

Evaluation of aperture efficiency by using ray-tracing software in designing a wide field-of-view telescope

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Abstract— A wide field-of-view telescope is essential for large area observations which need as high mapping speed as possible. The size of field of view is limited by aberrations in most cases. One of the figure of merit of a radio telescope is aperture efficiency. Analytical expression of aperture efficiency affected by aberrations has been obtained recently. In this paper, we have implemented the analytical expression to commercial ray tracing software so that we can use aperture efficiency as well as Strehl ratio in designing a radio telescope.

INTRODUCTION

Various plans of astronomical observation are proposed in a radio band recently, e.g. LiteBIRD, 10-m THz telescope in Antarctica, CCAT, LST, and so on. The target bands of these telescopes are sub-mm or THz bands, which are at higher frequencies than ever. In addition, these plans demand very wide field-of-view (FOV) covering 1 square degree or more.

A radio telescope with a single beam or multi-beam has been designed so far. For a feed located at the center of FOV, we can calculate performance both analytically and numerically by using a fundamental Gaussian beam and physical optics simulation. For a feed located at the other area of FOV than the center, physical optics simulation is only available. Physical optics can provide a figure-of-merit such as aperture efficiency but not provide an optical design. An optical design must be given by using another way and also we have difficulty in looking for an optimal design. In other words, we do not have a systematic method to design a wide FOV telescope for multi-beam.

Ray tracing based on geometrical optics can design a wide FOV telescope for optical and infrared astronomy. Geometrical optics helps us achieve a wide FOV radio telescope but cannot provide a figure-of-merit which is need in evaluating a radio telescope. References [1], [2], and [3] revealed the relation between geometrical optics and aperture efficiency. Therefore we made a tool based on ray tracing but it can provide aperture efficiency.

FORMULATION

In this section, formulation of aperture efficiency affected by Seidel aberrations is summarized according to [1], [2], and [3]. [1] revealed that aperture efficiency is intrinsically equal

to the product of two spillover efficiencies and coupling efficiency, i.e.,

$$\eta_A = \eta_{sp,en} \eta_{coup} \eta_{sp,ex}$$

where $\eta_{sp,en}$ is spillover efficiency at entrance pupil and $\eta_{sp,ex}$ is spillover efficiency at exit pupil. Coupling efficiency η_{coup} can be determined by the electric field of an incident wave and the virtual field from a feed at a pupil. In other words, coupling efficiency is given as a function of the direction of incident wave and the position of a feed because the electric fields are dependent on them.

References [2] and [3] demonstrated coupling efficiency as a function of the direction of incident wave and the position of a feed explicitly. The definition of coordinates is shown in Fig. 1. As a result, aperture efficiency is expressed as

$$\eta_A(\hat{\mathbf{p}}; \mathbf{r}_{det}) = \eta_{sp,en} \eta_{sp,ex} \frac{\lambda^2 P_n(\hat{\mathbf{p}}; \mathbf{r}_{det})}{A_{p,en} \Omega_A(\mathbf{r}_{det})}$$

where $\hat{\mathbf{p}}$ is incident direction, \mathbf{r}_{det} feed position, λ wavelength, $A_{p,en}$ the area of entrance pupil, Ω_A beam solid angle, and P_n normalized beam pattern. In order to observe effects of aberrations on aperture efficiency, wavefront error of the incident wave at the exit pupil is expressed as a form of summation of Zernike polynomials, i.e.,

$$\begin{aligned} W(\hat{\mathbf{p}}; \rho; \mathbf{r}_{ref}) = & A_1^{-1}(\hat{\mathbf{p}}; \mathbf{r}_{ref}) Z_1^{-1}(\rho) + B_1^{-1}(\hat{\mathbf{p}}; \mathbf{r}_{ref}) Z_1^{-1}(\rho) \\ & + A_2^0(\hat{\mathbf{p}}; \mathbf{r}_{ref}) Z_2^0(\rho) + A_2^2(\hat{\mathbf{p}}; \mathbf{r}_{ref}) Z_2^2(\rho) \\ & + B_2^2(\hat{\mathbf{p}}; \mathbf{r}_{ref}) Z_2^{-2}(\rho) + A_3^1(\hat{\mathbf{p}}; \mathbf{r}_{ref}) Z_3^1(\rho) \\ & + B_3^1(\hat{\mathbf{p}}; \mathbf{r}_{ref}) Z_3^{-1}(\rho) + A_4^0(\hat{\mathbf{p}}; \mathbf{r}_{ref}) Z_4^0(\rho) \end{aligned}$$

