

# Experimental Verification of the Fundamental Gaussian Beam Properties of Smooth-walled Feedhorns

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**Abstract**—We report the measurements of near field patterns for three smooth-walled feedhorns and a conventional corrugated feedhorn at different wavelengths. We fit the measurement results into fundamental Gaussian beams. We verify our measurement method by comparing the corrugated horn result with that from the theory calculation. The measurement result of the smooth-walled feedhorns are in good agreement with those from numerical calculations.

## I. INTRODUCTION

Feedhorns are widely used in THz applications for radiation coupling. The output beampatterns of these feedhorns are required to be symmetrical in the E- and the H- planes and have low sidelobe levels. A corrugated feedhorn [1] is the conventional design to achieve these goals. Corrugated feedhorns can offer highly symmetrical beampatterns over a very wide bandwidth. However, they are difficult to make, especially at high frequencies. In the past decades, in order to meet the requirement of focal planes with large format of feedhorn arrays, many alternative feedhorn designs have been proposed [2], [3], [4]. One design of special interest is the smooth-walled feedhorns. This class of horn have adiabatic monotonically increasing profiles from input waveguide to feedhorn aperture, which allows easy fabrication. The smooth-walled feedhorns can offer a close performance comparable to those of the corrugated feedhorns. However, the beams generated by these feedhorns have not been very well studied experimentally. In optics design, it is common to characterize and model the output beam into a fundamental Gaussian beam mode. In this paper, we show the near field measurement results for four feedhorns with different designs (see Figure 1), including a profile horn at 420 GHz (scaled from the design in [3]), a standard corrugated horn at 232 GHz and two smooth-walled feedhorns at 92.5 and 38 GHz (from the design described in [4]). Among them, the 232 GHz corrugated horn was used as a standard to verify our measurement. We fit the measured beampatterns to obtain the parameters for the fundamental Gaussian beam and compare them with the results from numerical calculations.

## II. NEAR FIELD MEASUREMENT SETUP

Our measurement setup varies a little depending on the measurement frequency. Figure 2 shows the block diagram of the setup for measuring the 92.5 GHz feedhorn. A Gunn oscillator,

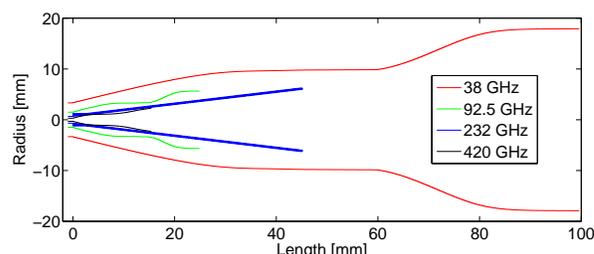


Fig. 1. Profiles of the feedhorns that are measured in this paper. They are smooth-walled feedhorns [4] at 38 and 92.5 GHz (shown in red and green color), a conventional corrugated horn at 232 GHz (blue) and a spline horn [3] at 420 GHz (black).

an isolator, a coupler and the feedhorn to be measured are mounted on the left side in the transmitter assembly. The Gunn oscillator is phase-locked to a reference signal. The receiver assembly on the right, shown in the blue box is mounted on an X-Y translation stage. It consists of a waveguide probe, a waveguide variable attenuator and a harmonic mixer. The IF signal from the harmonic mixer is amplified, filtered and passed to a vector voltmeter. The transmitter and receiver assemblies are masked by microwave absorber sheets, except for the openings of the feedhorn and waveguide probe.

The setups for measuring the other feedhorns are similar to the setup described above, except for the changes in the sources and LOs. The measurement parameters for all the feedhorns are listed in Table I. The distances from the feedhorn apertures to scan planes (defined by the tip of waveguide probe) and the sizes of the scan plane are chosen to make sure that we can measure the main beam down to  $< -30$  dB level except for the 420 GHz horn (down to  $\approx -20$  dB level). The scan resolutions are generally smaller than the sizes of the waveguide probes. Additional reference measurements are taken during the scan to ensure good data quality. As a result, we find that the measurement is very stable for both amplitude and phase [5].

## III. MEASUREMENT RESULTS

Figure 3 shows the 2-dimension amplitude beampatterns from our measurements. They are normalized to 0 dB in the center and with the contour steps of 5 dB. The dynamics range are greater than 30 dB except for the 420 GHz feedhorn.

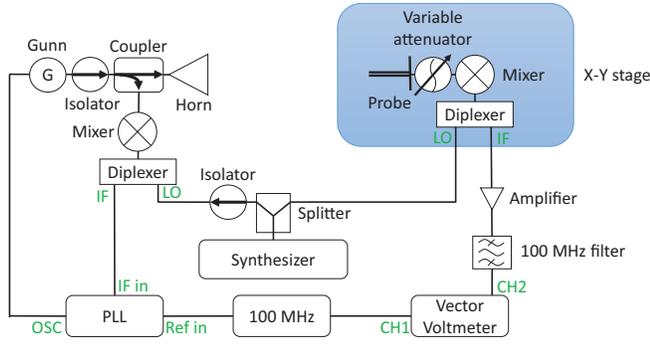


Fig. 2. Schematic of measurement setup for 92.5 GHz horn. The transmitter on the left side has a Gunn oscillator, an isolator, a coupler and the feedhorn to be measured. The Gunn oscillator is phase-locked to a reference signal. The receiver shown in the blue box is mounted on an X-Y translation stage. It consists of a waveguide probe, a variable attenuator and a harmonic mixer. The IF signal is amplified, filtered and passed to a vector voltmeter.

