Polarization in Interferometry

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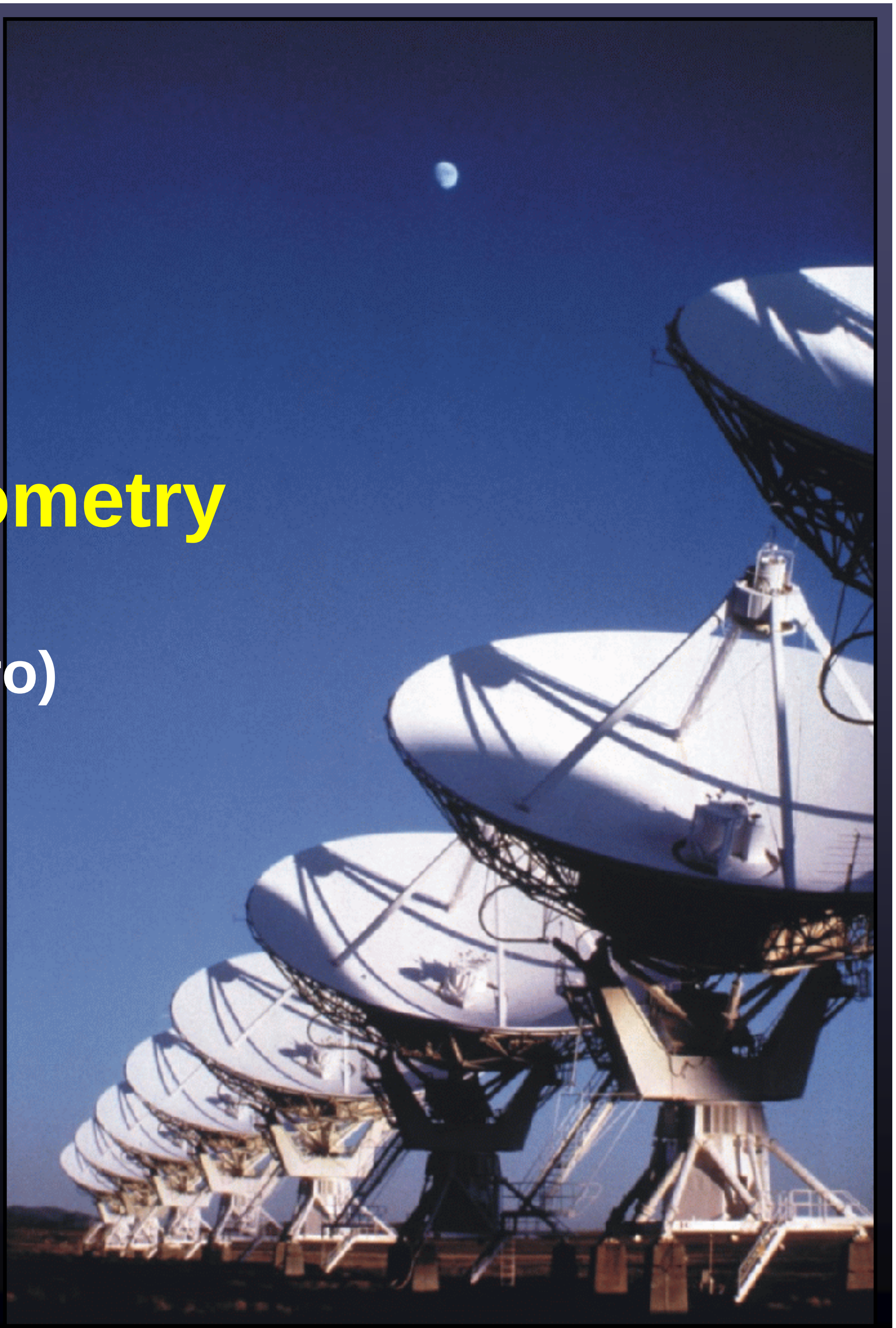

Ninth Synthesis Imaging Summer School
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Polarization in interferometry

- Physics of Polarization
- Interferometer Response to Polarization
- Polarization Calibration & Observational Strategies
- Polarization Data & Image Analysis
- Astrophysics of Polarization
- Examples

- References:
  - Synth Im. II lecture 6, also parts of 1, 3, 5, 32
  - "Tools of Radio Astronomy" Rohlfs & Wilson

WARNING!

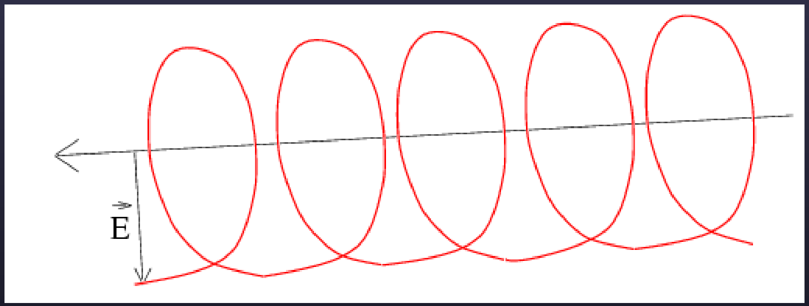
- Polarimetry is an exercise in bookkeeping!
  - many places to make sign errors!
  - many places with complex conjugation (or not)
  - possible different conventions (e.g. signs)
  - different conventions for notation!
  - lots of matrix multiplications
- And be assured…
  - I've mixed notations (by stealing slides ☺)
  - I've made sign errors ☹ (I call it "choice of convention" ☺)
  - I've probably made math errors ☹
  - I've probably made it too confusing by going into detail ☹
  - But … persevere (and read up on it later) ☺

DON'T PANIC !

Polarization Fundamentals

Physics of polarization

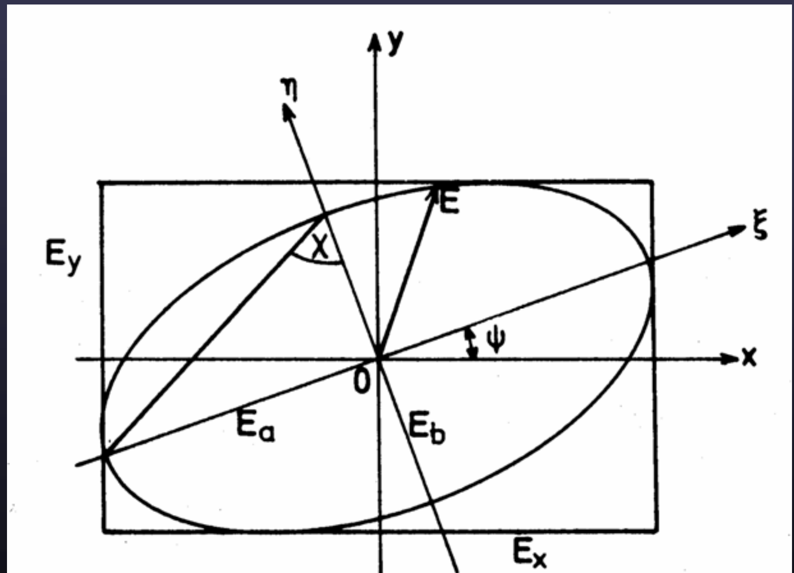
- Maxwell's Equations + Wave Equation
  - E•B=0 (perpendicular) ; $E_z = B_z = 0$ (transverse)
- Electric Vector – 2 orthogonal independent waves:
  - $E_x = E_1 \cos(k\,z - \omega\,t + \delta_1)$ $k = 2\pi / \lambda$
  - $E_y = E_2 \cos(k\,z - \omega\,t + \delta_2)$ $\omega = 2\pi\,\nu$
  - describes helical path on surface of a cylinder…



  - parameters E_1, E_2, $\delta = \delta_1 - \delta_2$ define ellipse

The Polarization Ellipse

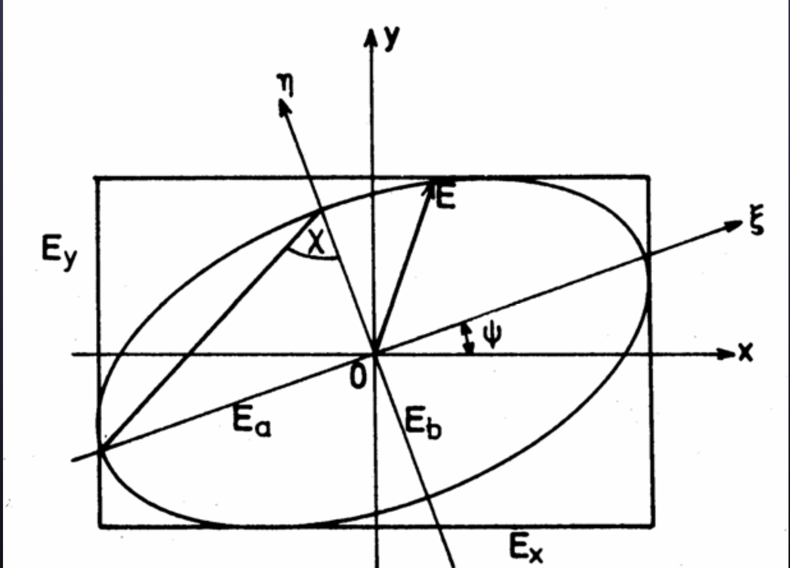
- Axes of ellipse E_a, E_b
  - $S_0 = E_1^2 + E_2^2 = E_a^2 + E_b^2$ Poynting flux
  - δ phase difference $\tau = k z - \omega t$
  - $E_\xi = E_a \cos(\tau + \delta) = E_x \cos\psi + E_y \sin\psi$
  - $E_\eta = E_b \sin(\tau + \delta) = -E_x \sin\psi + E_y \cos\psi$



Rohlfs & Wilson

The polarization ellipse continued…

- Ellipticity and Orientation
  - $E_1 / E_2 = \tan\alpha$ $\tan 2\psi = -\tan 2\alpha \cos\delta$
  - $E_a / E_b = \tan\chi$ $\sin 2\chi = \sin 2\alpha \sin\delta$
  - handedness ($\sin\delta > 0$ or $\tan\chi > 0$ ➔ *right-handed*)



Rohlfs & Wilson

Polarization ellipse – special cases

- Linear polarization
  - $\delta = \delta_1 - \delta_2 = m\,\pi \quad m = 0, \pm 1, \pm 2, \ldots$
  - ellipse becomes straight line
  - electric vector position angle $\psi = \alpha$
- Circular polarization
  - $\delta = ½\,(1 + m)\,\pi \quad m = 0, 1, \pm 2, \ldots$
  - equation of circle $E_x^2 + E_y^2 = E^2$
  - orthogonal linear components:
    - $E_x = E \cos \tau$
    - $E_y = \pm E \cos(\tau - \pi/2)$
    - note quarter-wave delay between E_x and E_y !

Orthogonal representation

- A monochromatic wave can be expressed as the superposition of *two orthogonal linearly polarized waves*
- A arbitrary elliptically polarizated wave can also equally well be described as the superposition of two orthogonal *circularly polarized* waves!
- We are free to choose the orthogonal basis for the representation of the polarization
- NOTE: Monochromatic waves MUST be (fully) polarized – IT'S THE LAW!

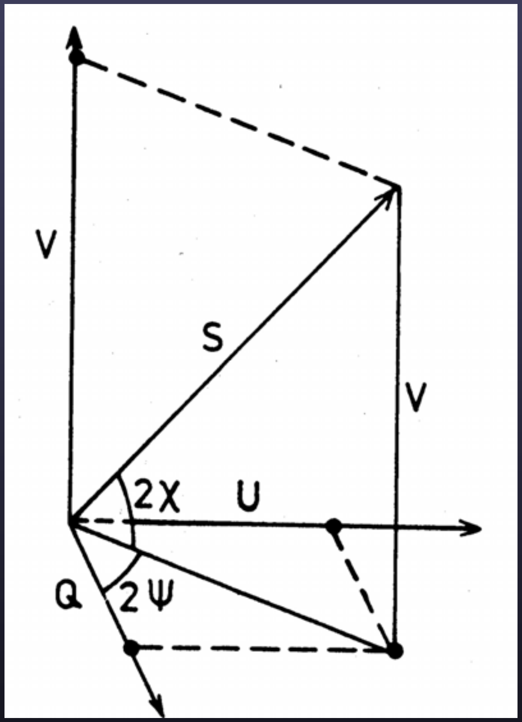
Linear and Circular representations

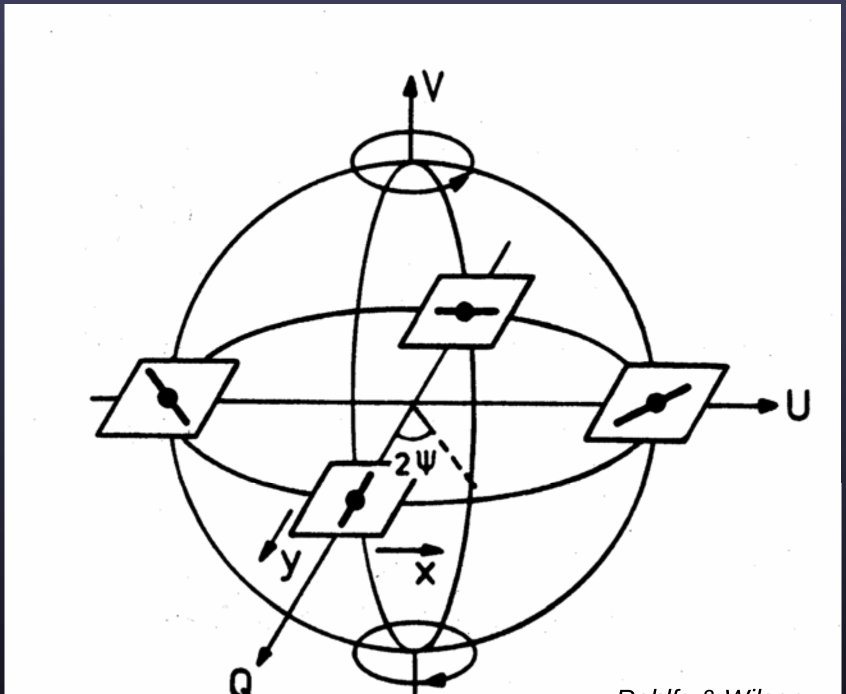
- Orthogonal Linear representation:
  - $E_\xi = E_a \cos(\tau + \delta) = E_x \cos\Psi + E_y \sin\Psi$
  - $E_\eta = E_b \sin(\tau + \delta) = -E_x \sin\Psi + E_y \cos\Psi$

- Orthogonal Circular representation:
  - $E_\xi = E_a \cos(\tau + \delta) = (E_r + E_l) \cos(\tau + \delta)$
  - $E_\eta = E_b \sin(\tau + \delta) = (E_r - E_l) \cos(\tau + \delta - \pi/2)$
  - $E_r = ½(E_a + E_b)$
  - $E_l = ½(E_a – E_b)$

The Poincare Sphere

- Treat 2ψ and 2χ as longitude and latitude on sphere of radius S_0





Rohlfs & Wilson

Stokes parameters

- Spherical coordinates: radius I, axes Q, U, V
  - $S_0 = I = E_a^2 + E_b^2$
  - $S_1 = Q = S_0 \cos 2\chi \cos 2\psi$
  - $S_2 = U = S_0 \cos 2\chi \sin 2\psi$
  - $S_3 = V = S_0 \sin 2\chi$
- Only 3 independent parameters:
  - $S_0^2 = S_1^2 + S_2^2 + S_3^2$
  - $I^2 = Q^2 + U^2 + V^2$
- Stokes parameters I,Q,U,V
  - form complete description of wave polarization
  - NOTE: above true for monochromatic wave!

Stokes parameters and polarization ellipse

- Spherical coordinates: radius I, axes Q, U, V
  - $S_0 = I = E_a^2 + E_b^2$
  - $S_1 = Q = S_0 \cos 2\chi \cos 2\psi$
  - $S_2 = U = S_0 \cos 2\chi \sin 2\psi$
  - $S_3 = V = S_0 \sin 2\chi$
- In terms of the polarization ellipse:
  - $S_0 = I = E_1^2 + E_2^2$
  - $S_1 = Q = E_1^2 - E_2^2$
  - $S_2 = U = 2 E_1 E_2 \cos \delta$
  - $S_3 = V = 2 E_1 E_2 \sin \delta$

Stokes parameters special cases

- Linear Polarization
  - $S_0 = I = E^2 = S$
  - $S_1 = Q = I \cos 2\psi$
  - $S_2 = U = I \sin 2\psi$
  - $S_3 = V = 0$

Note: cycle in 180°

- Circular Polarization
  - $S_0 = I = S$
  - $S_1 = Q = 0$
  - $S_2 = U = 0$
  - $S_3 = V = S$ (RCP) or $-S$ (LCP)

Quasi-monochromatic waves

- Monochromatic waves are fully polarized
- Observable waves (averaged over $\Delta\nu/\nu \ll 1$)
- Analytic signals for *x* and *y* components:
  - $E_x(t) = a_1(t)\, e^{\,i(\phi_1(t) - 2\pi\nu t)}$
  - $E_y(t) = a_2(t)\, e^{\,i(\phi_2(t) - 2\pi\nu t)}$
  - actual components are the real parts Re $E_x(t)$, Re $E_y(t)$
- Stokes parameters
  - $S_0 = I = \langle a_1^2 \rangle + \langle a_2^2 \rangle$
  - $S_1 = Q = \langle a_1^2 \rangle - \langle a_2^2 \rangle$
  - $S_2 = U = 2 \langle a_1 a_2 \cos\delta \rangle$
  - $S_3 = V = 2 \langle a_1 a_2 \sin\delta \rangle$

Stokes parameters and intensity measurements

- If phase of E_y is retarded by ε relative to E_x , the electric vector in the orientation θ is:
  - $E(t; \theta, \varepsilon) = E_x \cos\theta + E_y e^{i\varepsilon} \sin\theta$
- Intensity measured for angle θ:
  - $I(\theta, \varepsilon) = < E(t; \theta, \varepsilon) E^*(t; \theta, \varepsilon) >$
- Can calculate Stokes parameters from 6 intensities:
  - $S_0 = I = I(0°,0) + I(90°,0)$
  - $S_1 = Q = I(0°,0) + I(90°,0)$
  - $S_2 = U = I(45°,0) – I(135°,0)$
  - $S_3 = V = I(45°,\pi/2) – I(135°,\pi/2)$
  - this can be done for single-dish (intensity) polarimetry!

