Noise Analysis of SIS Receivers Using Chain Noise Correlation Matrices

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Modern SIS receivers are built from a series of components including waveguide elements, multiple SIS devices, IF matching network, isolators and low noise amplifiers. A simple noise analysis of the receiver is generally performed with the well-known Friis formula for noise:

$$T_{eq} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \cdots \quad (1)$$

Although this method provides the correct result if each of the components is well matched, this is generally not the case for SIS mixers. A more rigorous approach is based on the cascade of chain noise correlation matrices:

$$\boldsymbol{C}_{A_{eq}} = \boldsymbol{C}_{A_1} + \boldsymbol{A}_1 \boldsymbol{C}_{A_2} \boldsymbol{A}_1^+ + \boldsymbol{A}_1 \boldsymbol{A}_2 \boldsymbol{C}_{A_3} \boldsymbol{A}_2^+ \boldsymbol{A}_1^+ + \cdots \quad (2)$$

In the above equation, $\boldsymbol{A}_m$ and $\boldsymbol{A}_m^+$ are the chain circuit transmission matrix (also known as ABCD matrix) of the *m*-th component and its Hermitian conjugate respectively, while $\boldsymbol{C}_{A_m}$ is the corresponding chain noise correlation matrix. For an SIS mixer, there are at least 3 frequencies involved. A tri-frequency chain transmission matrix formalism has been introduced [1] to extend the single frequency circuit theory to cover the operation of series connected SIS mixer arrays. A similar treatment can be applied to extend the single frequency chain noise correlation matrix to general multi-junction mixer circuits [2]. In this case, the chain noise correlation matrix becomes a block matrix, with its entries appearing as 3x3 matrices,

$$\boldsymbol{C}_A = \begin{pmatrix} [\underline{V_n}\,\underline{V_n}^+] & [\underline{V_n}\,\underline{I_n}^+] \\ [\underline{V_n}^+\underline{I_n}] & [\underline{I_n}\,\underline{I_n}^+] \end{pmatrix} \quad (3)$$

where the noise voltages, $\underline{V_n}$, and noise currents, $\underline{I_n}$, are column vectors:

$$\underline{V_n} = [V_n^{\mathrm{USB}} \quad V_n^{\mathrm{IF}} \quad V_n^{\mathrm{LSB}}]^T \quad (4)$$

$$\underline{I_n} = [I_n^{\mathrm{USB}} \quad I_n^{\mathrm{IF}} \quad I_n^{\mathrm{LSB}}]^T \quad (5)$$

Note that the matrix $[\underline{I_n}\,\underline{I_n}^+]$ corresponds to the noise correlation matrix $\boldsymbol{H}$ for the SIS mixer in Tucker's Theory and a single SIS junction with one terminal connected to ground would have a chain noise correlation matrix of $\begin{pmatrix} 0 & 0 \\ 0 & \boldsymbol{H} \end{pmatrix}$. Therefore, this formalism allows a simple build-up of a noise analysis of a complete multi-junction SIS receiver using equation (2).

In this paper, we will develop the equations needed to develop a full noise model for an SIS receiver. This model incorporates the interaction between the SIS mixer, which has generally a high output impedance, and the IF stage, the noise temperature of which is a function of its input impedance. As a result, the model can easily be applied to the optimization of the SIS receiver with a given IF configuration. In addition, since the SIS mixer has generally a poor input match, it would interact with the waveguide components in front of it, causing frequency ripples in receiver noise temperature. The model can also be used to account for such effects. Finally, it can be shown that for an SIS mixer based on a distributed array, the chain noise correlation matrix for the array would have non-null matrix entries for each of its 4 constituting matrices. Thus, this model is well suited for the noise analysis of SIS receivers based on distributed series SIS junction array

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References

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