega\ \mu m^2)^{-1}$ is the conductance of the RF quantum admittance real part that originates from the photon-assisted current through the barrier per area unit, whereas B_{RF} is the reactive part of the RF quantum admittance per area unit. The specific surface impedance Z_S per square of a superconducting film with thickness d is expressed as follows [11].

The real and imaginary part of the propagation factor at 100 GHz vs. bias voltage for different pumping levels are compared to the propagation factor when $B_{RF}=B_{11}=0$ (cf. Fig.6 and Fig. 7).

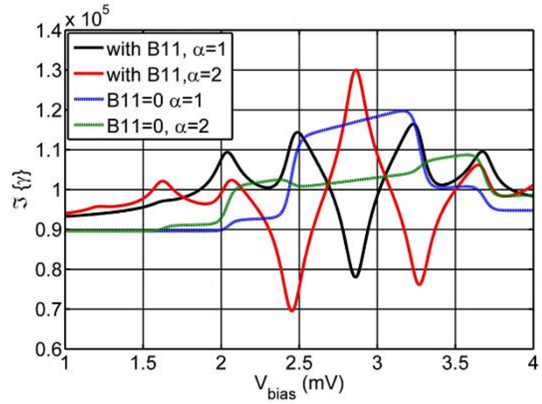

Fig. 6: The imaginary part of the propagation factor at 100 GHz vs. bias voltage for different pumping levels at B_{11}=0 and $B_{11}\neq 0$.

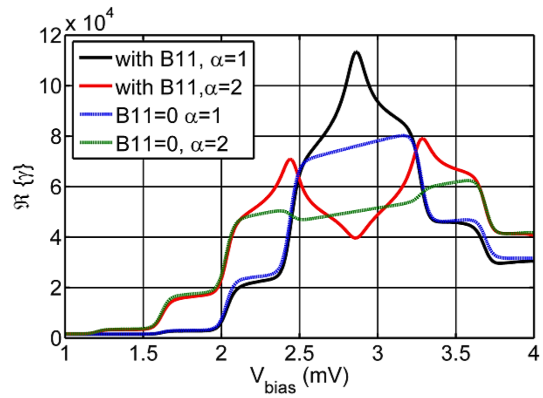

Fig.7**:** The real part of the propagation factor at 100 GHz vs. bias voltage at different pumping levels for B_{11}=0 and $B_{11}\neq 0$.

Comparing the curves in Fig. 7 reveals that the loss factor can increase up to approximately 55% depending on the bias voltage. This would yield the electrical length of the distributed SIS microstrip line junction resonator would be substantially longer than that with the assumption of B_{11}=0, which would make the center frequency significantly lower than designed value.

Fig.8 shows when the propagation factor vs frequency for a fixed bias voltage (Vb=2.35 V), and α = 2. It can be seen that the losses increase up to approximately 40 % at this particular bias voltage as compared to when the imaginary part of the SIS quantum admittance is neglected. On the other hand, the imaginary part becomes almost constant from 70-140 GHz, which is about one octave fractional bandwidth. This could be used for example to tune the center frequency of a distributed SIS junction resonance frequency by properly setting the bias voltage.

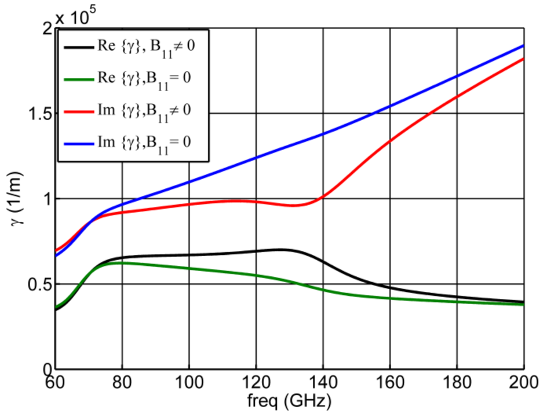

Fig. 8. The imaginary part of the propagation factor at V_b=2.35 mV and α = 2 for two cases, one with B_{11}=0 and B_{11}≠0. The dashed lines are the real parts whereas the solid lines are the imaginary parts.

III. Conclusions

The Niobium based distributed SIS microstrip tunnel junction model presented in [7] was modified by including the reactive part of the SIS microstrip line tunnel junction quantum admittance.

The new model was used to investigate the propagation factor of at frequencies 70-200 GHz. It was shown that imaginary part of SIS quantum admittance had significant effects on both the loss factor, Re (γ), and phase factor, Im (γ). The SIS quantum susceptance changes the effective wavelength depending on the DC voltage bias.

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