Partial polarization

- The observable electric field need not be fully polarized as it is the superposition of monochromatic waves
- On the Poincare sphere:
  - $S_0^2 \geq S_1^2 + S_2^2 + S_3^2$
  - $I^2 \geq Q^2 + U^2 + V^2$
- Degree of polarization *p* :
  - $p^2 S_0^2 = S_1^2 + S_2^2 + S_3^2$
  - $p^2 I^2 = Q^2 + U^2 + V^2$

Summary – Fundamentals

- Monochromatic waves are polarized
- Expressible as 2 orthogonal independent transverse waves
  - elliptical cross-section ➔ polarization ellipse
  - 3 independent parameters
  - choice of basis, e.g. linear or circular
- Poincare sphere convenient representation
  - Stokes parameters I, Q, U, V
  - I intensity; Q,U linear polarization, V circular polarization
- Quasi-monochromatic "waves" in reality
  - can be partially polarized
  - still represented by Stokes parameters

Antenna & Interferometer Polarization

Interferometer response to polarization

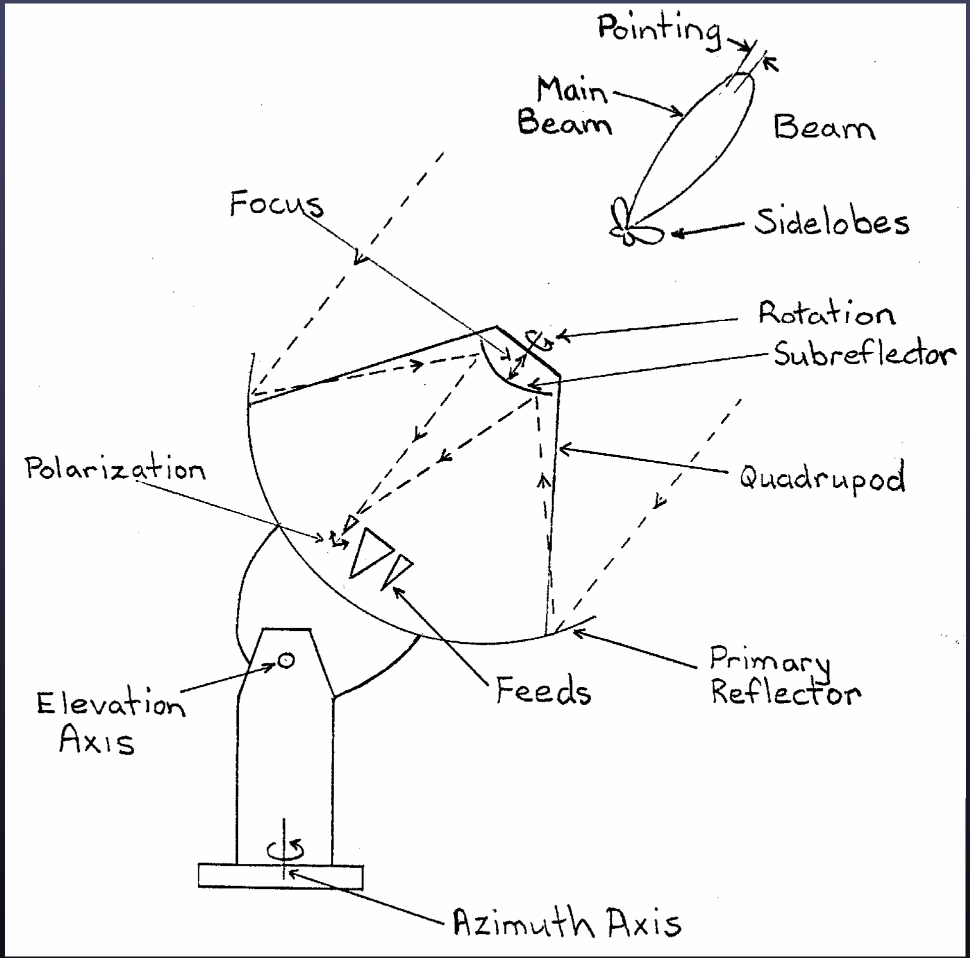
- Stokes parameter recap:
  - intensity I
  - fractional polarization $(p\ I)^2 = Q^2 + U^2 + V^2$
  - linear polarization Q,U $(m\ I)^2 = Q^2 + U^2$
  - circular polarization V $(v\ I)^2 = V^2$
- Coordinate system dependence:
  - I independent
  - V depends on choice of "handedness"
    - V > 0 for RCP
  - Q,U depend on choice of "North" (plus handedness)
    - Q "points" North, U 45 toward East
    - EVPA $\Phi = ½ \tan^{-1} (U/Q)$ (North through East)

Reflector antenna systems

- Reflections
  - turn RCP ⇔ LCP
  - E-field allowed only in plane of surface
- Curvature of surfaces
  - introduce cross-polarization
  - effect increases with curvature (as f/D decreases)
- Symmetry
  - on-axis systems see linear cross-polarization
  - off-axis feeds introduce asymmetries & R/L squint
- Feedhorn & Polarizers
  - introduce further effects (e.g. “leakage”)

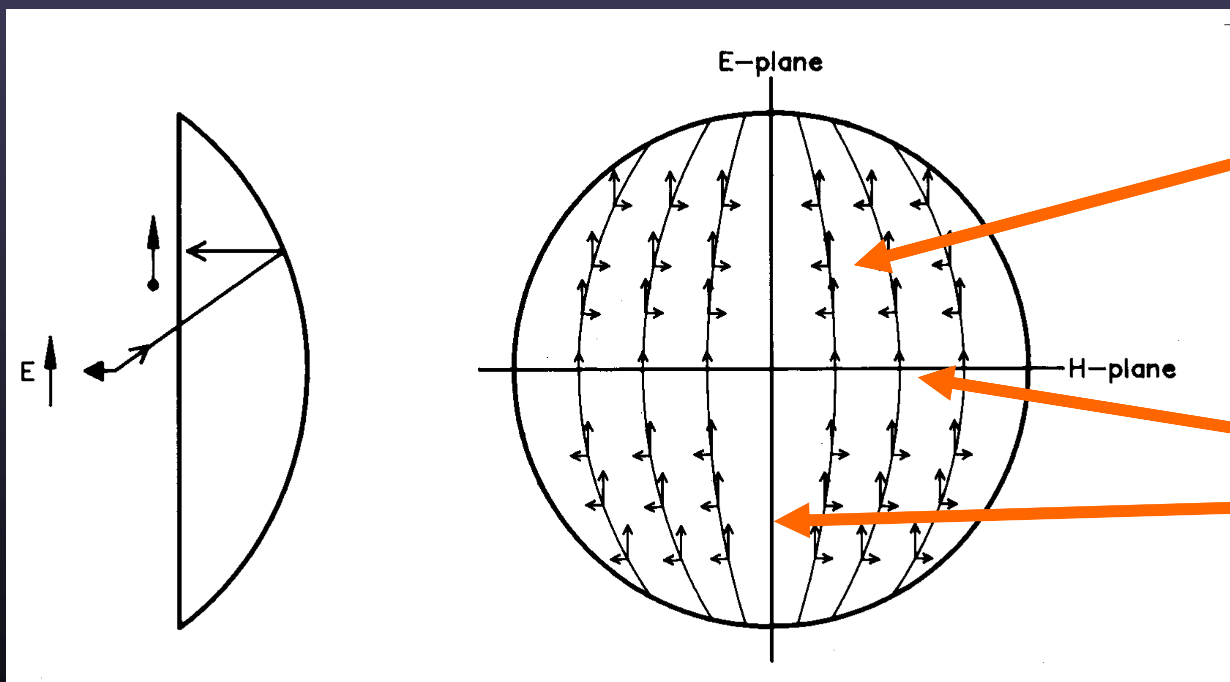
Optics – Cassegrain radio telescope

- Paraboloid illuminated by feedhorn:



Optics – telescope response

- Reflections
  - turn RCP ⇔ LCP
  - E-field (currents) allowed only in plane of surface
- “Field distribution” on aperture for E and B planes:

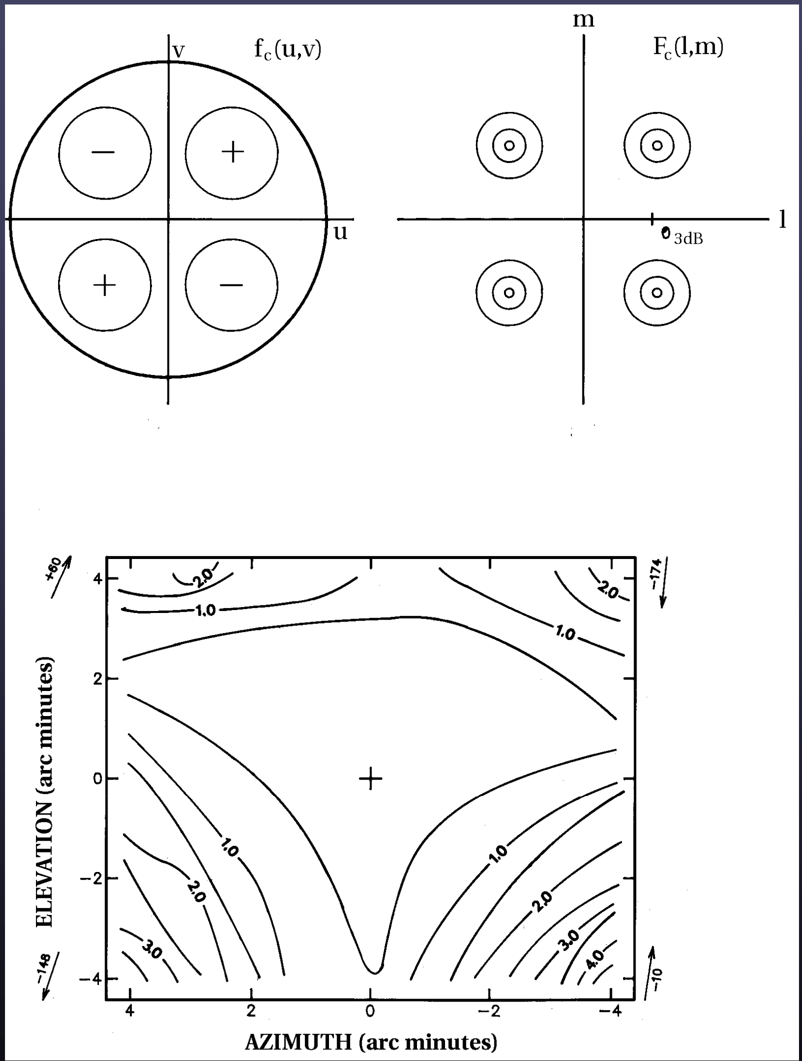


Cross-polarization at 45°

No cross-polarization on axes

Polarization field pattern

- Cross-polarization
  - 4-lobed pattern
- Off-axis feed system
  - perpendicular elliptical linear pol. beams
  - R and L beams diverge (beam squint)
- See also:
  - “Antennas” lecture by P. Napier

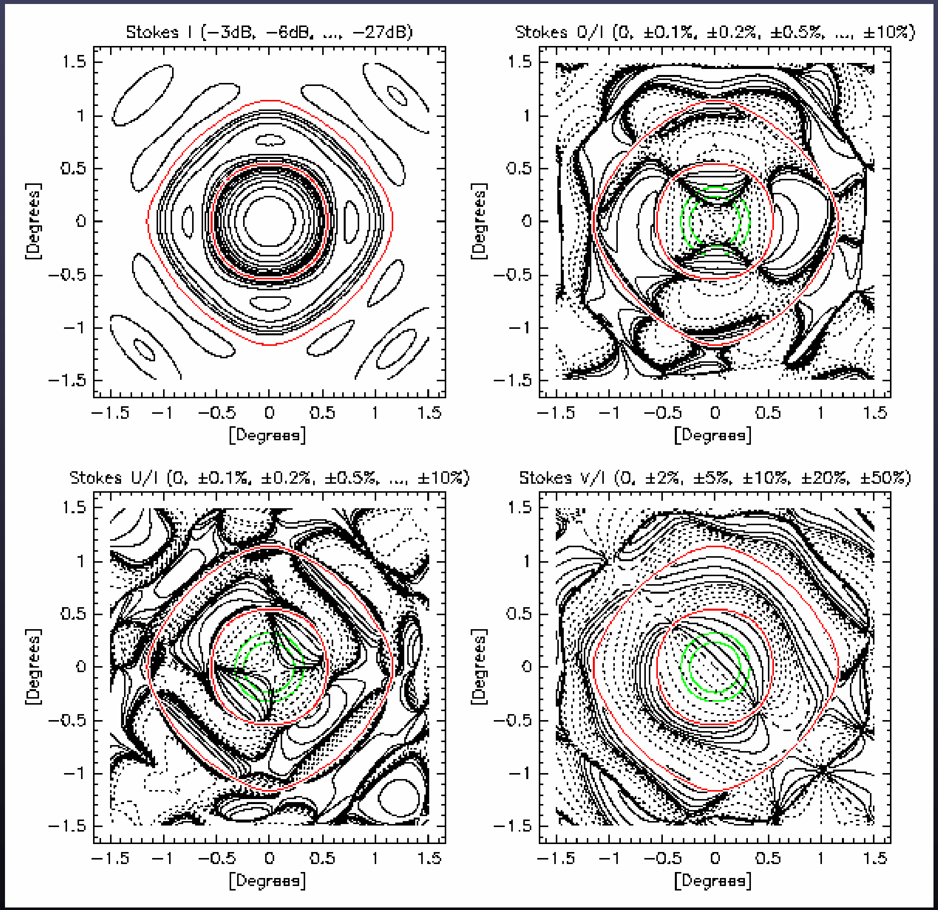


Feeds – Linear or Circular?

- The VLA uses a circular feedhorn design
  - plus (quarter-wave) polarizer to convert circular polarization from feed into linear polarization in rectangular waveguide
  - correlations will be between R and L from each antenna
    - RR RL LR RL form complete set of correlations
- Linear feeds are also used
  - e.g. ATCA, ALMA (and possibly EVLA at 1.4 GHz)
  - no need for (lossy) polarizer!
  - correlations will be between X and Y from each antenna
    - XX XY YX YY form complete set of correlations
- Optical aberrations are the same in these two cases
  - but different response to electronic (e.g. gain) effects

Example – simulated VLA patterns

- EVLA Memo 58 "Using Grasp8 to Study the VLA Beam" W. Brisken



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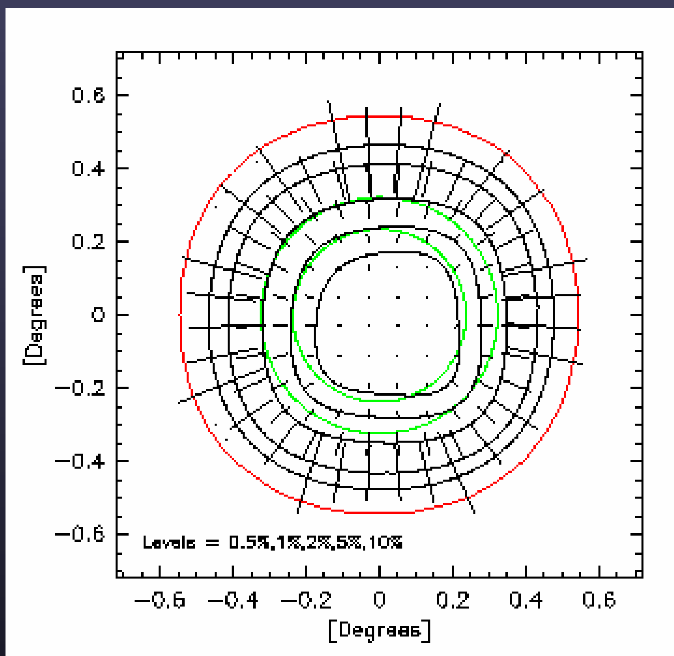
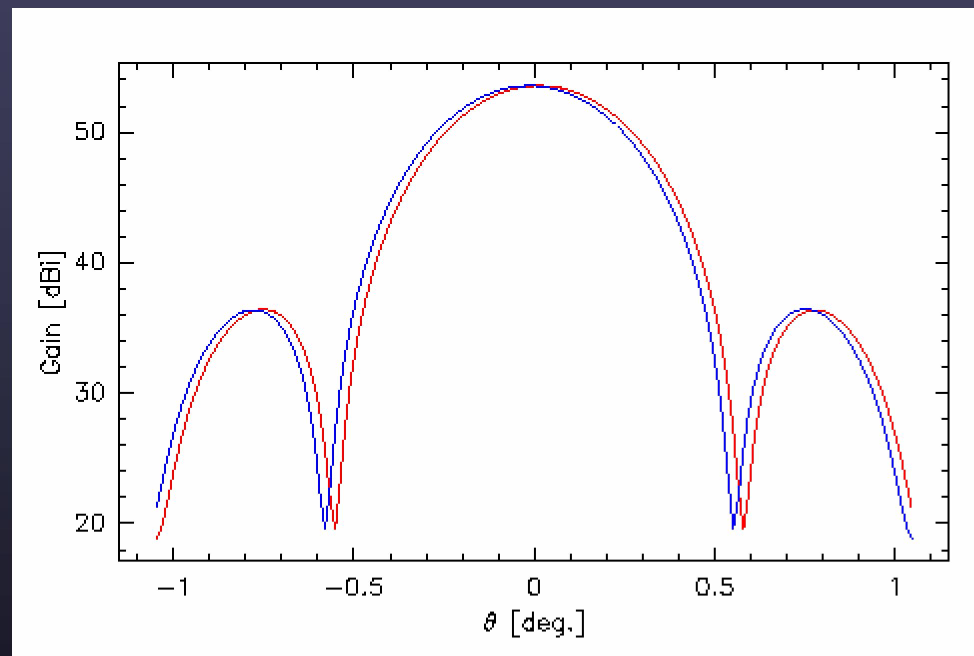