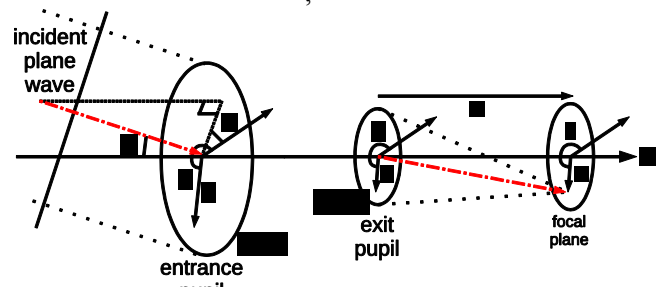


Fig. 1 Definition of coordinates.

where A_n^m and B_n^m are coefficients and Z_n^m is Zernike polynomial. The meaning of the coefficients is summarized in Table I. Then, the electric field of the incident wave at the exit pupil is written as

$$E_{\text{ex}}(\hat{\mathbf{p}}; \boldsymbol{\rho}) = C \exp \left[\sqrt{-1} k \frac{f}{z_{\text{ref}}} R_{\text{p,ex}} \rho \sin \theta \cos(\psi - \Phi) + \sqrt{-1} \frac{k R_{\text{p,ex}}^2}{2 z_{\text{ref}}} \rho^2 \right] \sum_{m,n} C_n^m(\hat{\mathbf{p}}; \mathbf{r}_{\text{ref}}) Z_n^m(\boldsymbol{\rho})$$

C_n^m is a function of A_n^m and B_n^m which are determined by using ray tracing software. The virtual field of a feed is also expanded into the summation of Zernike polynomials as well as the incident wave.

$$E_{\text{det}}(\boldsymbol{\rho}; \mathbf{r}_0) = C' \exp \left[\sqrt{-1} k \frac{f}{z_{\text{ref}}} R_{\text{p,ex}} \rho \sin \theta \cos(\psi - \Phi) + \sqrt{-1} \frac{k R_{\text{p,ex}}^2}{2 z_{\text{ref}}} \rho^2 \right] \sum_{p,q} D_p^q(\mathbf{r}_{\text{ref}}; \mathbf{r}_0; w_0) Z_p^q(\boldsymbol{\rho})$$

D_p^q is determined by beam pattern measurement or simulation such as HFSS. Using coefficients C_n^m and D_p^q , we obtain coupling efficiency as follows:

$$\eta_{\text{coup}} = \frac{\left| \sum_{m,n} \sum_{p,q} \delta_{np} \delta_{mq} C_n^m(\hat{\mathbf{p}}; \mathbf{r}_{\text{ref}}) (D_p^q(\mathbf{r}_{\text{ref}}; \mathbf{r}_0; w_0)) \right|^2}{\sum_{m,n} |C_n^m(\hat{\mathbf{p}}; \mathbf{r}_{\text{ref}})|^2 \sum_{p,q} |D_p^q(\mathbf{r}_{\text{ref}}; \mathbf{r}_0; w_0)|^2}$$

If a feed pattern is assumed to be an axially symmetric fundamental Gaussian beam, then coefficients D_p^q is determined by

$$\begin{aligned} E_{\text{det}}(\boldsymbol{\rho}; \mathbf{r}_0) &= C' \exp \left[\sqrt{-1} k \frac{f}{z_{\text{ref}}} R_{\text{p,ex}} \rho \sin \theta \cos(\psi - \Phi) + \sqrt{-1} \frac{k R_{\text{p,ex}}^2}{2 z_{\text{ref}}} \rho^2 \right] \sum_{p,q} D_p^q(\mathbf{r}_{\text{ref}}; \mathbf{r}_0; w_0) Z_p^q(\boldsymbol{\rho}) \\ &= C' \exp \left[\sqrt{-1} k \frac{f}{z_{\text{ref}}} R_{\text{p,ex}} \rho \sin \theta \cos(\psi - \Phi) \right] \exp \left[-\frac{R_{\text{p,ex}}^2 \rho^2}{w^2} + \sqrt{-1} \frac{k R_{\text{p,ex}}^2 \rho^2}{2R} \right] \end{aligned}$$