TABLE I  
FEEDHORN MEASUREMENT PARAMETERS

Freq (GHz)	Scan Dist (mm)	Scan Res (mm)	Scan size (mm × mm)	Dynamic range (dB)
38.0	105.8	3.0	138 × 138	>30
92.5	31.8	2.0	60 × 60	>30
232.0	22.0	1.0	34 × 34	>30
420.0	28.8	0.5	20 × 20	≈20

The complex beampatterns are fit into a fundamental Gaussian beam mode [6]

$$\Phi(x, y) = \left( \frac{2}{\pi\omega^2} \right)^{0.5} \exp\left( -\frac{x^2 + y^2}{\omega^2} \right) \times \exp\left[ \frac{2\pi j}{\lambda} \left( \frac{x^2 + y^2}{2R} + \delta_x x + \delta_y y \right) \right], \quad (1)$$

where  $x = X - X_c$  and  $y = Y - Y_c$  are the coordinates from the center of the beampattern ( $X_c, Y_c$ ).  $\omega$  is the beam radius,  $R$  is the radius of curvature of wave front and  $\delta_x$  and  $\delta_y$  are the tilt angles of the beam with respect to the normal of scan plane. By maximizing the power coupling coefficient to this fundamental Gaussian beam (see the last column of Table II), we obtain the best fit value for the parameters of  $X_c, Y_c, \omega, R, \delta_x$  and  $\delta_y$ . From these we can derive the beamwaist and its location inside the feedhorn by considering the relations among the different parameters of the fundamental Gaussian mode

$$\omega_0 = \frac{\omega}{\left[ 1 + \left( \frac{\pi\omega^2}{\lambda R} \right)^2 \right]^{0.5}}, \quad (2)$$

$$z = d + z_0 = \frac{R}{1 + \left( \frac{\lambda R}{\pi\omega^2} \right)^2},$$

where  $\omega_0$  is the feedhorn beamwaist,  $z$  is the propagation distance from beamwaist to scan plane,  $d$  is the distance from feedhorn aperture to scan plane (see the "Scan Dist" column in Table I) and  $z_0$  is the beamwaist offset from feedhorn aperture.

To verify our measurement results, We use the conventional method to calculate the theoretical value of 232 GHz corrugated feedhorn beamwaist and its offset ( $\omega'_0$  and  $z'_0$ ).

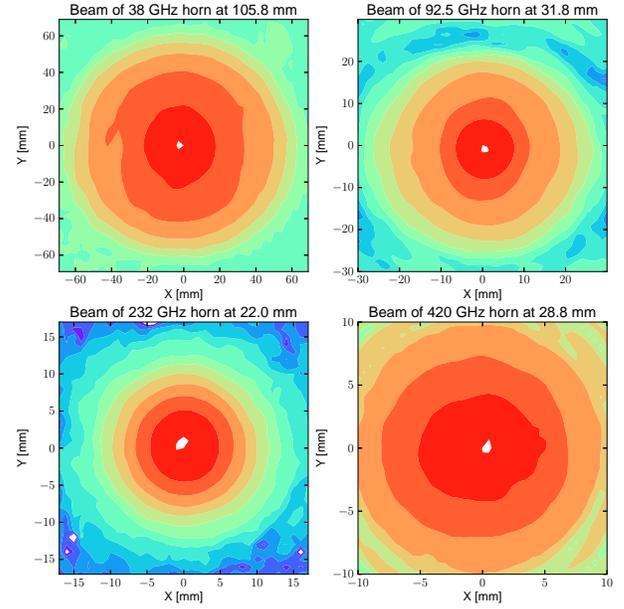


Fig. 3. 2-D amplitude beampatterns from the near field beam measurement. The distances in the titles are from scanning planes to feedhorn apertures. The intensity in the center of plots are normalized to 0 dB. The contours are in 5 dB steps.

TABLE II  
FEEDHORN MEASUREMENT DATA FIT RESULTS

Freq (GHz)	$\omega'_0$ (mm)	$\omega_0$ (mm)	$Z'_0$ (mm)	$Z_0$ (mm)	Coupling to Gaussian (%)
-	calc	meas	calc	meas	
38.0	9.41	9.52	38.70	39.27	96.09
92.5	3.08	3.27	9.36	10.76	95.85
232.0	3.22	3.32	18.87	19.50	98.38
420.0	1.18	1.14	6.73	5.90	97.49

We assume that the beamsize at its aperture is  $\omega = 0.644a$ , where  $a$  is the radius of feedhorn aperture [6]. After that, we calculate  $\omega'_0$  and  $z'_0$  using Equation 2, with  $d = 0$ . For the other feedhorns, we calculated the electric field at the horn aperture using numerical calculation described in [4] and fit the electric field distribution at the horn aperture into a fundamental Gaussian beam. We obtain a set of beamwaist and offset ( $\omega'_0$  and  $z'_0$ ) by inverting the propagation of this Gaussian beam.

Table II lists the theoretical values and the values derived from the near field measurements for both the size of the beamwaist and its location relative the horn aperture. The good agreement between the theoretical and measure results of the 232 GHz horn verifies that the near field measurement method is a good way to obtain the beam properties for feedhorns. The coupling coefficients of the measured beam to the fundamental Gaussian mode are > 95% for the smooth-walled feedhorns. This demonstrated that a fundamental Gaussian beam mode is a good approximation to the beam generated by a smooth-walled feedhorn.

## IV. CONCLUSION

We propose an experimental method to characterize the property of a fundamental Gaussian beam generated by various feedhorns in different designs and different wavebands. Our results are in good agreement with the numerical calculation results.

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