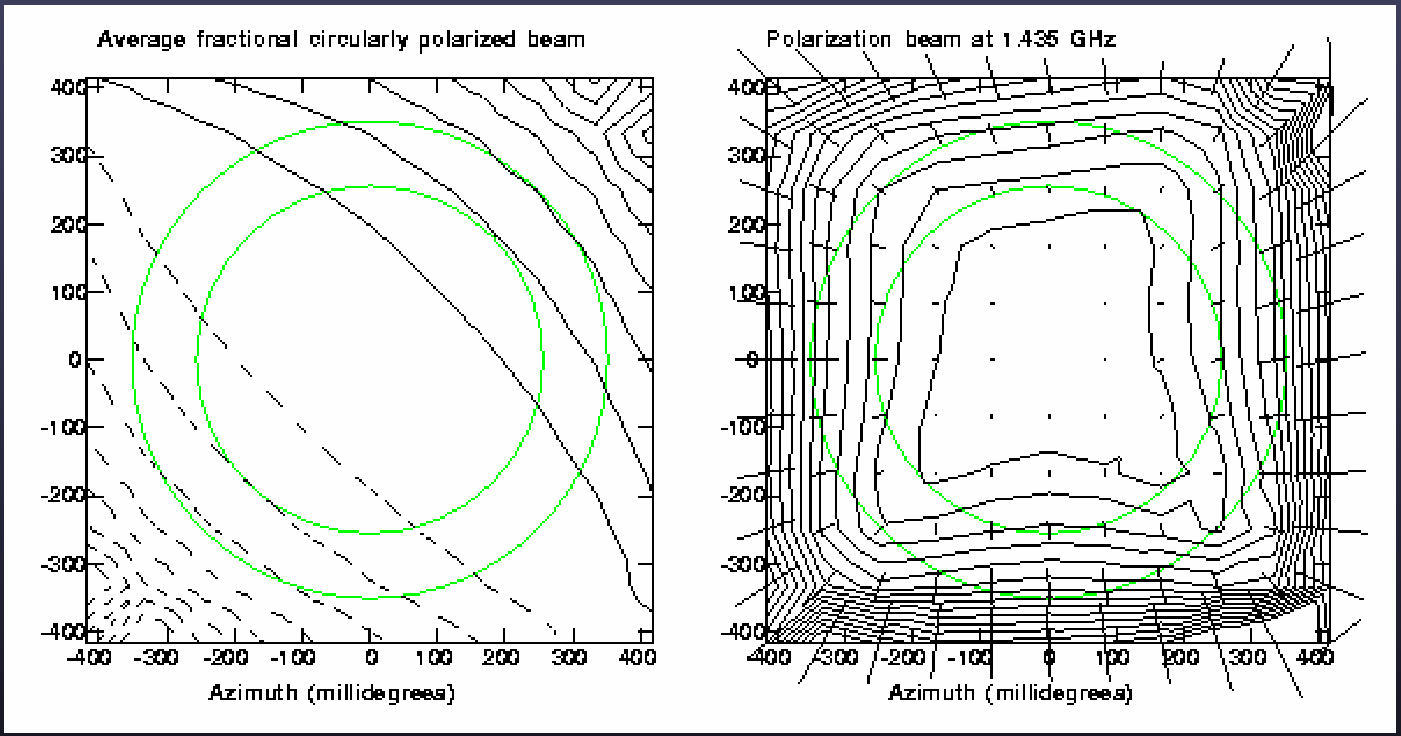
Linear Polarization



Circular Polarization cuts in R & L

Example – measured VLA patterns

- AIPS Memo 86 “Widefield Polarization Correction of VLA Snapshot Images at 1.4 GHz” W. Cotton (1994)

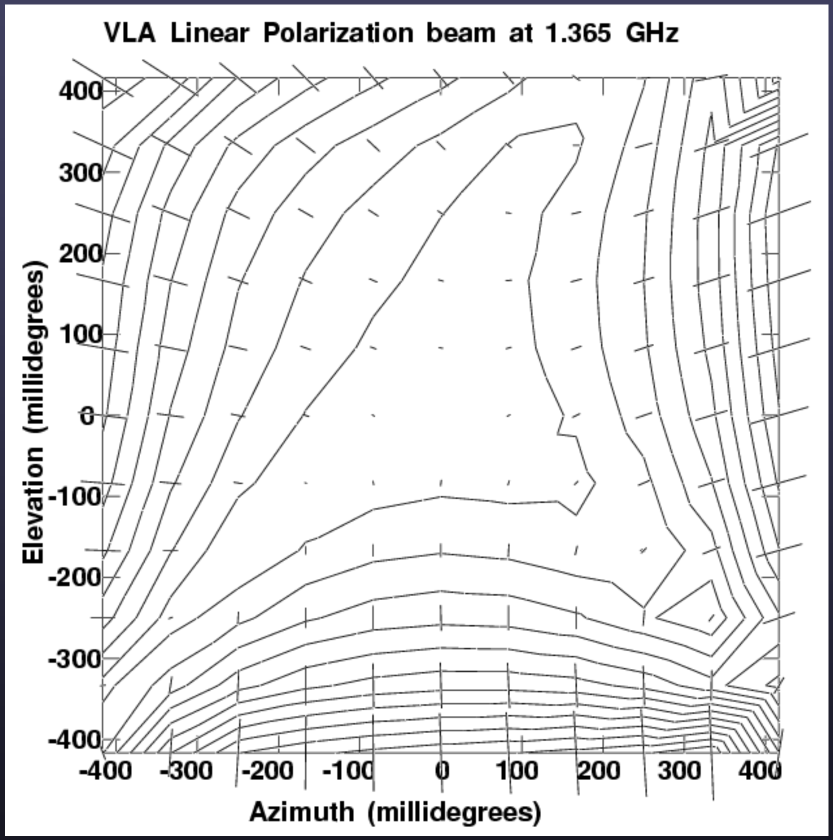


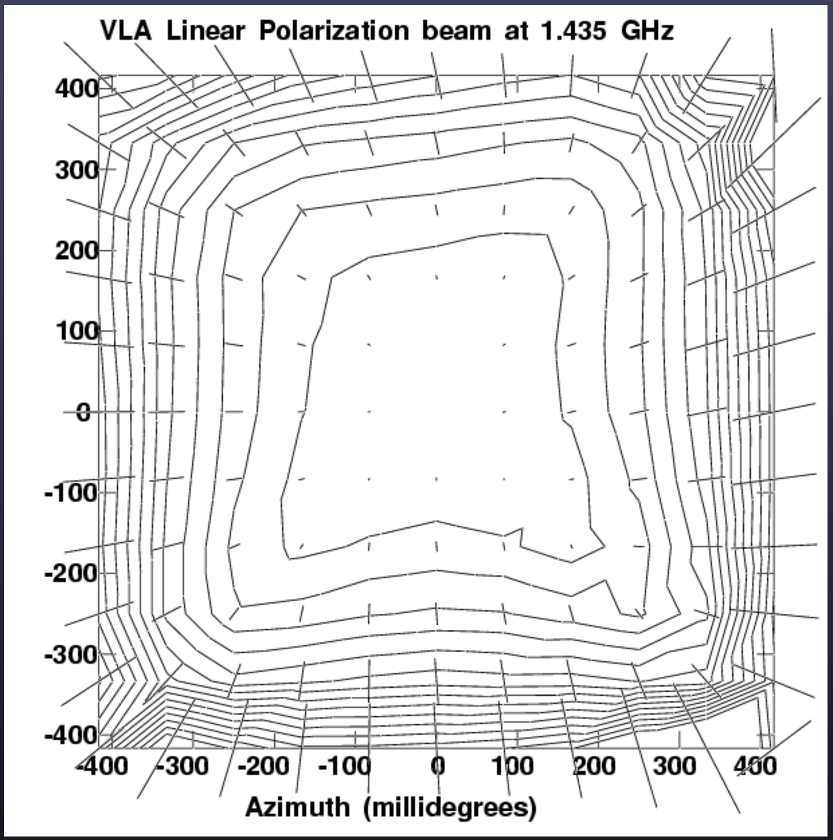
Circular Polarization

Linear Polarization

Example – measured VLA patterns

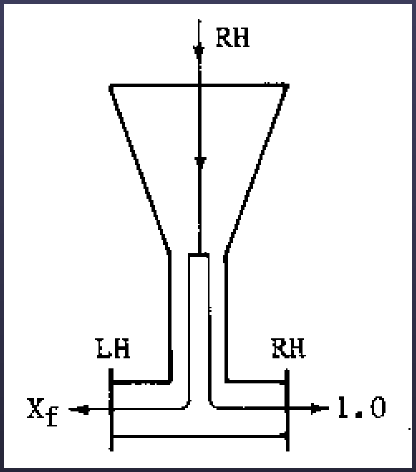
- frequency dependence of polarization beam :





Beyond optics – waveguides & receivers

- Response of polarizers
  - convert R & L to X & Y in waveguide
  - purity and orthogonality errors
- Other elements in signal path:
  - Sub-reflector & Feedhorn
    - symmetry & orientation
  - Ortho-mode transducers (OMT)
    - split orthogonal modes into waveguide
  - Polarizers
    - retard one mode by quarter-wave to convert LP ➔ CP
    - frequency dependent!
  - Amplifiers
    - separate chains for R and L signals



Back to the Measurement Equation

- Polarization effects in the signal chain appear as error terms in the Measurement Equation
  - e.g. "Calibration" lecture, G. Moellenbrock:
    - *F* = ionospheric Faraday rotation
    - *T* = tropospheric effects
    - *P* = parallactic angle
    - *E* = antenna voltage pattern
    - *D* = polarization leakage
    - *G* = electronic gain
    - *B* = bandpass response

Antenna *i*

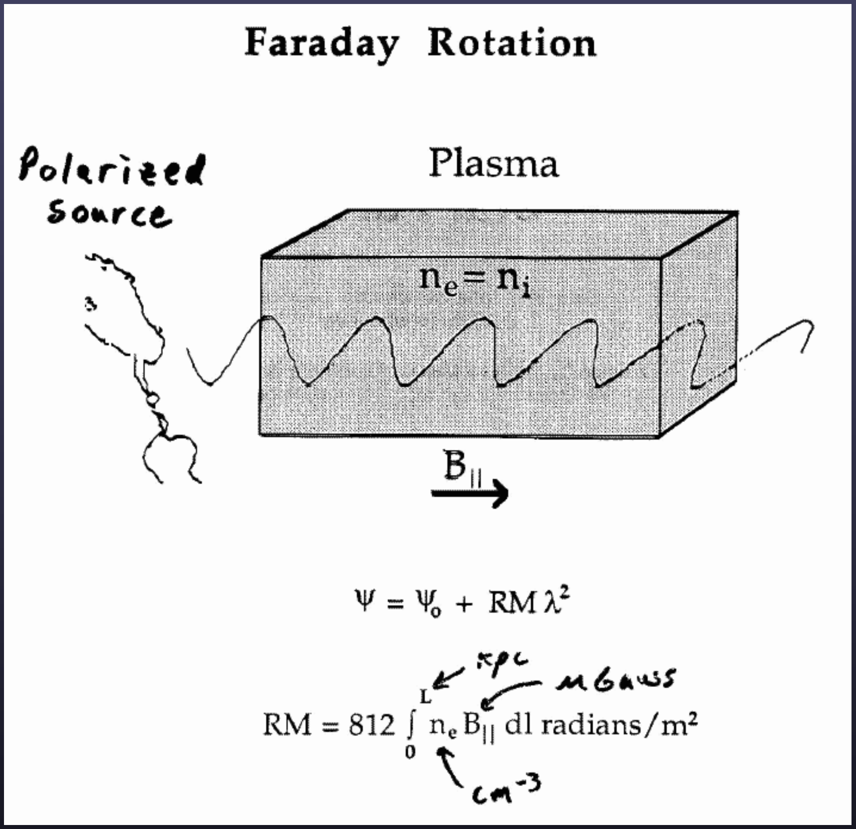
$$\overleftrightarrow{J}_i = \overleftrightarrow{B}_i \overleftrightarrow{G}_i \overleftrightarrow{D}_i \overleftrightarrow{E}_i \overleftrightarrow{P}_i \overleftrightarrow{T}_i \overleftrightarrow{F}_i$$

Baseline *ij* (outer product)

$$\begin{aligned}
\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}_j^* &= \left(\overleftrightarrow{B}_i \overleftrightarrow{G}_i \overleftrightarrow{D}_i \overleftrightarrow{E}_i \overleftrightarrow{P}_i \overleftrightarrow{T}_i \overleftrightarrow{F}_i \otimes \overleftrightarrow{B}_j^* \overleftrightarrow{G}_j^* \overleftrightarrow{D}_j^* \overleftrightarrow{E}_j^* \overleftrightarrow{P}_j^* \overleftrightarrow{T}_j^* \overleftrightarrow{F}_j^* \right) \\
&= \left(\overleftrightarrow{B}_i \otimes \overleftrightarrow{B}_j^* \right) \left(\overleftrightarrow{G}_i \otimes \overleftrightarrow{G}_j^* \right) \left(\overleftrightarrow{D}_i \otimes \overleftrightarrow{D}_j^* \right) \left(\overleftrightarrow{E}_i \otimes \overleftrightarrow{E}_j^* \right) \left(\overleftrightarrow{P}_i \otimes \overleftrightarrow{P}_j^* \right) \left(\overleftrightarrow{T}_i \otimes \overleftrightarrow{T}_j^* \right) \left(\overleftrightarrow{F}_i \otimes \overleftrightarrow{F}_j^* \right) \\
&= \overleftrightarrow{B}_{ij} \overleftrightarrow{G}_{ij} \overleftrightarrow{D}_{ij} \overleftrightarrow{E}_{ij} \overleftrightarrow{P}_{ij} \overleftrightarrow{T}_{ij} \overleftrightarrow{F}_{ij}
\end{aligned}$$

Ionospheric Faraday Rotation, *F*

- Birefringency due to magnetic field in ionospheric plasma



  - also present in radio sources!

Ionospheric Faraday Rotation, *F*

- The ionosphere is *birefringent*; one hand of circular polarization is delayed w.r.t. the other, introducing a phase shift:

$$\Delta\phi \approx 0.15\ \lambda^2 \int B_{\|} n_e ds\ \ \text{deg} \quad (\lambda \text{ in cm},\ n_e ds \text{ in } 10^{14}\text{cm}^{-2},\ B_{\|} \text{ in G})$$

  - rotates the linear polarization position angle :

$$\vec{F}^{RL} = \begin{pmatrix} e^{i\Delta\phi} & 0 \\ 0 & e^{-i\Delta\phi} \end{pmatrix};\ \vec{F}^{XY} = \begin{pmatrix} \cos\Delta\phi & -\sin\Delta\phi \\ \sin\Delta\phi & \cos\Delta\phi \end{pmatrix}$$

  - more important at longer wavelengths:

$$TEC = \int n_e ds\ \sim 10^{14}\text{cm}^{-2}; \quad B_{\|} \sim 1\text{G}; \quad \lambda = 20\text{cm} \rightarrow \Delta\phi \sim 60^{\circ}$$

  - ionosphere most active at solar maximum and sunrise/sunset
  - watch for direction dependence (in-beam)
  - see "Low Frequency Interferometry" (C. Brogan)

Parallactic Angle, *P*

- Orientation of sky in telescope's field of view
  - Constant for equatorial telescopes
  - Varies for alt-az-mounted telescopes:

$$\chi(t) = \arctan\left(\frac{\cos(l)\sin(h(t))}{\sin(l)\cos(\delta) - \cos(l)\sin(\delta)\cos(h(t))}\right)$$

$$l = \text{latitude},\ \ h(t) = \text{hour angle},\ \ \delta = \text{declination}$$