Finally, aperture efficiency for a fundamental Gaussian feed is expressed as

$$\begin{aligned} \eta_A &= \eta_{\text{sp,en}} \eta_{\text{coup}} \eta_{\text{sp,ex}} \\ &= \frac{2R_{\text{p,ex}}^2}{w^2} \left| D_0^0 + \sqrt{-1} k (A_2^0 D_2^0 + A_4^0 D_4^0) - \frac{k^2}{2} \left[\left(D_0^0 + \frac{D_2^0}{\sqrt{3}} \right) (A_1^1)^2 + 2\sqrt{2} \left(\frac{D_2^0}{\sqrt{3}} + \frac{D_4^0}{\sqrt{5}} \right) A_1^1 A_3^1 \right. \right. \\ &\quad \left. \left. + \left(D_0^0 + \frac{D_2^0}{5\sqrt{3}} + \frac{D_4^0}{\sqrt{5}} + \frac{9D_6^0}{5\sqrt{7}} \right) (A_3^1)^2 + \left(D_0^0 + \frac{2D_4^0}{\sqrt{5}} \right) (A_2^0)^2 + \left(\frac{4D_2^0}{\sqrt{5}} + \frac{6\sqrt{15}D_6^0}{5\sqrt{7}} \right) A_2^0 A_4^0 \right. \right. \\ &\quad \left. \left. + \left(D_0^0 + \frac{2\sqrt{5}D_4^0}{7} + \frac{6D_8^0}{7} \right) (A_4^0)^2 + \left(D_0^0 + \frac{\sqrt{3}D_2^0}{2} + \frac{D_4^0}{2\sqrt{5}} \right) (B_2^2)^2 \right] \right|^2 \end{aligned}$$

IMPLEMENTATION

We implemented a script to calculate aperture efficiency expressed in the above equation to Zemax 13. The script can calculate aperture efficiency from the coefficients of a Zernike polynomial series. Also the root sum square of the difference

TABLE I ABERRATION COEFFICIENTS.

$A_1^1(\hat{\mathbf{p}}; \mathbf{r}_{\text{ref}}), B_1^1(\hat{\mathbf{p}}; \mathbf{r}_{\text{ref}})$	Tip, tilt, distortion
$A_2^0(\hat{\mathbf{p}}; \mathbf{r}_{\text{ref}})$	Defocus, curvature of field
$A_2^2(\hat{\mathbf{p}}; \mathbf{r}_{\text{ref}}), B_2^2(\hat{\mathbf{p}}; \mathbf{r}_{\text{ref}})$	Astigmatism
$A_3^1(\hat{\mathbf{p}}; \mathbf{r}_{\text{ref}}), B_3^1(\hat{\mathbf{p}}; \mathbf{r}_{\text{ref}})$	Coma

$A_4^0(\hat{\mathbf{p}}; \mathbf{r}_{\text{ref}})$	spherical
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between the analytical expression and a theoretical maximum determined by the edge taper. Constrains can be taken into

account, e.g., aperture efficiency to be achieved, weighted by the position on the focal plane, and so on.

DESIGNING A WIDE FOV TELESCOPE WITH SCRIPT

A wide FOV telescope was designed by using the implemented script. The telescope consisted of two axially symmetric conic mirrors. Initial values for their parameters are shown in TABLE II and the system before optimization is shown in Fig. 2. The parameters to be changed during optimization were the conic constants of both of the mirrors and the radius of curvature of the secondary mirror. A stop was located at the secondary mirror. The center of FOV was weighted more than the edge in order to attain higher aperture efficiency at the center.

Two types of optimization were carried out. One adopted aperture efficiency as a figure of merit and the other adopted

Strehl ratio. Figs. 3 and 4 show the systems optimized by aperture efficiency and Strehl ratio, respectively. The parameters obtained after optimization are shown in Tables III and IV. The parameters after optimization by aperture efficiency are different from those by Strehl ratio. Figs. 5 and 6 shows the maps of aperture efficiency as a function of an incident angle. In the case where aperture efficiency is adopted as a figure of merit, the aperture efficiency for the center of FOV is lower than those for the other incident angles, which arises from failure to weigh the center of FOV more than the others in evaluating aperture efficiencies.

TABLE III PARAMETERS AND INITIAL VALUES.

	conic constant	Radius of curvature	diameter	Thickness
Primary	-1	-1000 mm	-600 mm	-400 mm
secondary	-1.8	-230 mm	-100 mm	600 mm

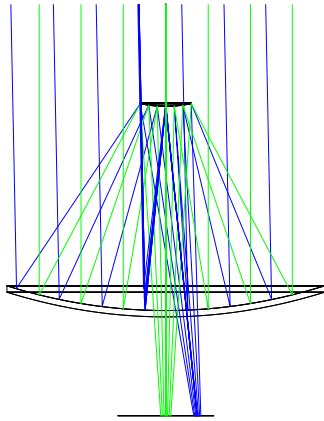


Fig. 2 An optical system composed of two axially symmetric conic mirrors before optimization.

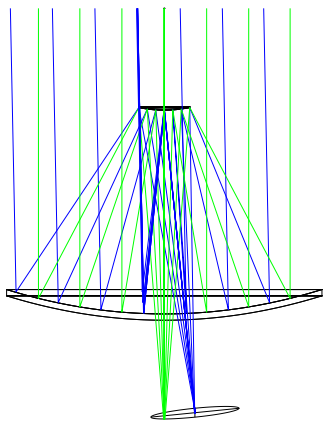


Fig. 3 After optimization by evaluating aperture efficiency.

