Cosmology:
Polarization of the Cosmic Microwave Background

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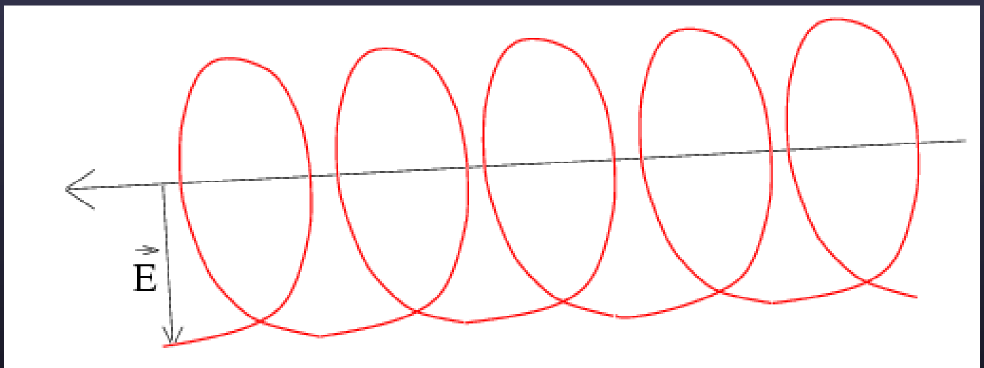
Polarization basics

Physics of polarization

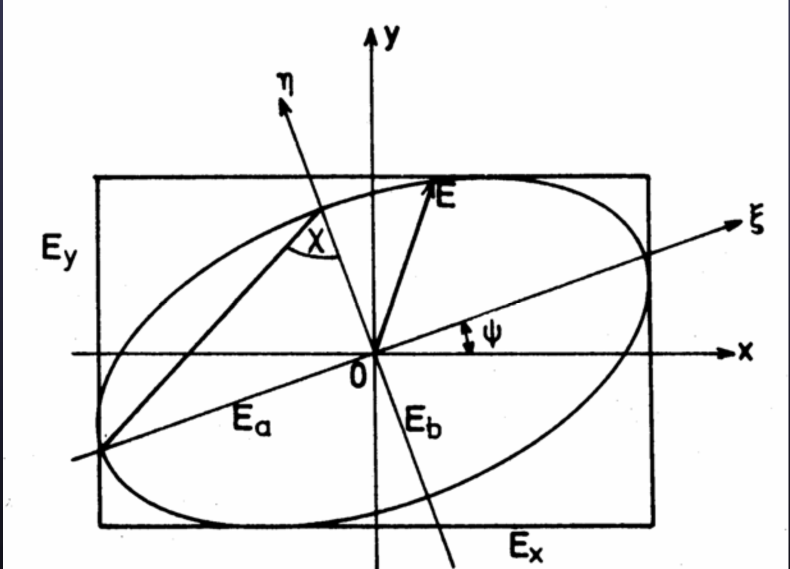
- Maxwell's Equations + Wave Equation
  - E•B=0 (perpendicular) ; $E_Z = B_Z = 0$ (transverse)
- Electric Vector – 2 orthogonal independent waves:
  - $E_x = E_1 \cos(k z - \omega t + \delta_1)$ $\qquad k = 2\pi / \lambda$
  - $E_y = E_2 \cos(k z - \omega t + \delta_2)$ $\qquad \omega = 2\pi \nu$
  - describes helical path on surface of a cylinder…



  - parameters E_1, E_2, $\delta = \delta_1 - \delta_2$ define ellipse
    - electric vector traces ellipse viewed along ***k*** direction