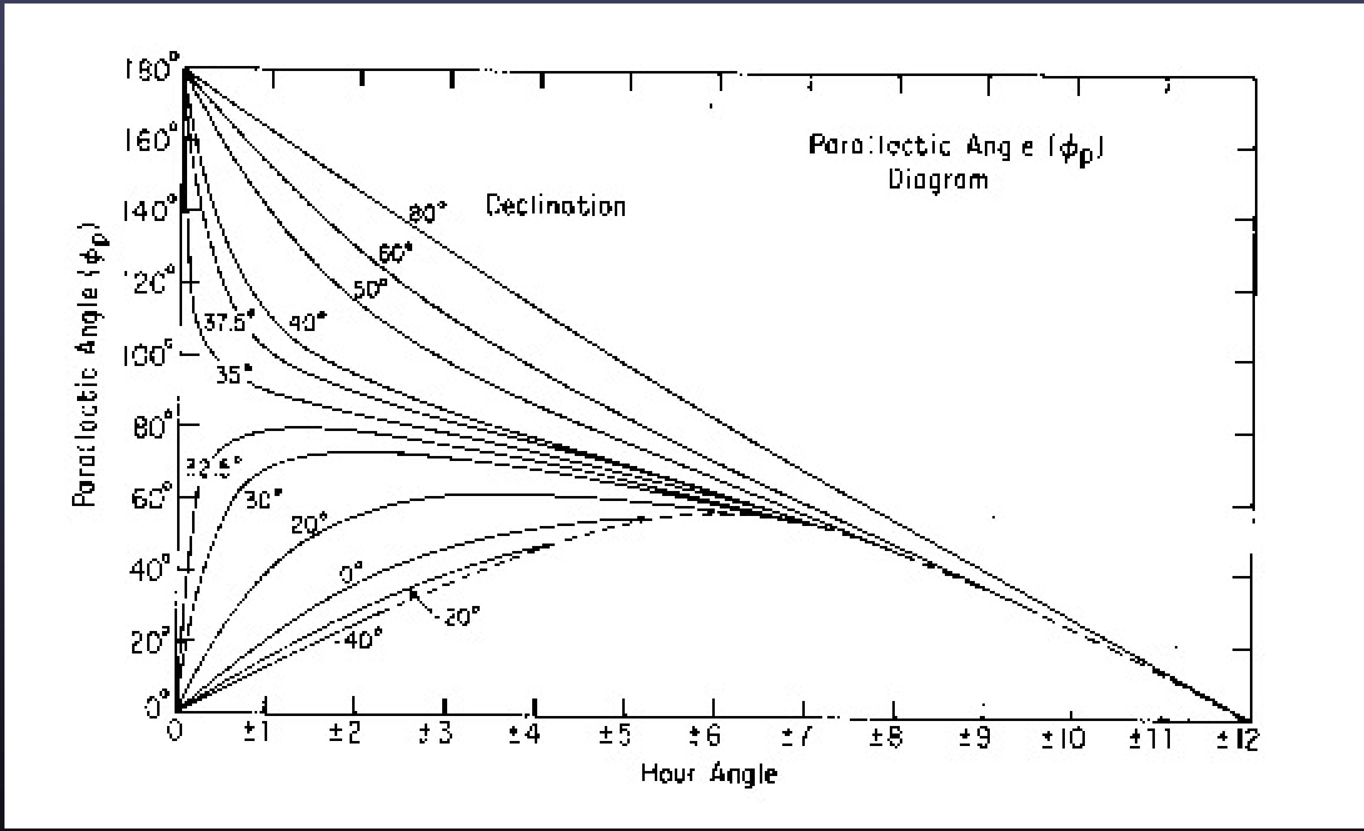
  - Rotates the position angle of linearly polarized radiation (c.f. *F*)

$$\vec{\overleftrightarrow{P}}^{RL} = \begin{pmatrix} e^{i\chi} & 0 \\ 0 & e^{-i\chi} \end{pmatrix};\ \ \vec{\overleftrightarrow{P}}^{XY} = \begin{pmatrix} \cos\chi & -\sin\chi \\ \sin\chi & \cos\chi \end{pmatrix}$$

  - defined per antenna (often same over array)
  - P modulation can be used to aid in calibration

Parallactic Angle, *P*

- Parallactic angle versus hour angle at VLA :
  - fastest swing for source passing through zenith



Antenna voltage pattern, E

- Direction-dependent gain and polarization
  - includes primary beam
    - Fourier transform of cross-correlation of antenna voltage patterns
    - includes polarization asymmetry (squint)

$$E^{pq} = \begin{pmatrix} e^p(l,m) & 0 \\ 0 & e^q(l,m) \end{pmatrix}$$

  - can include off-axis cross-polarization (leakage)
    - convenient to reserve D for on-axis leakage
    - will then have off-diagonal terms
  - important in wide-field imaging and mosaicing
    - when sources fill the beam (e.g. low frequency)

Polarization Leakage, *D*

- Polarizer is not ideal, so orthogonal polarizations not perfectly isolated
  - Well-designed systems have *d* < 1-5%
  - A geometric property of the antenna, feed & polarizer design
    - frequency dependent (e.g. quarter-wave at center ν)
    - direction dependent (in beam) due to antenna
  - For *R*,*L* systems
    - parallel hands affected as d•Q + d•U , so only important at high dynamic range (because *Q*,*U*~*d*, typically)
    - cross-hands affected as d•I so almost always important

$$\overleftrightarrow{D}^{pq} = \begin{pmatrix} 1 & d^p \\ d^q & 1 \end{pmatrix}$$

Leakage of q into p (e.g. L into R)

Coherency vector and correlations

- Coherency vector:

$$\mathbf{e} = \left\langle \vec{s}_i \otimes \vec{s}_j^{\,*} \right\rangle = \left\langle \begin{pmatrix} s^p \\ s^q \end{pmatrix}_i \otimes \begin{pmatrix} s^p \\ s^q \end{pmatrix}_j^* \right\rangle = \begin{pmatrix} \left\langle s_i^p \cdot s_j^{*p} \right\rangle \\ \left\langle s_i^p \cdot s_j^{*q} \right\rangle \\ \left\langle s_i^q \cdot s_j^{*p} \right\rangle \\ \left\langle s_i^q \cdot s_j^{*q} \right\rangle \end{pmatrix}$$

  – e.g. for circularly polarized feeds:

$$\mathbf{e}_{circ} = \left\langle \vec{s}_i \otimes \vec{s}_j^{\,*} \right\rangle = \left\langle \begin{pmatrix} s^R \\ s^L \end{pmatrix}_i \otimes \begin{pmatrix} s^R \\ s^L \end{pmatrix}_j^* \right\rangle = \begin{pmatrix} \left\langle s_i^R \cdot s_j^{*R} \right\rangle \\ \left\langle s_i^R \cdot s_j^{*L} \right\rangle \\ \left\langle s_i^L \cdot s_j^{*R} \right\rangle \\ \left\langle s_i^L \cdot s_j^{*L} \right\rangle \end{pmatrix}$$

Coherency vector and Stokes vector

- Example: circular polarization (e.g. VLA)

$$\mathbf{e}_{circ} = \overleftrightarrow{S}_{circ}\mathbf{e}^S = \begin{pmatrix} RR \\ RL \\ LR \\ LL \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & i & 0 \\ 0 & 1 & -i & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+V \\ Q+iU \\ Q-iU \\ I-V \end{pmatrix}$$

- Example: linear polarization (e.g. ATCA)

$$\mathbf{e}_{lin} = \overleftrightarrow{S}_{lin}\mathbf{e}^S = \begin{pmatrix} XX \\ XY \\ YX \\ YY \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & 1 & -i \\ 1 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+Q \\ U+iV \\ U-iV \\ I-Q \end{pmatrix}$$

Visibilities and Stokes parameters

- Convolution of sky with measurement effects:

$$\vec{V}_{ij}^{obs} = \int_{sky} \left(\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}_j^*\right) \overleftrightarrow{S}\vec{I}(l,m)\, e^{-i2\pi\left(u_{ij}l+v_{ij}m\right)}\, dl\, dm$$

Instrumental effects, including "beam" E(l,m)

coordinate transformation to Stokes parameters

(I, Q, U, V)

- e.g. with (polarized) beam E :

$$\vec{V}_{ij}^{obs} = \int_{sky} \left(\overleftrightarrow{E}_i \otimes \overleftrightarrow{E}_j^*\right) \overleftrightarrow{S}\vec{I}(l,m)\, e^{-i2\pi\left(u_{ij}l+v_{ij}m\right)}\, dl\, dm$$

$$\vec{V}_{ij}^{obs} = \int_{uv} \left(\widetilde{E}_i \otimes \widetilde{E}_j^*\right) \overleftrightarrow{S}\widetilde{\vec{I}}(u,v)\, e^{i2\pi\left(u_{ij}l+v_{ij}m\right)}\, du\, dv$$

– imaging involves inverse transforming these

Example: RL basis

- Combining E, etc. (no D), expanding P,S:

$$V_{ij}^{RR} = \int_{sky} E_{ij}^{RR}(l,m)[\mathrm{I}(l,m) + \mathrm{V}(l,m)]e^{i(\chi_i - \chi_j)}e^{-i2\pi(u_{ij}l + v_{ij}m)}dldm$$

$$V_{ij}^{RL} = \int_{sky} E_{ij}^{RL}(l,m)[\mathrm{Q}(l,m) + i\mathrm{U}(l,m)]e^{i(\chi_i + \chi_j)}e^{-i2\pi(u_{ij}l + v_{ij}m)}dldm$$

$$V_{ij}^{LR} = \int_{sky} E_{ij}^{LR}(l,m)[\mathrm{Q}(l,m) - i\mathrm{U}(l,m)]e^{-i(\chi_i + \chi_j)}e^{-i2\pi(u_{ij}l + v_{ij}m)}dldm$$

$$V_{ij}^{LL} = \int_{sky} E_{ij}^{LL}(l,m)[\mathrm{I}(l,m) - \mathrm{V}(l,m)]e^{-i(\chi_i - \chi_j)}e^{-i2\pi(u_{ij}l + v_{ij}m)}dldm$$

2χ for co-located array

0 for co-located array

Example: RL basis imaging

- Parenthetical Note:
  - can make a pseudo-I image by gridding RR+LL on the Fourier half-plane and inverting to a real image
  - can make a pseudo-V image by gridding RR-LL on the Fourier half-plane and inverting to real image
  - can make a pseudo-(Q+iU) image by gridding RL to the full Fourier plane (with LR as the conjugate) and inverting to a complex image
  - does not require having full polarization RR,RL,LR,LL for every visibility
- More on imaging (& deconvolution) tomorrow!

Leakage revisited…

- Primary on-axis effect is "leakage" of one polarization into the measurement of the other (e.g. R ⇔ L)
  - but, direction dependence due to polarization beam!
- Customary to factor out on-axis leakage into D and put direction dependence in "beam"
  - example: expand RL basis with on-axis leakage

$$\hat{V}_{ij}^{RR} = V_{ij}^{RR} + d_i^R V_{ij}^{LR} + d_j^{*R} V_{ij}^{RL} + d_i^R d_j^{*R} V_{ij}^{LL}$$

$$\hat{V}_{ij}^{RL} = V_{ij}^{RL} + d_i^R V_{ij}^{LL} + d_j^{*L} V_{ij}^{RR} + d_i^R d_j^L V_{ij}^{LR}$$

  - similarly for XY basis

Example: RL basis leakage

- In full detail:

$$V_{ij}^{RR} = \int_{sky} E_{ij}^{RR}(l,m)[(\mathrm{I}+\mathrm{V})e^{i(\chi_i-\chi_j)}$$
$$+ d_i^R e^{-i(\chi_i+\chi_j)}(\mathrm{Q}-i\mathrm{U}) + d_j^{*R} e^{i(\chi_i+\chi_j)}(\mathrm{Q}+i\mathrm{U})$$
$$+ d_i^R d_j^{*R} e^{-i(\chi_i-\chi_j)}(\mathrm{I}-\mathrm{V})](l,m)e^{-i2\pi(u_{ij}l+v_{ij}m)}dldm$$

$$V_{ij}^{RL} = \int_{sky} E_{ij}^{RL}(l,m)[(\mathrm{Q}+i\mathrm{U})e^{i(\chi_i+\chi_j)}$$
$$+ d_i^R(\mathrm{I}-\mathrm{V})e^{-i(\chi_i-\chi_j)} + d_j^{*L}(\mathrm{I}+\mathrm{V})e^{i(\chi_i-\chi_j)}$$
$$+ d_i^R d_j^{*L}(\mathrm{Q}-i\mathrm{U})e^{-i(\chi_i+\chi_j)}](l,m)e^{-i2\pi(u_{ij}l+v_{ij}m)}dldm$$

"true" signal

2nd order: D•P into I

2nd order: D²•I into I

1st order: D•I into P

3rd order: D²•P* into P

Example: Linearized response

- Dropping terms in d^2, dQ, dU, dV (and expanding G)

- **For crossed linearly polarized feeds**

$$
\begin{aligned}
v_{pp} &= \tfrac{1}{2} g_{ip} g^*_{kp} (I + Q\ \cos 2\chi + U\ \sin 2\chi), \\
v_{pq} &= \tfrac{1}{2} g_{ip} g^*_{kq} ((d_{ip} - d^*_{kq}) I - Q\ \sin 2\chi + U\ \cos 2\chi + jV), \\
v_{qp} &= \tfrac{1}{2} g_{iq} g^*_{kp} ((d^*_{kp} - d_{iq}) I - Q\ \sin 2\chi + U\ \cos 2\chi - jV), \\
v_{qq} &= \tfrac{1}{2} g_{iq} g^*_{kq} (I - Q\ \cos 2\chi - U\ \sin 2\chi),
\end{aligned}
$$

- **for circularly polarized feeds:**

$$
\begin{aligned}
v_{pp} &= \tfrac{1}{2} g_{ip} g^*_{kp} (I + V), \\
v_{pq} &= \tfrac{1}{2} g_{ip} g^*_{kq} ((d_{ip} - d^*_{kq}) I + \ e^{-2j\chi} (Q + jU)), \\
v_{qp} &= \tfrac{1}{2} g_{iq} g^*_{kp} ((d^*_{kp} - d_{iq}) I + \ e^{2j\chi} (Q - jU)), \\
v_{qq} &= \tfrac{1}{2} g_{iq} g^*_{kq} (I - V).
\end{aligned}
$$

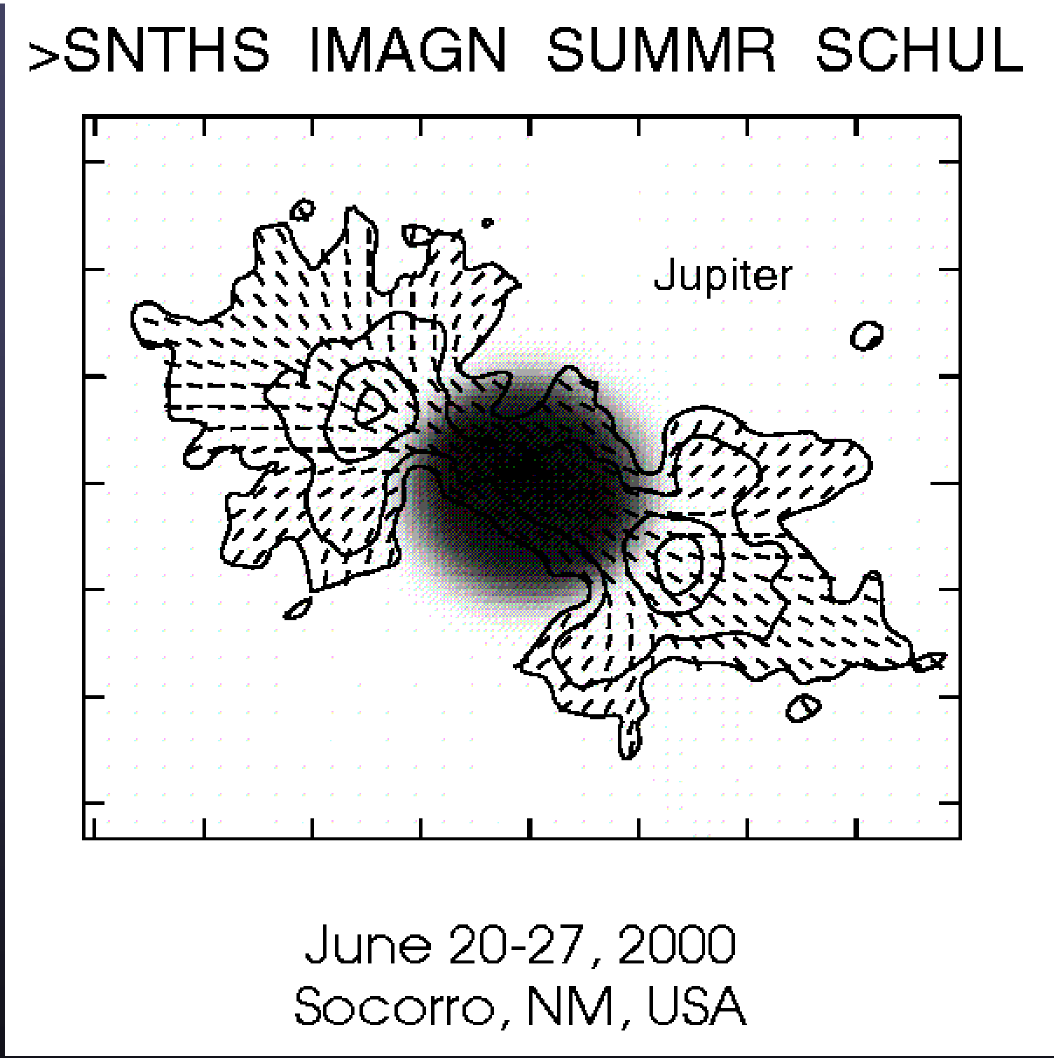
– warning: using linear order can limit dynamic range!