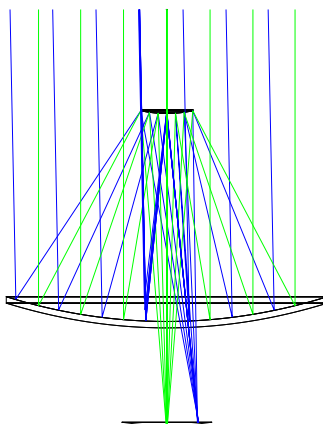


Fig. 4 After optimization by evaluating Strehl ratio.

TABLE IIIII PARAMETERS OPTIMIZED BY APERTURE EFFICIENCY.

	conic constant	Radius of curvature
Primary	-1.042528	-1000 mm
secondary	-3.018659	-237.151 mm

TABLE IVV PARAMETERS OPTIMIZED BY STREHL RATIO.

	conic constant	Radius of curvature
Primary	-1.162993	-1000 mm
secondary	-3.256167	-239.866 mm

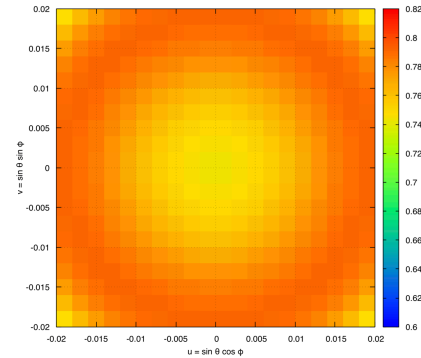


Fig. 5 Map of aperture efficiency optimized by evaluating aperture efficiency.

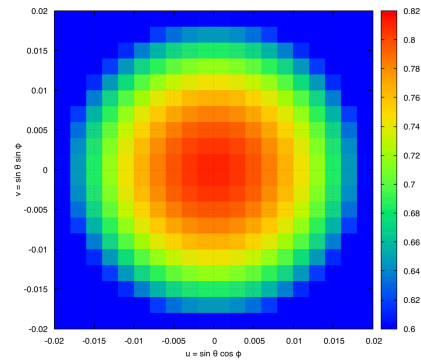
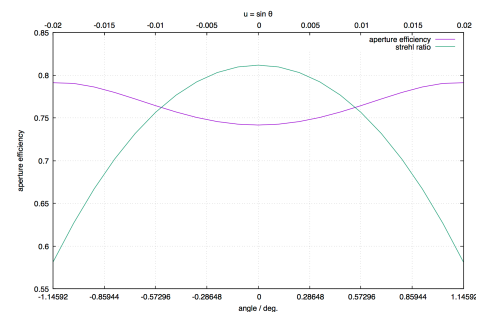


Fig. 6 Map of aperture efficiency optimized by evaluating Strehl ratio.



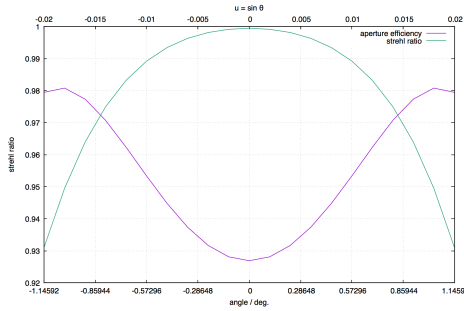


Fig. 7 Upper panel: Aperture efficiency vs. incident angle. Lower panel: Strehl ratio vs. incident angle. Purple lines represent optimization by aperture efficiency and green ones by Strehl ratio.

Fig. 7 shows the aperture efficiency and Strehl ratio obtained through the optimization as a function of incident angle. The angle zero corresponds to the center of FOV. When focusing on the angle of 0 degrees and 1.1 degrees, the Strehl ratios are about 0.93 but the aperture efficiencies are 0.74 for 0 degrees and 0.59 for 1.1 degrees, respectively. The fact that aperture efficiency depends on the types of aberrations is responsible for this discrepancy between aperture efficiency and Strehl ratio. The results in Fig. 7 imply that it is possible to fail to obtain target aperture efficiency when Strehl ratio is evaluated only.

CONCLUSIONS

We have developed a tools to calculate aperture efficiency based on the formulation in [1], [2], and [3] and implemented to the commercial ray tracing software, Zemax. Designing a radio telescope can be carried out by adopting aperture efficiency as a figure of merit. It has been revealed that aperture efficiency can be a different value for the same Strehl ratio. It arises from the dependence of aperture efficiency on the types of aberrations.

ACKNOWLEDGMENT

We are grateful to Professor Naomasa Nakai and Professor Nario Kuno at University of Tsukuba for supporting simulation software.

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