The Polarization Ellipse

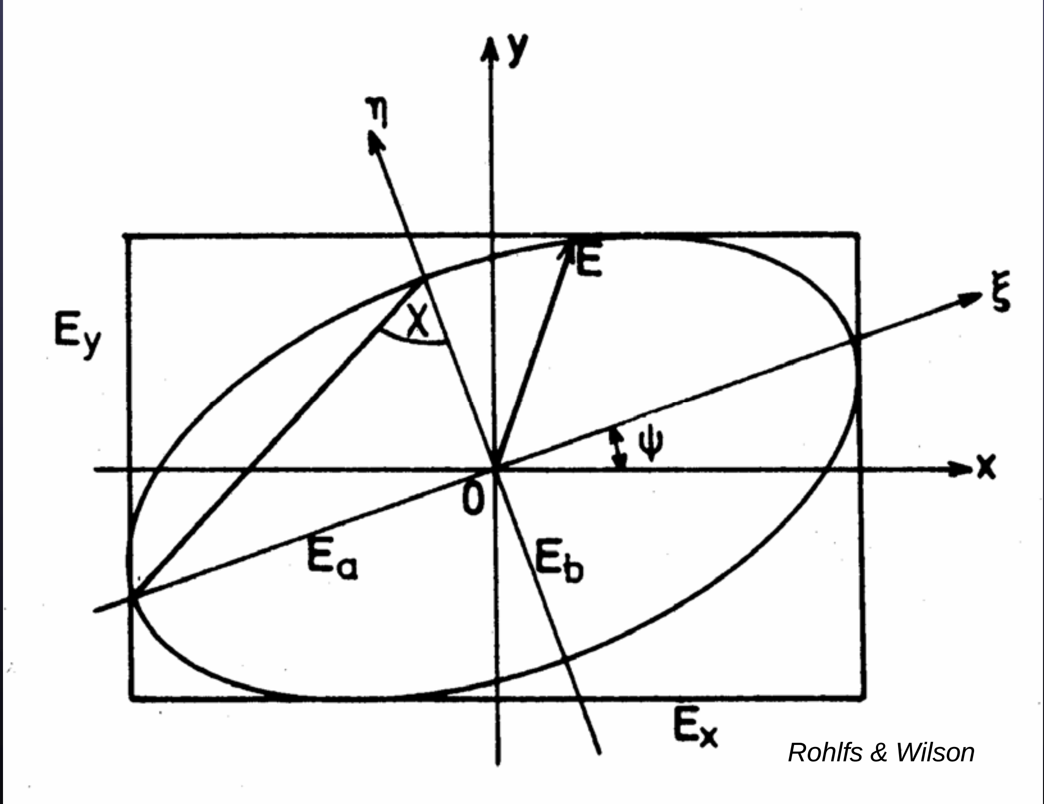
- Axes of ellipse E_a, E_b
  - $S_0 = E_1^2 + E_2^2 = E_a^2 + E_b^2$ Poynting flux
  - δ phase difference $\tau = k\,z - \omega\,t$
  - $E_\xi = E_a \cos(\tau + \delta) = E_x \cos\Psi + E_y \sin\Psi$
  - $E_\eta = E_b \sin(\tau + \delta) = -E_x \sin\Psi + E_y \cos\Psi$



Rohlfs & Wilson

The polarization ellipse continued…

- Ellipticity and Orientation
  - $E_1 / E_2 = \tan \alpha$ $\tan 2\psi = - \tan 2\alpha \cos \delta$
  - $E_a / E_b = \tan \chi$ $\sin 2\chi = \sin 2\alpha \sin \delta$
  - handedness ($\sin \delta > 0$ or $\tan \chi > 0$ ➔ *right-handed*)



Rohlfs & Wilson

Polarization ellipse – special cases

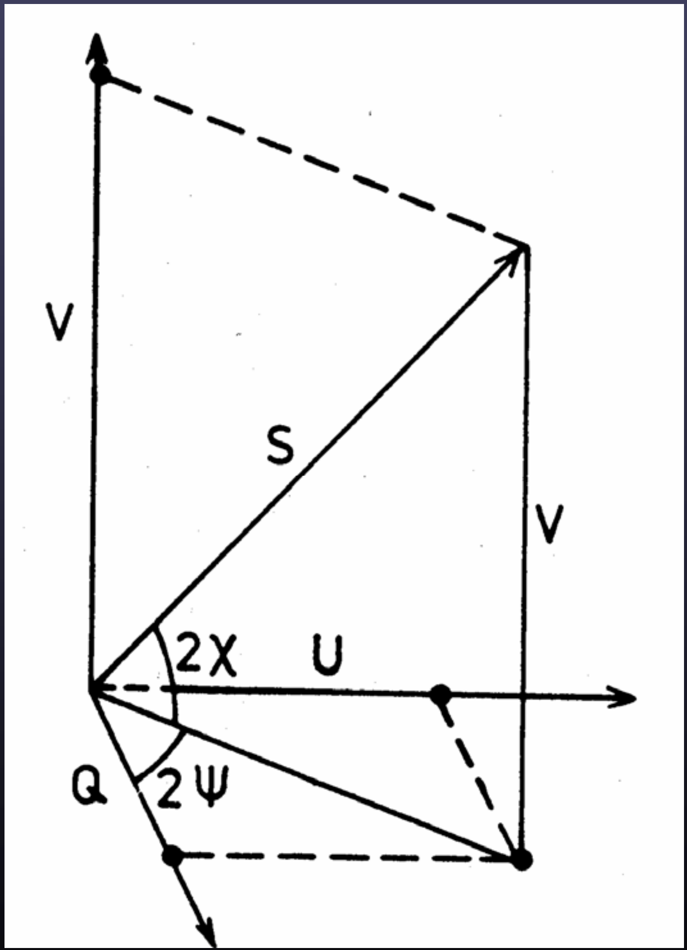
- Linear polarization
  - $\delta = \delta_1 - \delta_2 = m\,\pi \qquad m = 0, \pm 1, \pm 2, \ldots$
  - ellipse becomes straight line
  - electric vector position angle $\Psi = \alpha$
- Circular polarization
  - $\delta = \frac{1}{2}\,(\,1 + m\,)\,\pi \qquad m = 0, 1, \pm 2, \ldots$
  - equation of circle $E_x^2 + E_y^2 = E^2$
  - orthogonal linear components:
    - $E_x = E \cos \tau$
    - $E_y = \pm E \cos(\tau - \pi/2)$
    - note quarter-wave delay between E_x and E_y !