Summary – polarization interferometry

- Choice of basis: CP or LP feeds
- Follow the Measurement Equation
  - ionospheric Faraday rotation F at low frequency
  - parallactic angle P for coordinate transformation to Stokes
  - "leakage" D varies with ν and over beam (mix with E)
- Leakage
  - use full (all orders) D solver when possible
  - linear approximation OK for low dynamic range

Polarization Calibration & Observation

So you want to make a polarization map…



Strategies for polarization observations

- Follow general calibration procedure (last lecture)
  - will need to determine leakage D (if not known)
  - often will determine G and D together (iteratively)
  - procedure depends on basis and available calibrators
- Observations of polarized sources
  - follow usual rules for sensitivity, uv coverage, etc.
  - remember polarization fraction is usually low! (few %)
  - if goal is to map E-vectors, remember to calculate noise in $\Phi = \frac{1}{2} \tan^{-1} U/Q$
  - watch for gain errors in V (for CP) or Q,U (for LP)
  - for wide-field high-dynamic range observations, will need to correct for polarized primary beam (during imaging)

Strategies for leakage calibration

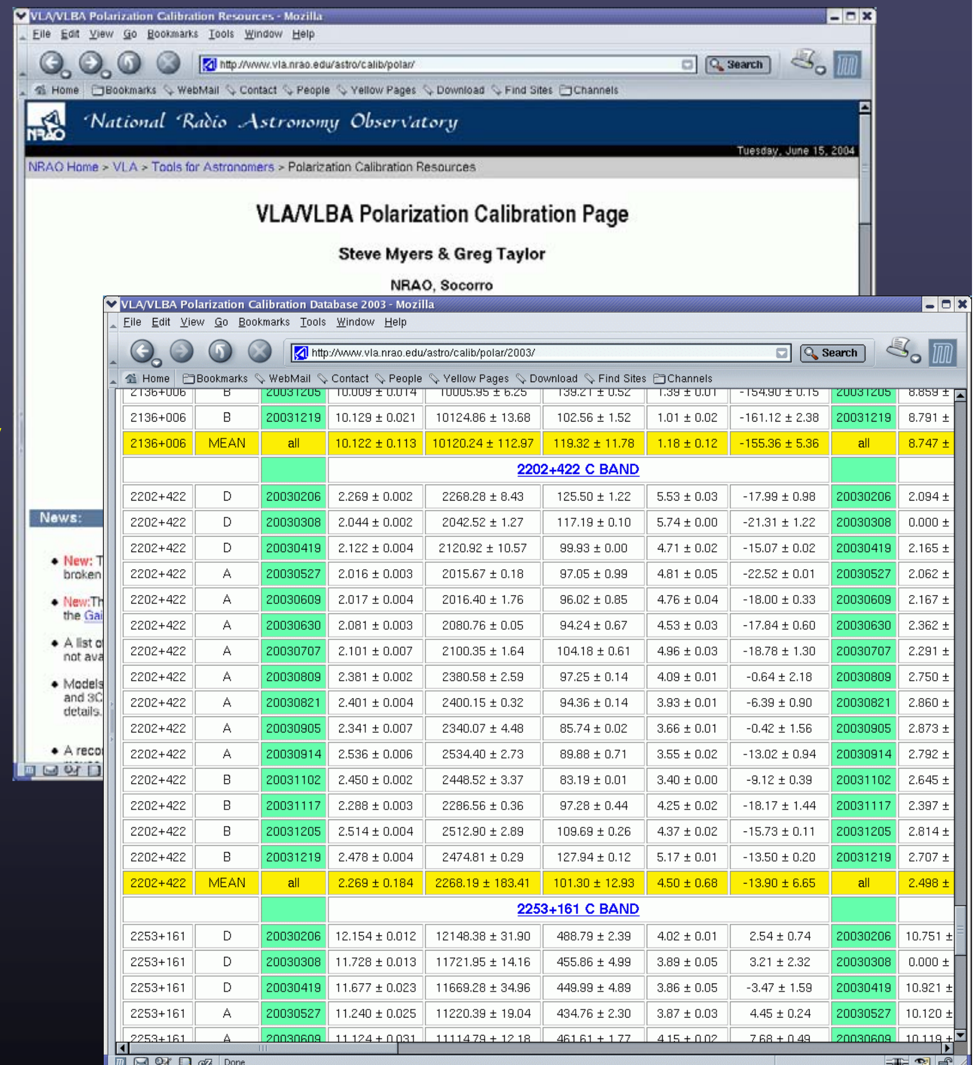
- Need a bright calibrator! Effects are low level…
  – determine gains G (mostly from parallel hands)
  – use cross-hands (mostly) to determine leakage
  – general ME D solver (e.g. aips++) uses all info
- Calibrator is unpolarized
  – leakage directly determined (ratio to I model), but only to an overall constant
  – need way to fix phase *p-q* (*ie.* R-L phase difference), e.g. using another calibrator with known EVPA
- Calibrator of known polarization
  – leakage can be directly determined (for I,Q,U,V model)
  – unknown *p-q* phase can be determined (from U/Q etc.)

Other strategies

- Calibrator of unknown polarization
  - solve for model IQUV and D simultaneously or iteratively
  - need good parallactic angle coverage to modulate sky and instrumental signals
    - in instrument basis, sky signal modulated by $e^{i2\chi}$
- With a very bright strongly polarized calibrator
  - can solve for leakages and polarization per baseline
  - can solve for leakages using parallel hands!
- With no calibrator
  - hope it averages down over parallactic angle
  - transfer D from a similar observation
    - usually possible for several days, better than nothing!
    - need observations at same frequency

Finding polarization calibrators

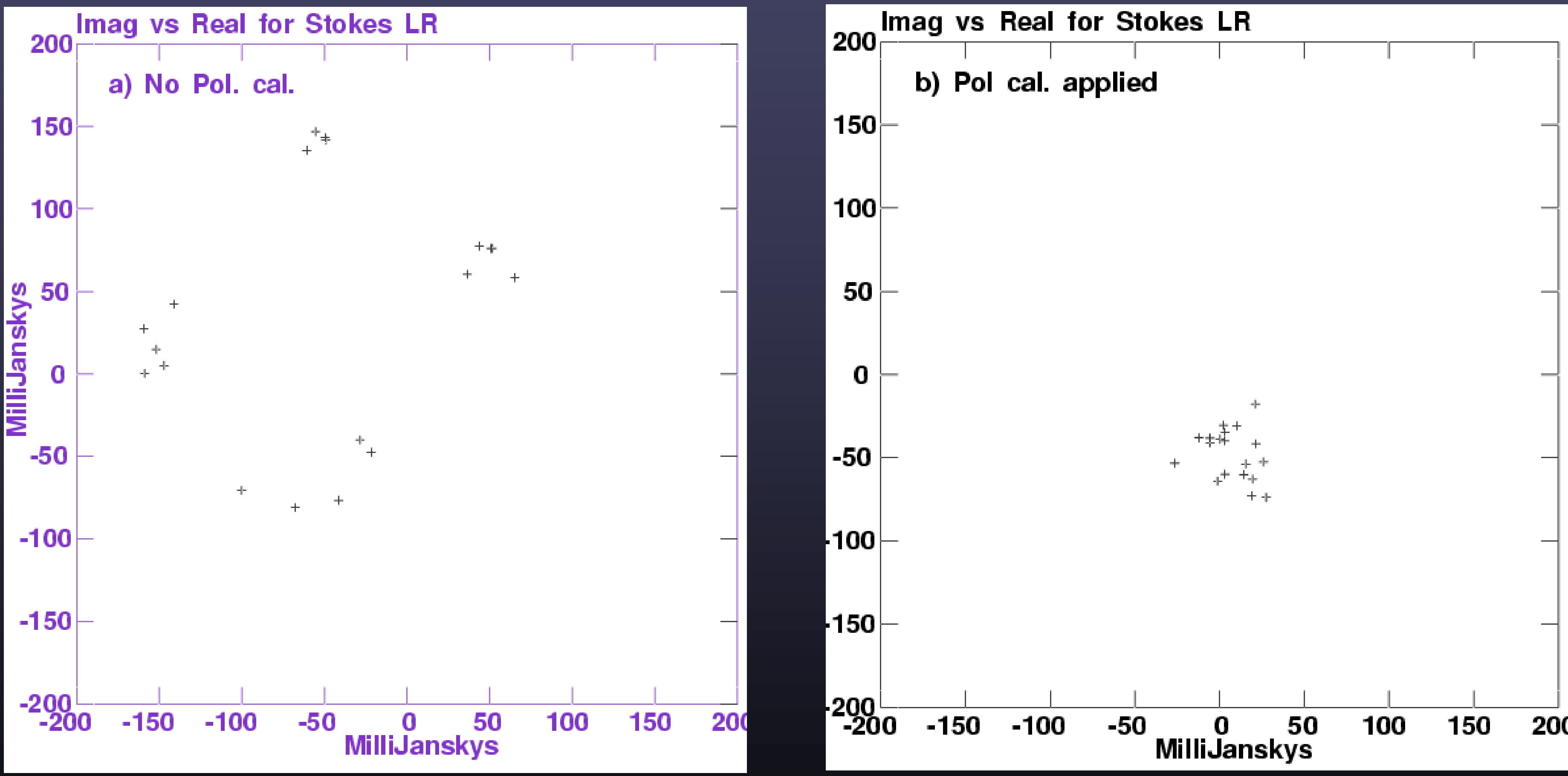
- Standard sources
  - planets (unpolarized if unresolved)
  - 3C286, 3C48, 3C147 (known IQU, stable)
  - sources monitored (e.g. by VLA)
  - other bright sources (bootstrap)



http://www.vla.nrao.edu/astro/calib/polar/

Example: D-term calibration

- D-term calibration effect on RL visibilities :



Example: D-term calibration

- D-term calibration effect in image plane :

Bad D-term solution

Good D-term solution

Example: "standard" procedure for CP feeds

Calibration of Circular Feeds

- Parallel correlations sensitive to Stokes I & V

$$
\begin{aligned}
v_{pp} &= \tfrac{1}{2} g_{ip} g^*_{kp} (I + V), \\
v_{qq} &= \tfrac{1}{2} g_{iq} g^*_{kq} (I - V).
\end{aligned}
$$

- Assume $V = 0$ for calibrator
- Can seperate and solve for gains for p and q
- Instrumental (d) and source polarization (Q,U) sum of two vectors:

$$
\begin{aligned}
v_{pq} &= \tfrac{1}{2} g_{ip} g^*_{kq} ((d_{ip} - d^*_{kq}) I + \; e^{-2j\chi} (Q + jU)), \\
v_{qp} &= \tfrac{1}{2} g_{iq} g^*_{kp} ((d^*_{kp} - d_{iq}) I + \; e^{2j\chi} (Q - jU))
\end{aligned}
$$

- Calibrator observations of a range of PA gives clean separation
- Independent gain calibration for p and q allows arbitrary phase offset – refer all phases to same "reference" antenna
- $p - q$ phase difference is that of the reference antenna
- Need observations of calibrator of known polarization angle aka Electric Vector Position Angle (EVPA)

Example: "standard" procedure for LP feeds

Calibration of Linear Feeds

- Parallel correlations sensitive to I, Q, & U

$$v_{pp} = \tfrac{1}{2} g_{ip} g^*_{kp} (I + Q\ \cos 2\chi + U\ \sin 2\chi),$$
$$v_{qq} = \tfrac{1}{2} g_{iq} g^*_{kq} (I - Q\ \cos 2\chi - U\ \sin 2\chi),$$

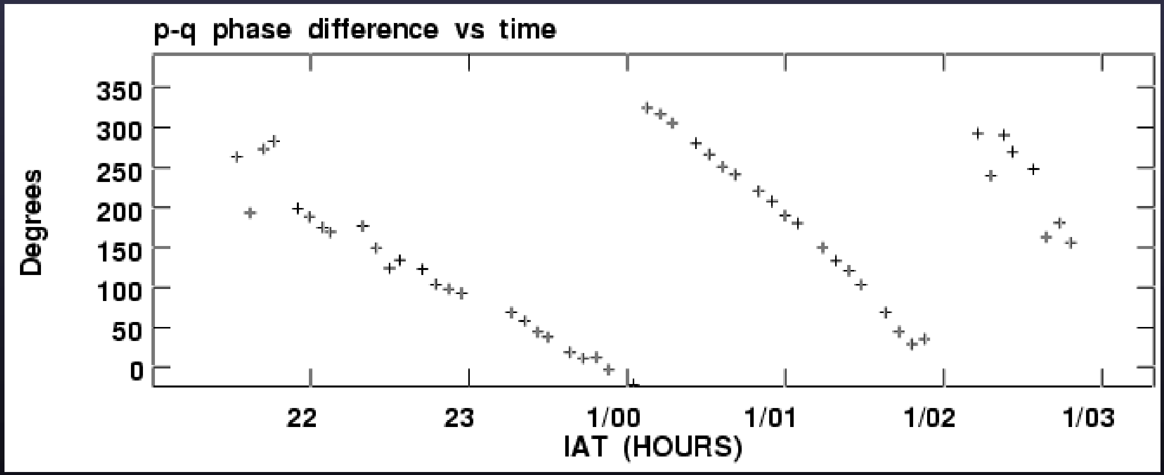
- Calibrator Q and U usually cannot be ignored (few %)
- Phase unaffected by polarization of a point source at the phase center
- Cannot seperate p, q gains and calibrator polarization
- $p - q$ phase offset not known
- May be unknown orientation error of p and q
- Need obs of source with known polarization

$$v_{pq} = \tfrac{1}{2} g_{ip} g^*_{kq} ((d_{ip} - d^*_{kq}) I - Q\ \sin 2\chi + U\ \cos 2\chi + jV),$$
$$v_{qp} = \tfrac{1}{2} g_{iq} g^*_{kp} ((d^*_{kp} - d_{iq}) I - Q\ \sin 2\chi + U\ \cos 2\chi - jV),$$

- Calibrator Q and U affect real part of cross pol. correlations
- Calibrator V affects imaginary part of cross pol. correlations but unaffected by PA

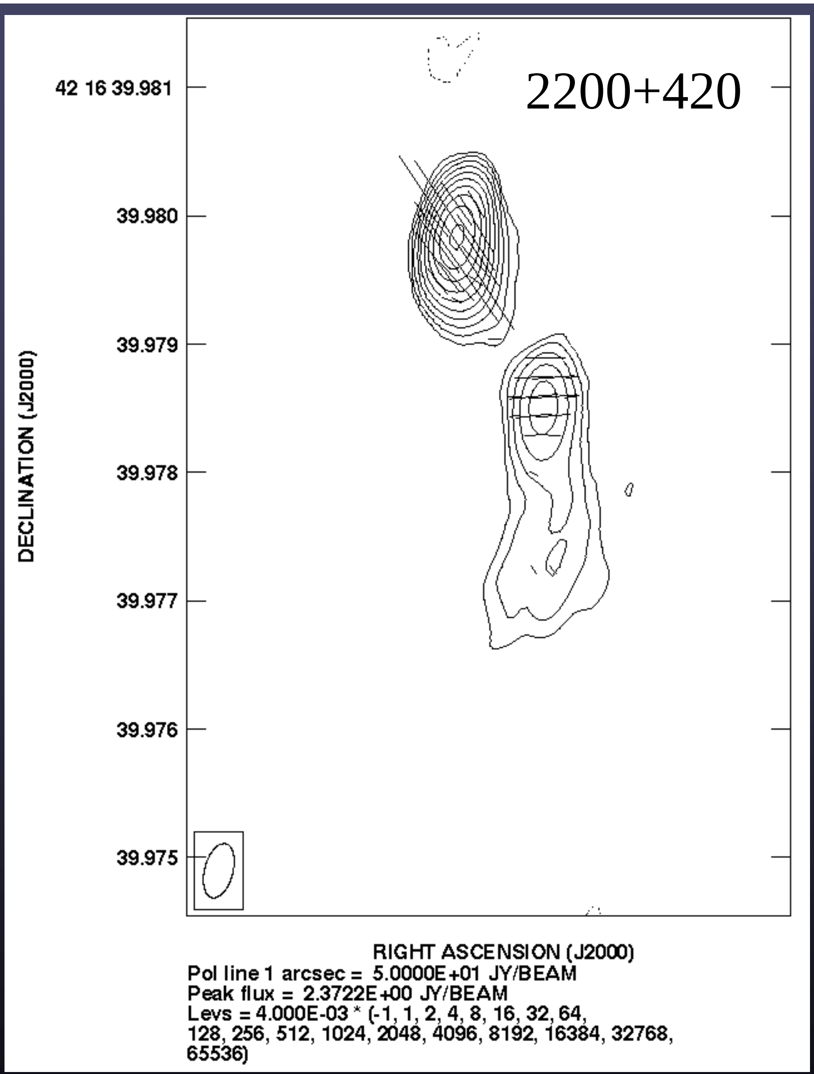
Special Issues

- Low frequency – ionospheric Faraday rotation
  - important for 2 GHz and below (sometimes higher too)
  - λ^2 dependence (separate out using multi-frequency obs.)
  - depends on time of day and solar activity (& observatory location)
  - external calibration using zenith TEC (plus gradient?)
  - self-calibration possible (e.g. with snapshots)



Special issues – continued…

- VLBI polarimetry
  - follows same principles
  - will have different parallactic angle at each station!
  - can have heterogeneous feed geometry (e.g. CP & LP)
  - harder to find sources with known polarization
    - calibrators resolved!
    - transfer EVPA from monitoring program



Subtleties …

- Antenna-based D solutions
  - closure quantities ➔ undetermined parameters
  - different for parallel and cross-hands
  - e.g. can add d to R and d* to L
  - need for reference antenna to align and transfer D solutions
- Parallel hands
  - are D solutions from cross-hands appropriate here?
  - what happens in full D solution (weighting issues?)

Special Issues – observing circular polarization

- Observing circular polarization V is straightforward with LP feeds (from Re and Im of cross-hands)
- With CP feeds:
  - gain variations can masquerade as (time-variable) V signal
    - helps to switch signal paths through back-end electronics
  - R vs. L beam squint introduces spurious V signal
    - limited by pointing accuracy
  - requires careful calibration
    - relative R and L gains critical
    - average over calibrators (be careful of intrinsic V)
  - VLBI somewhat easier
    - different systematics at stations help to average out

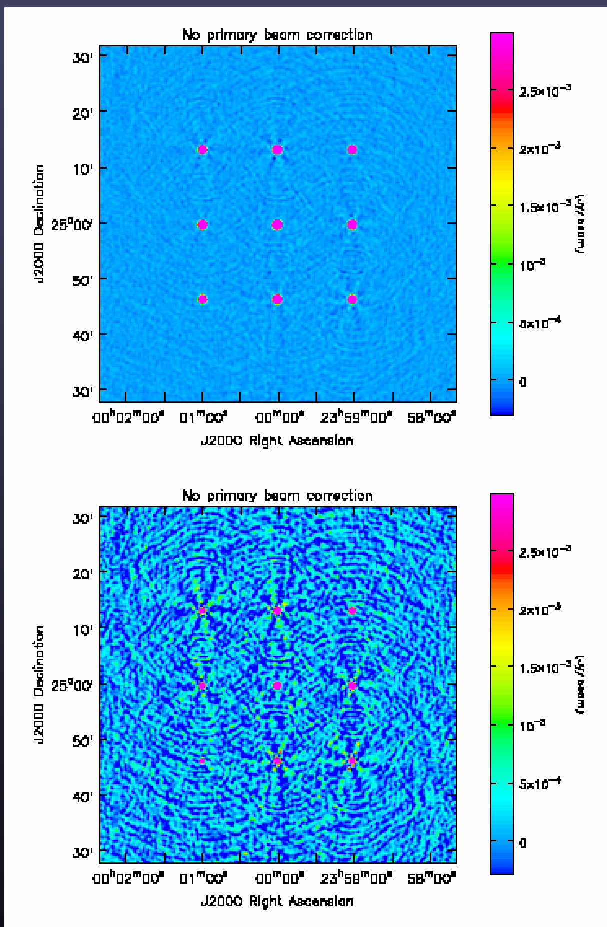
Special Issues – wide field polarimetry

- Actually an imaging & deconvolution issue
  - assume polarized beam D'•E is known
- Deal with direction-dependent effects
  - beam squint (R,L) or beam ellipticity (X,Y)
  - primary beam
- Iterative scheme (e.g. CLEAN)
  - implemented in aips++
  - see lectures by Bhatnagar & Cornwell
- Described in EVLA Memo 62 "Full Primary Beam Stokes IQUV Imaging" T. Cornwell (2003) :

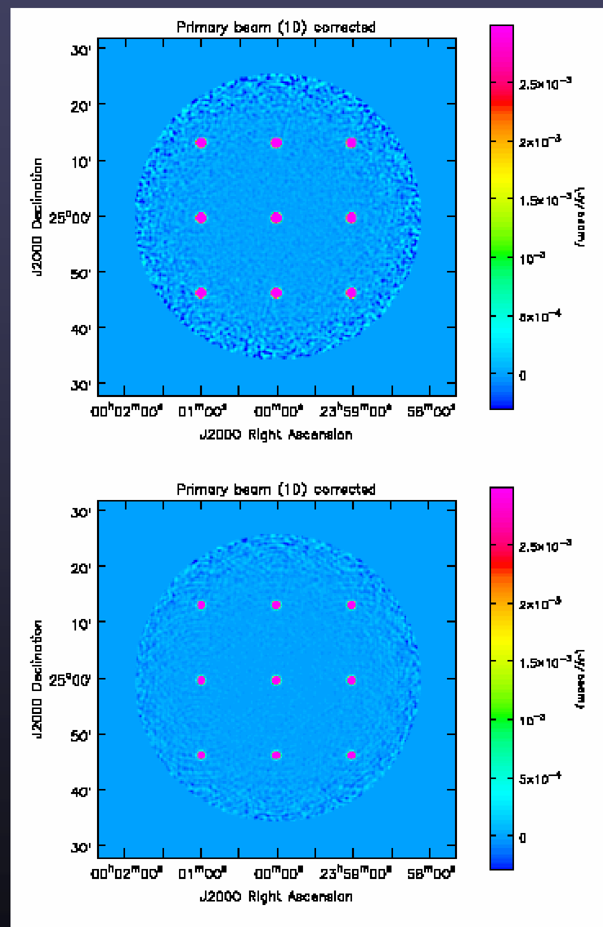
Example: wide field polarimetry (Cornwell 2003)

- Simulated array of point sources

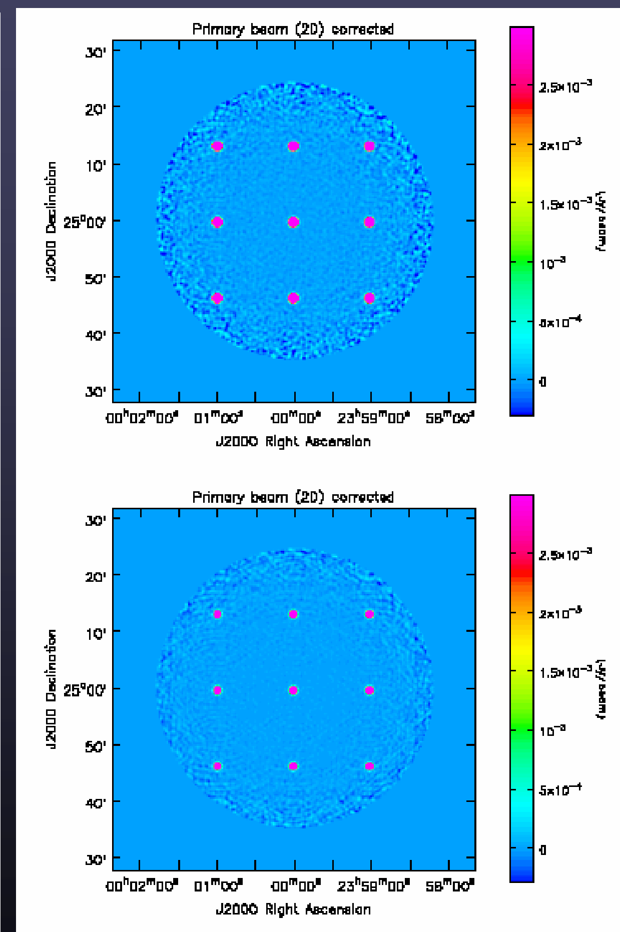
No beam correction



1D beam + squint

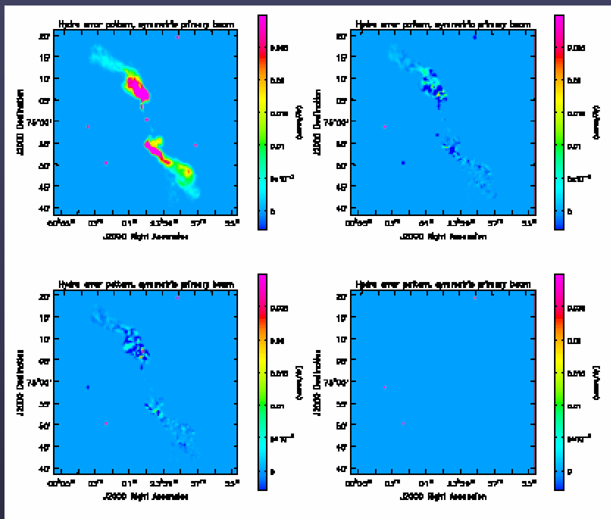


Full 2D beam



Example: wide field polarimetry continued…

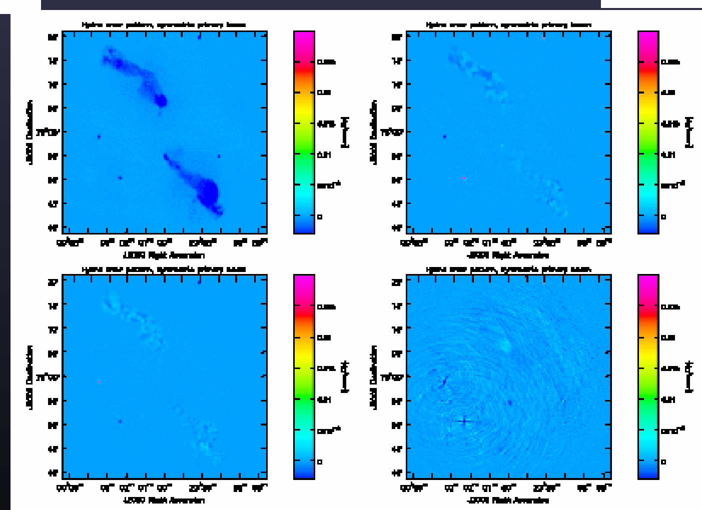
- Simulated Hydra A image

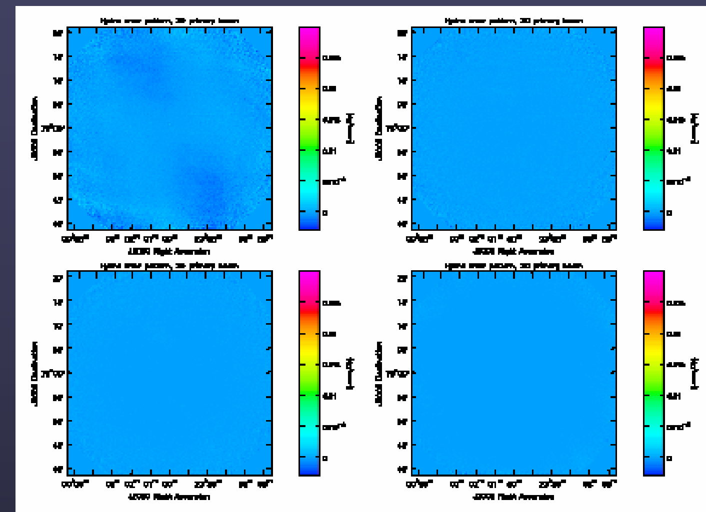


Model

Panels: I Q
U V

Errors 1D sym.beam





Errors full beam

Summary – Observing & Calibration

- Follow normal calibration procedure (previous lecture)
- Need bright calibrator for leakage D calibration
  - best calibrator has strong known polarization
  - unpolarized sources also useful
- Parallactic angle coverage useful
  - necessary for unknown calibrator polarization
- Need to determine unknown *p-q* phase
  - CP feeds need EVPA calibrator for R-L phase
  - if system stable, can transfer from other observations
- Special Issues
  - observing CP difficult with CP feeds
  - wide-field polarization imaging (needed for EVLA & ALMA)

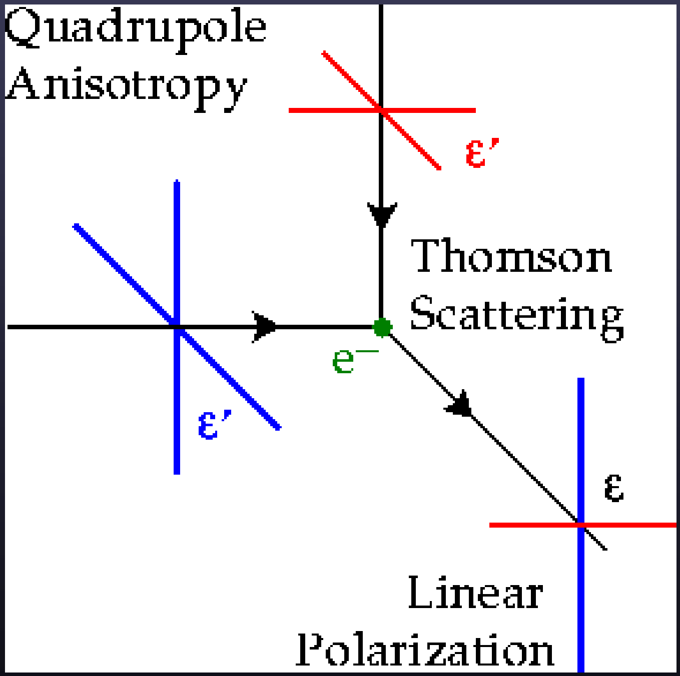
Polarization data analysis

- Making polarization images
  - follow general rules for imaging & deconvolution
  - image & deconvolve in I, Q, U, V (e.g. CLEAN, MEM)
  - note: Q, U, V will be positive and negative
  - in absence of CP, V image can be used as check
  - joint deconvolution (e.g. aips++, wide-field)
- Polarization vector plots
  - use “electric vector position angle” (EVPA) calibrator to set angle (e.g. R-L phase difference)
  - Φ = ½ tan-1 U/Q for E vectors (B vectors $\perp$ E)
  - plot E vectors with length given by p
- Faraday rotation: determine $\Delta\Phi$ vs. λ^2

Polarization Astrophysics

Astrophysical mechanisms for polarization

- Magnetic fields
  - synchrotron radiation ➔ LP (small amount of CP)
  - Zeeman effect ➔ CP
  - Faraday rotation (of background polarization)
  - dust grains in magnetic field
  - maser emission
- Electron scattering
  - incident radiation with quadrupole
  - e.g. Cosmic Microwave Background
- and more…

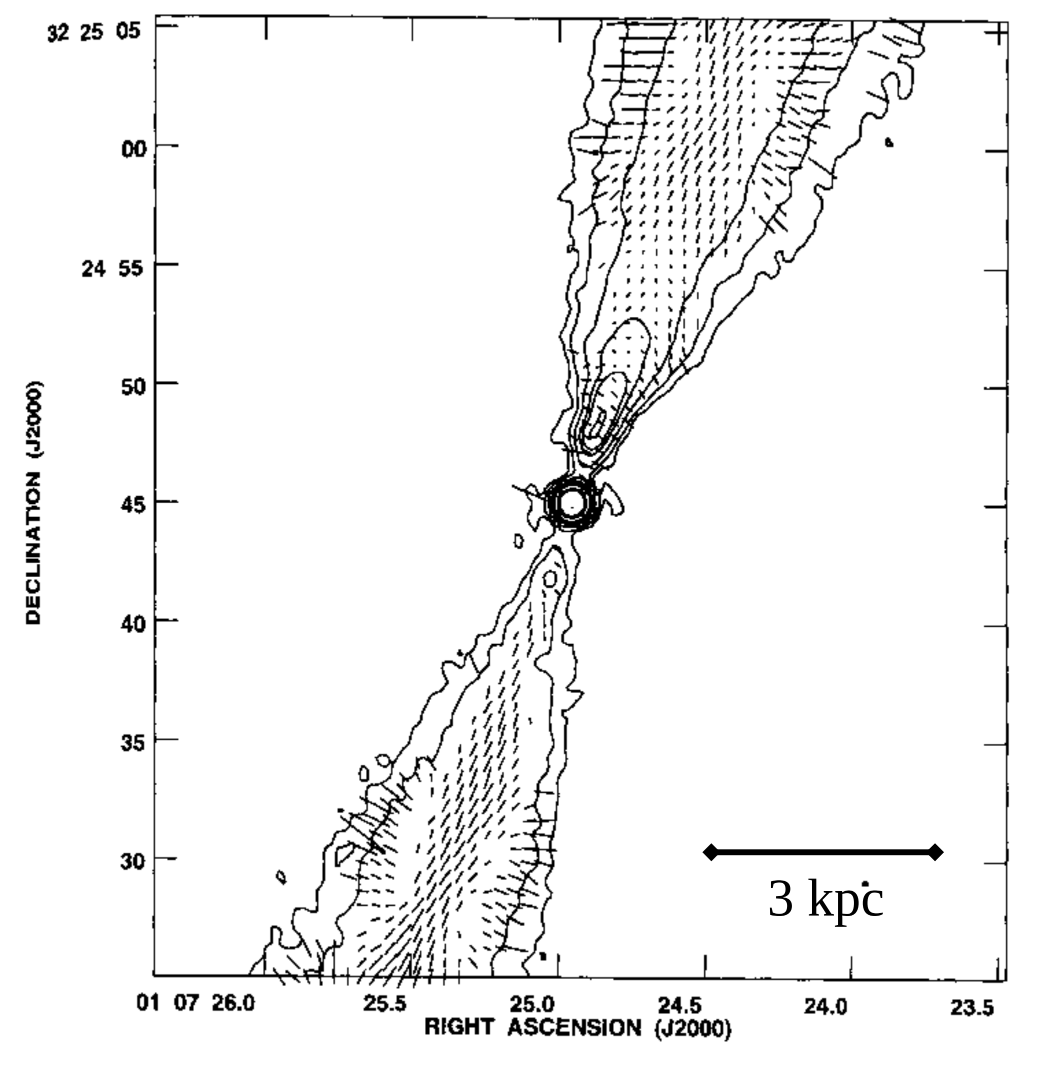


Astrophysical sources with polarization

- Magnetic objects
  - active galactic nuclei (AGN) (accretion disks, MHD jets, lobes)
  - protostars (disks, jets, masers)
  - clusters of galaxies IGM
  - galaxy ISM
  - compact objects (pulsars, magnetars)
  - planetary magnetospheres
  - the Sun and other (active) stars
  - the early Universe (primordial magnetic fields???)
- Other objects
  - Cosmic Microwave Background (thermal)
- Polarization levels
  - usually low (<1% to 5-10% typically)

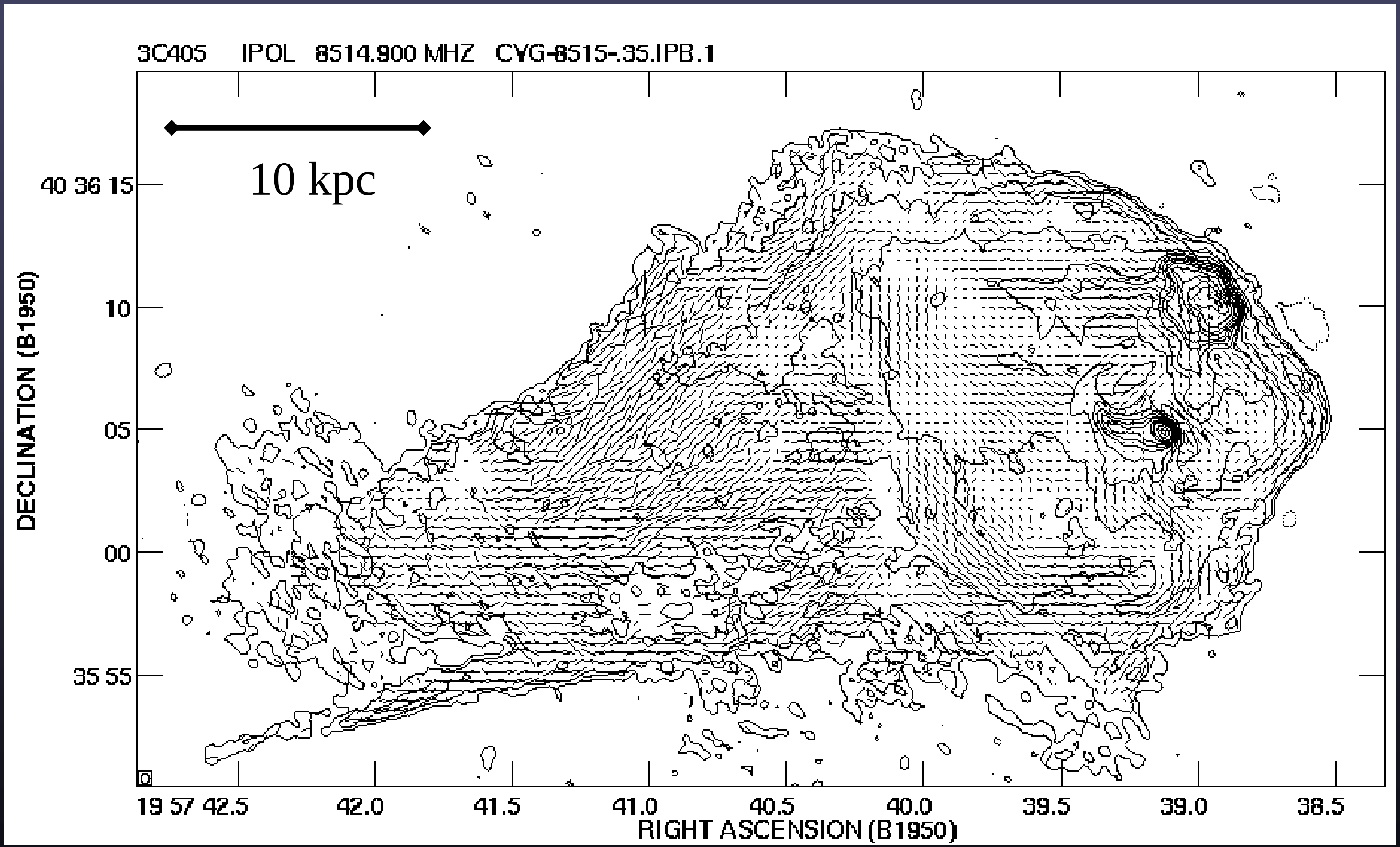
Example: 3C31

- VLA @ 8.4 GHz
- E-vectors
- Laing (1996)



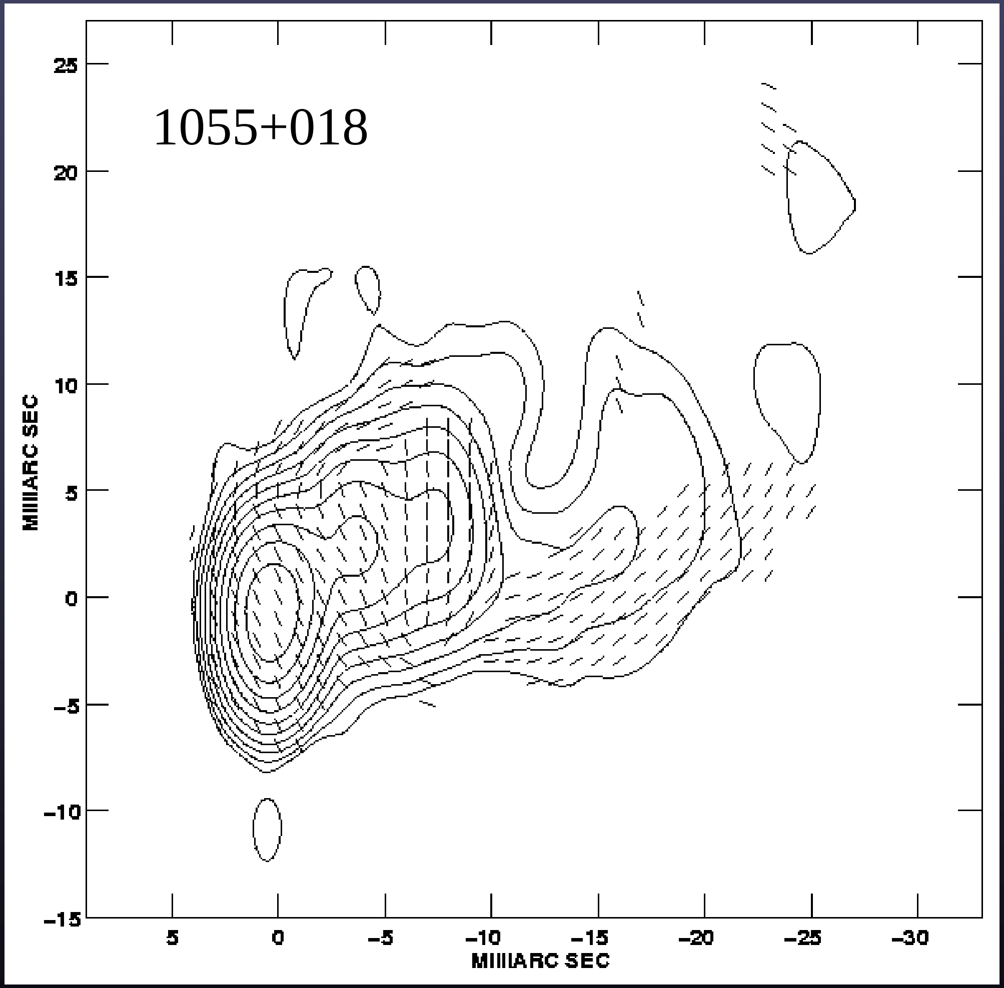
Example: Cygnus A

- VLA @ 8.5 GHz   B-vectors   Perley & Carilli (1996)



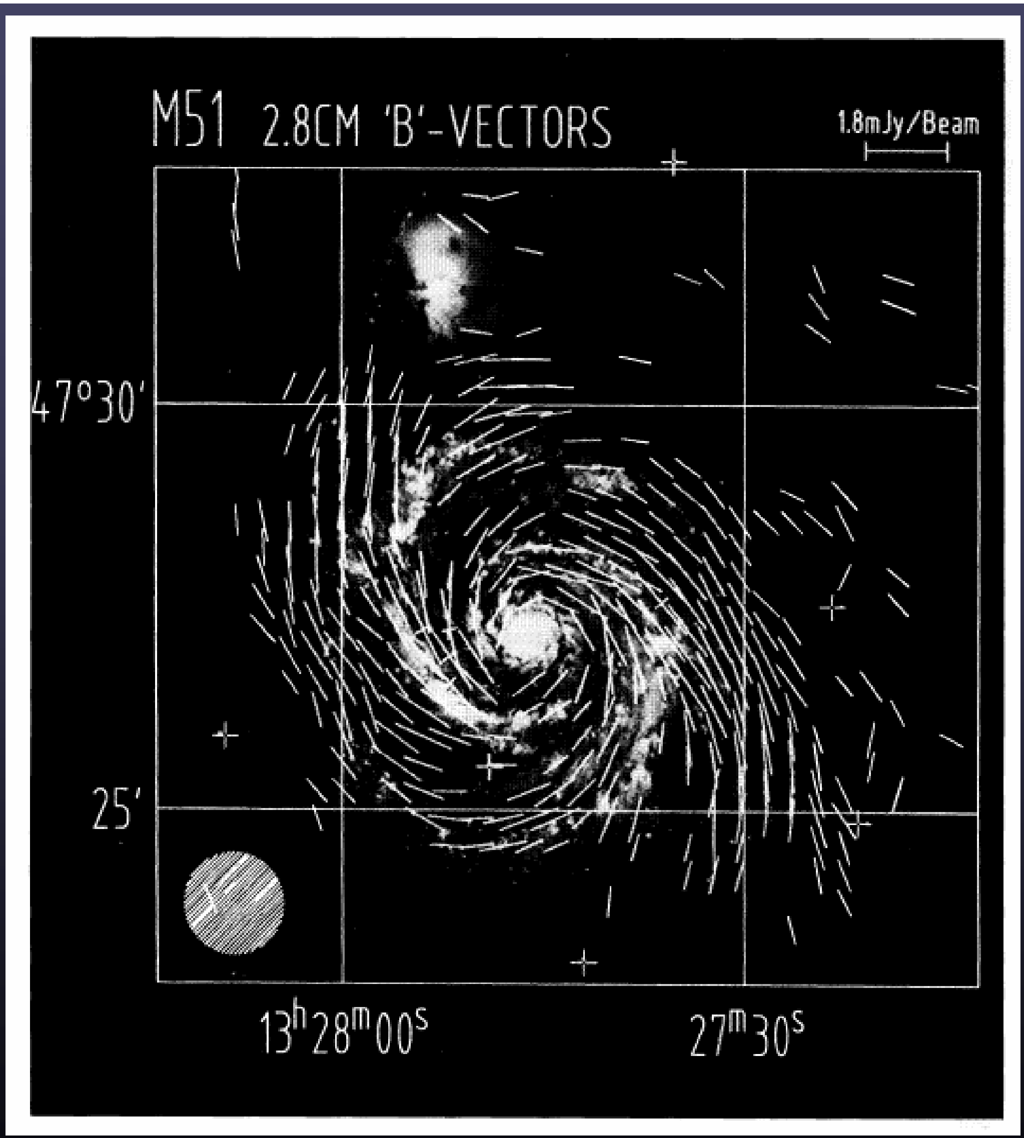
Example: Blazar Jets

- VLBA @ 5 GHz Attridge et al. (1999)



Example: the ISM of M51

Neininger (1992)



Example: Zeeman effect

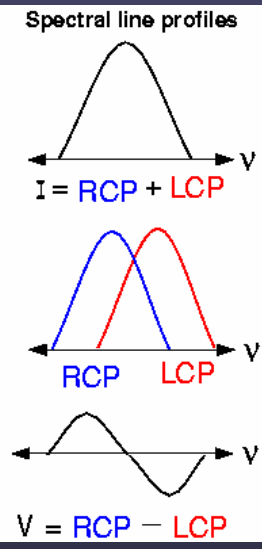
Zeeman Effect

Atoms and molecules with a net magnetic moment will have their energy levels split in the presence of a magnetic field.

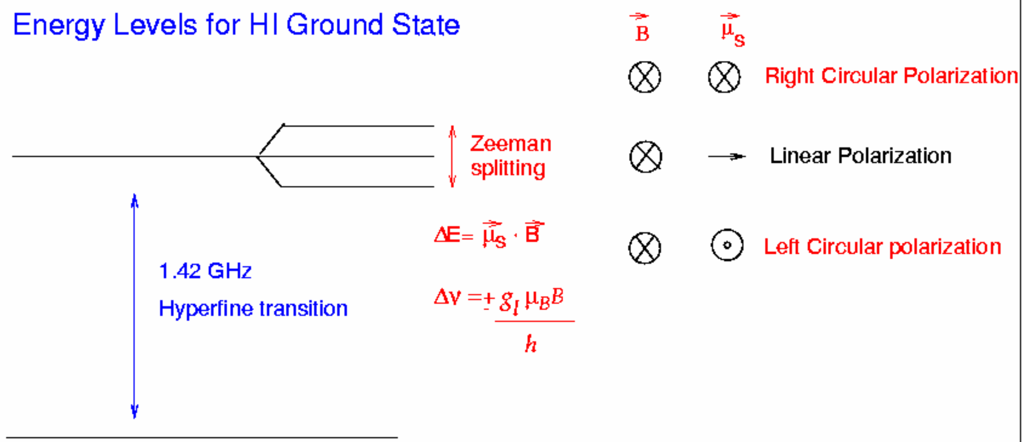
⇒ HI, OH, CN, H_2O

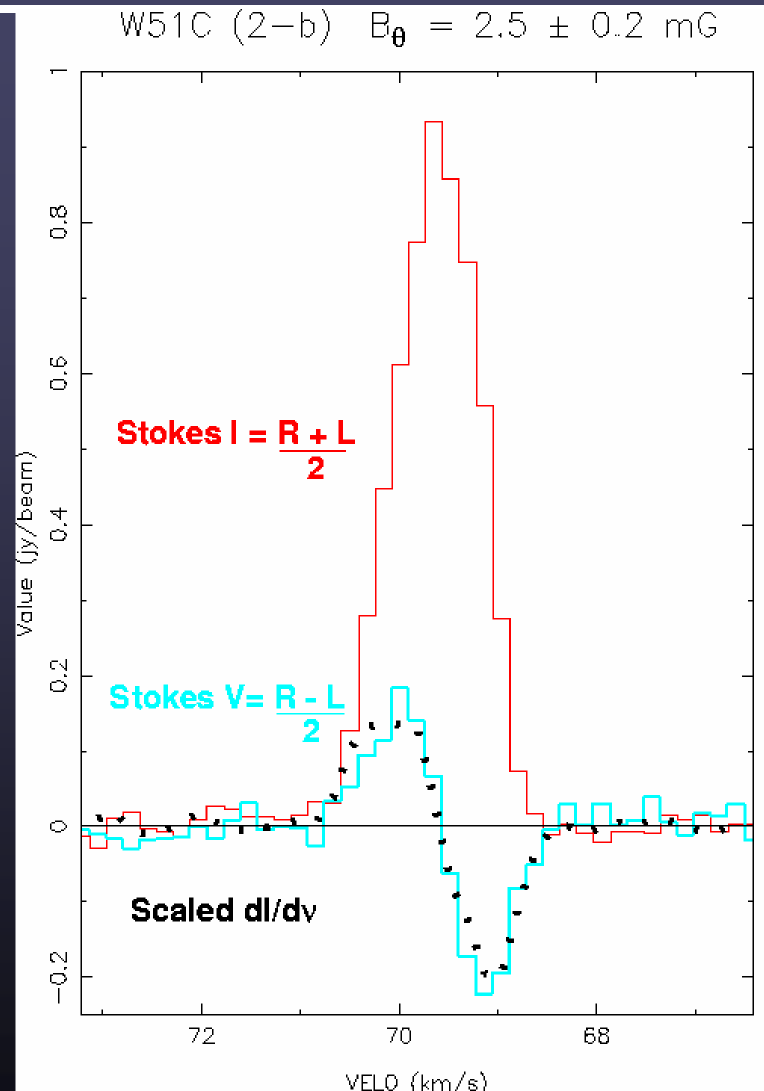
⇒ Detected by observing the frequency shift between right and left circularly polarized emission

⇒ V = RCP − LCP ∝ B_{los}

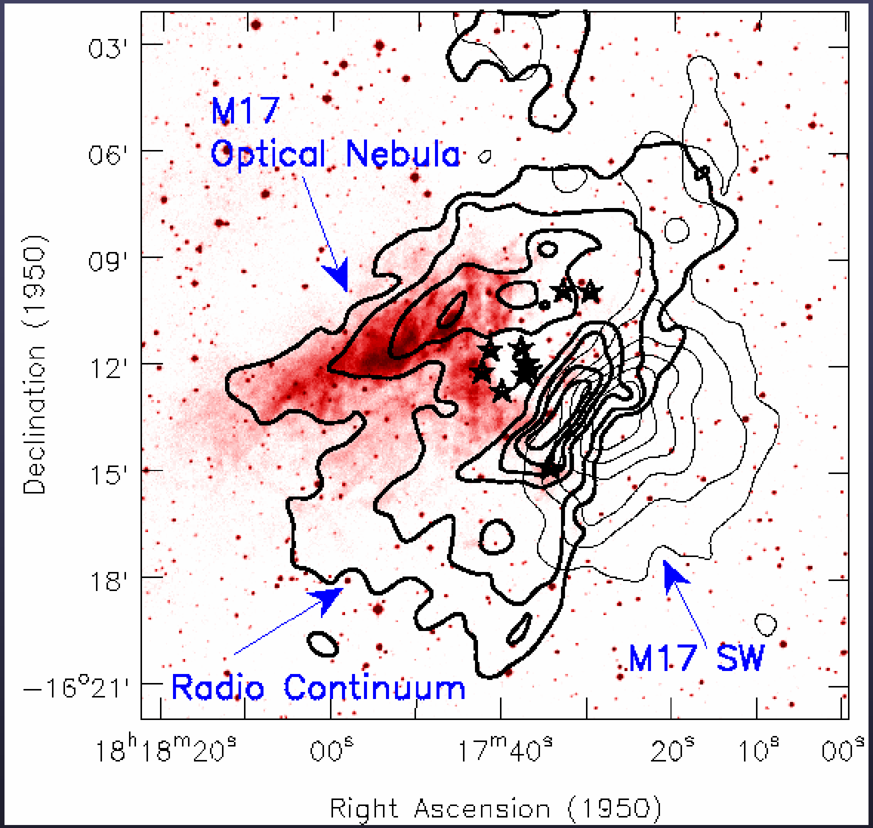


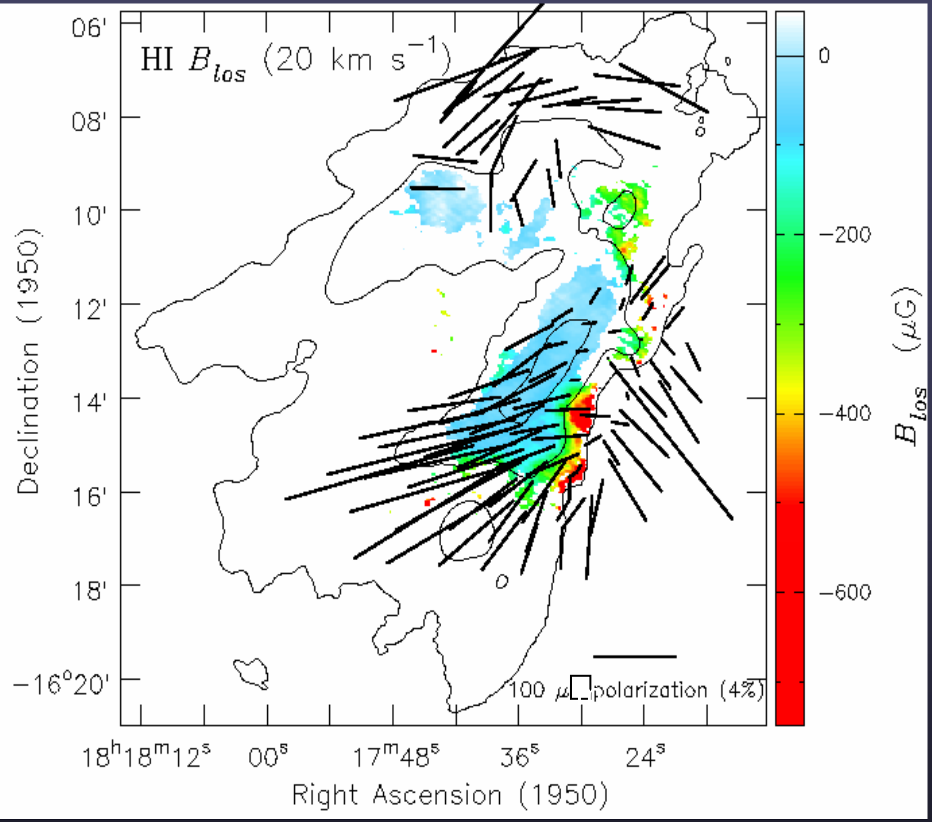
Energy Levels for HI Ground State





Example: Zeeman in M17



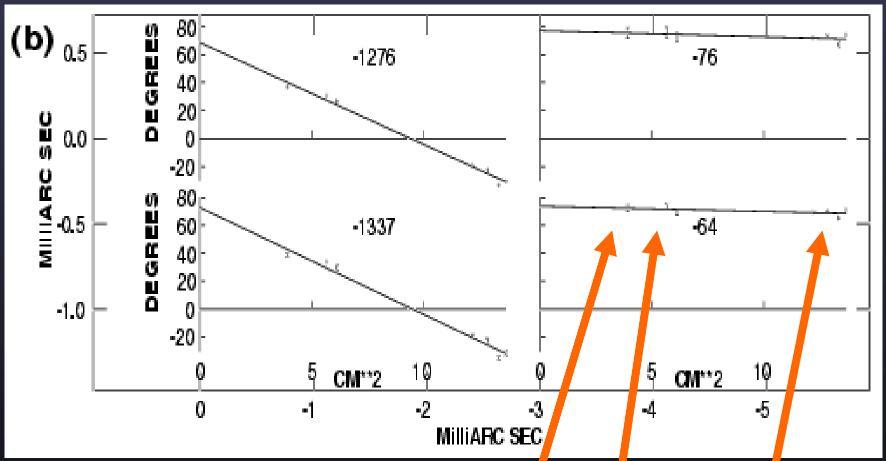


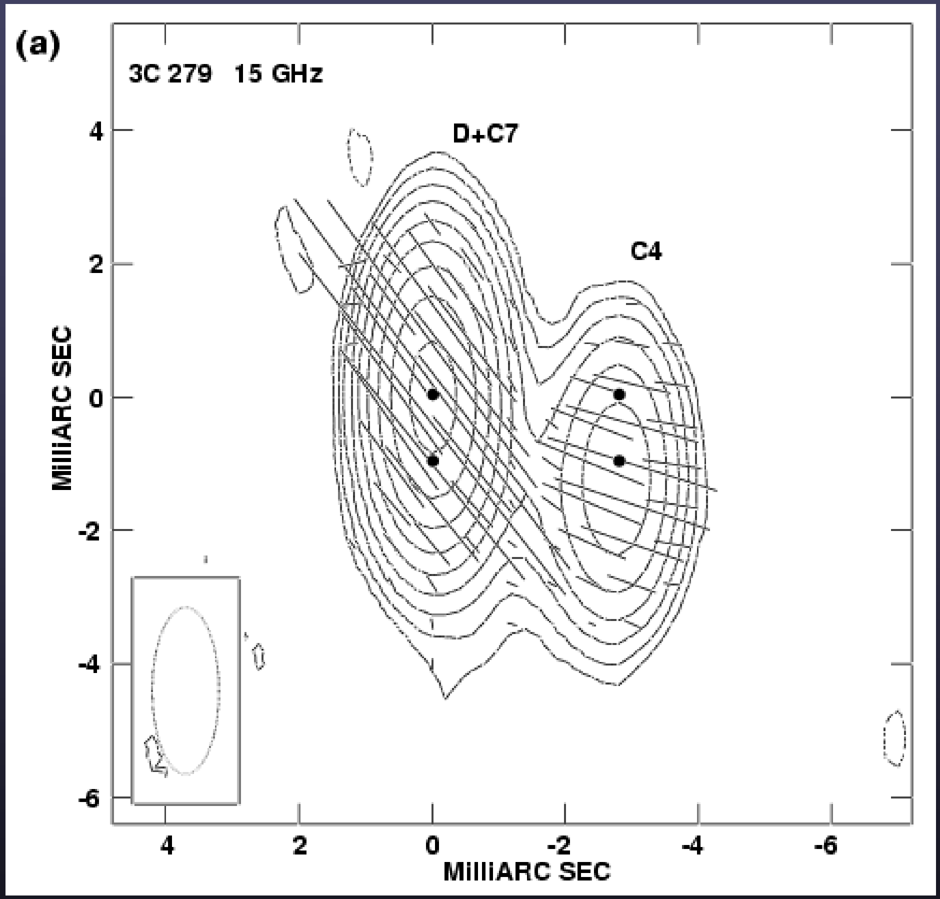
Color: optical from the *Digitized Sky Survey*
Thick contours: radio continuum from *Brogan & Troland (2001)*
Thin contours: ^{13}CO from *Wilson et al. (1999)*

Zeeman B_{los}: colors (*Brogan & Troland 2001*)
Polarization B_{perp}: lines (*Dotson 1996*)

Example: Faraday Rotation

- VLBA
- Taylor et al. 1998
- intrinsic vs. galactic

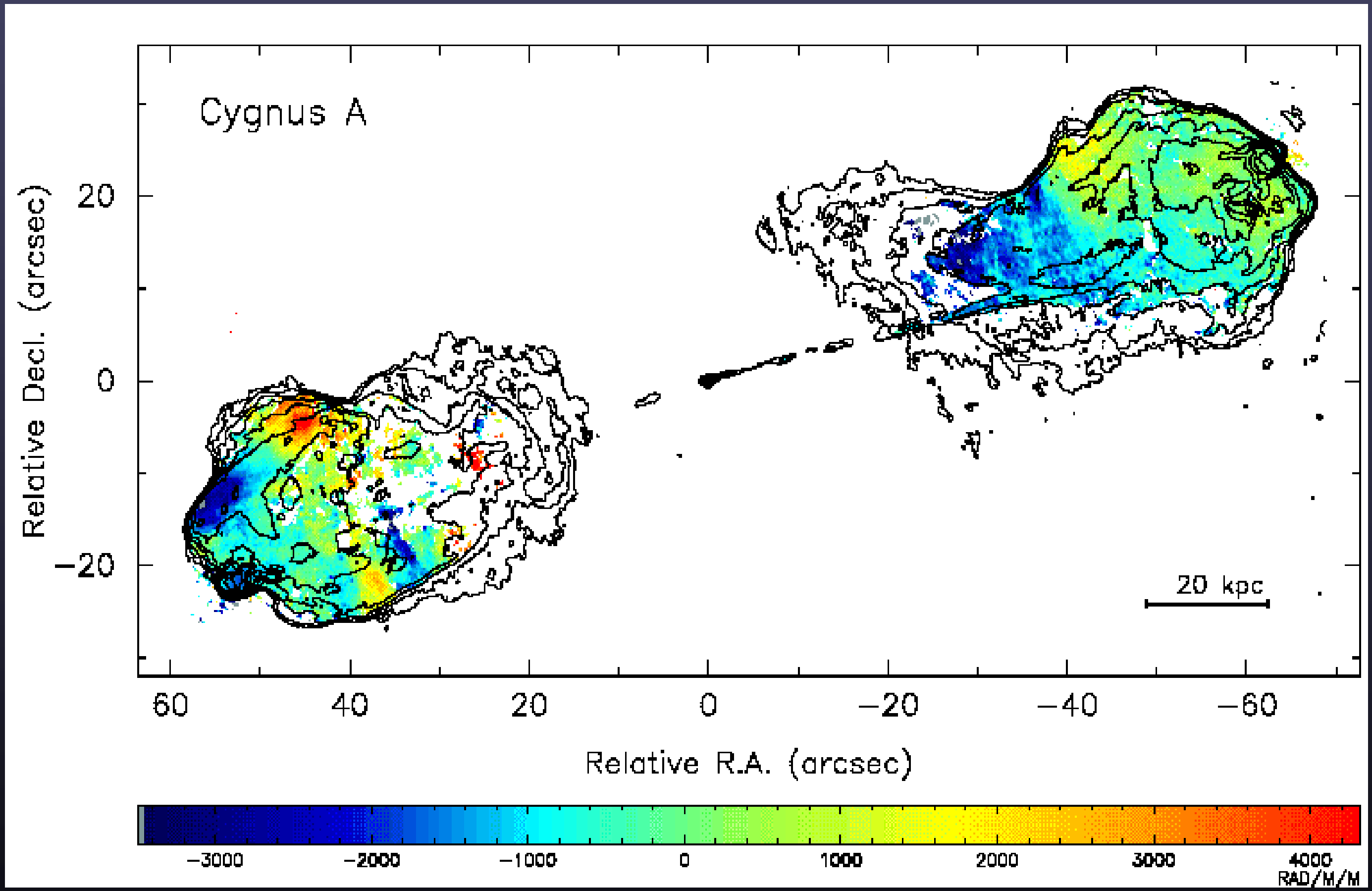




15 12 8 GHz

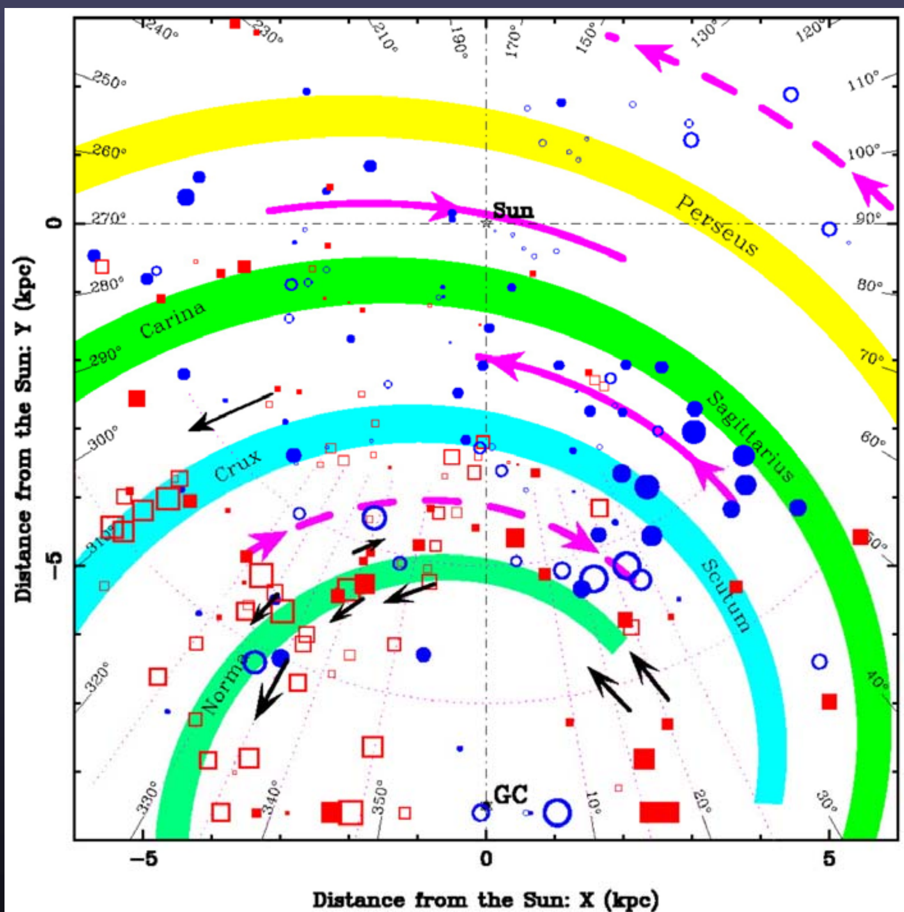
Example: more Faraday rotation

– See review of “Cluster Magnetic Fields” by Carilli & Taylor 2002 (ARAA)



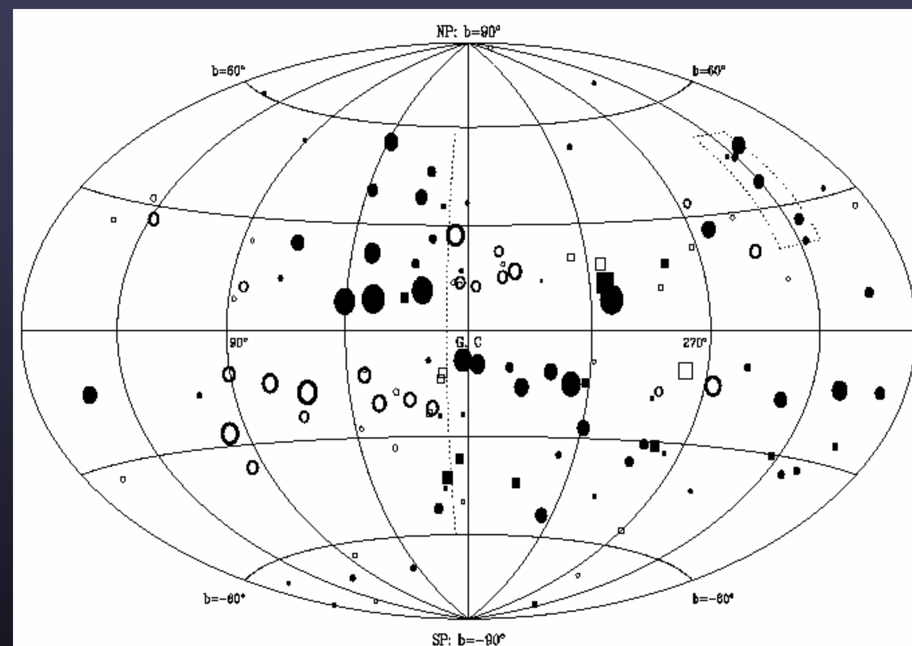
Example: Galactic Faraday Rotation

- Mapping galactic magnetic fields with FR



Han, Manchester, & Qiao (1999)

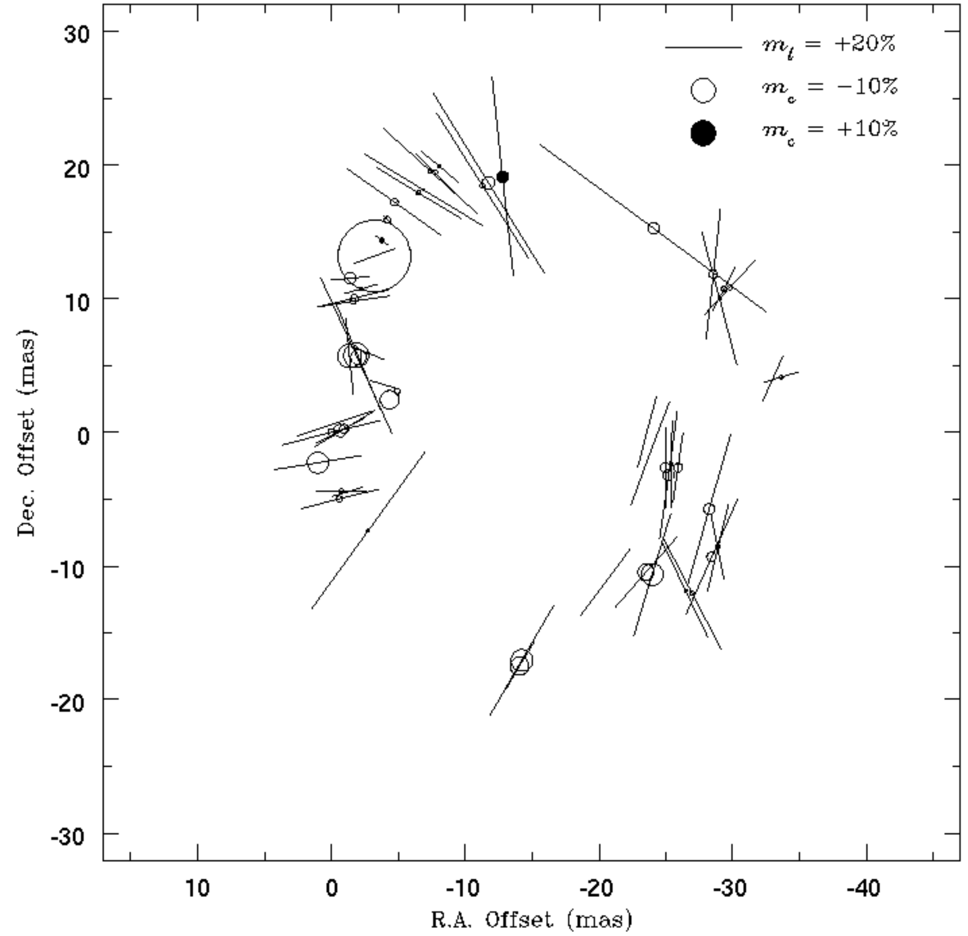
Han et al. (2002)



Filled: positive RM Open: negative RM

Example: Stellar SiO Masers

- R Aqr
- VLBA @ 43 GHz
- Boboltz et al. 1998



(b) Epoch 2 (29 Dec. 1995, $\phi = 0.78$)