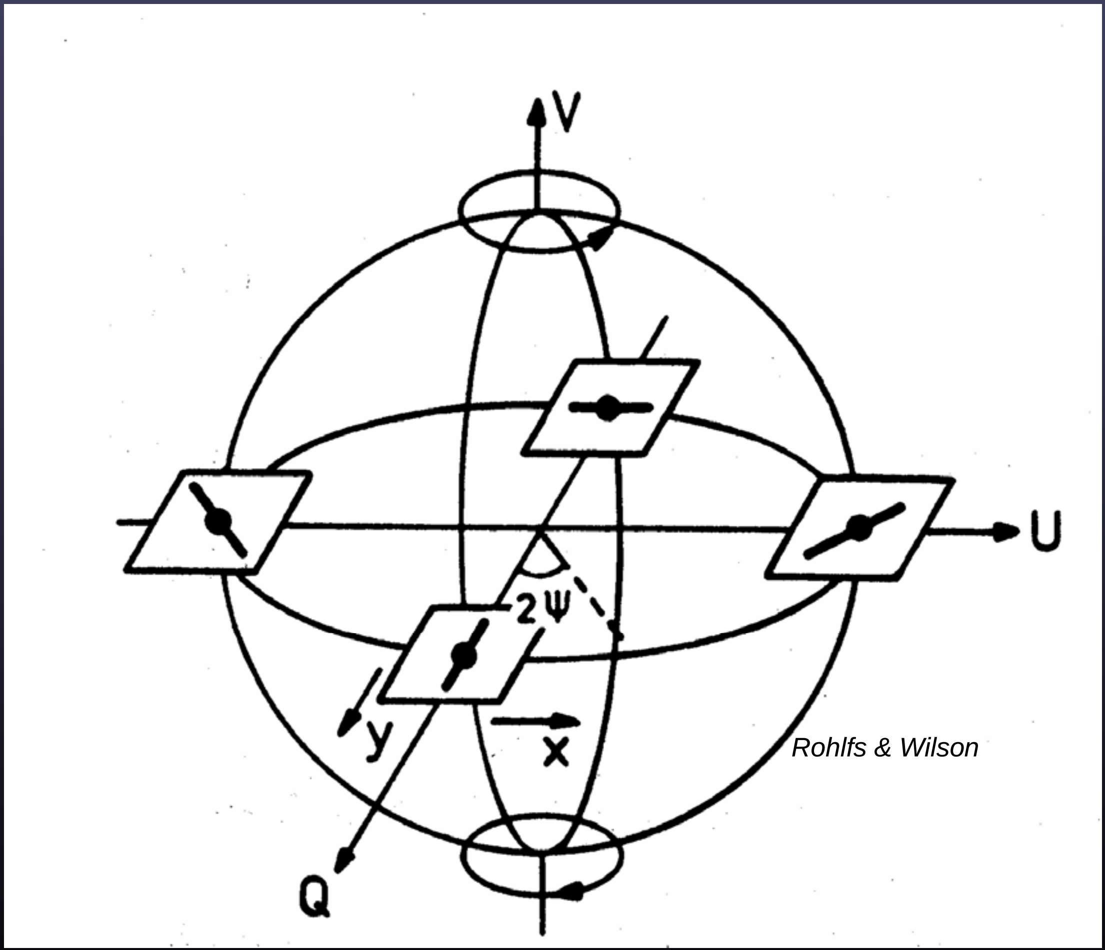
Linear and Circular representations

- Orthogonal Linear representation (E_x, E_y) :
  - $E_\xi = E_a \cos(\tau + \delta) = E_x \cos\psi + E_y \sin\psi$
  - $E_\eta = E_b \sin(\tau + \delta) = -E_x \sin\psi + E_y \cos\psi$
- Orthogonal Circular representation (E_r, E_l) :
  - $E_\xi = E_a \cos(\tau + \delta) = (E_r + E_l)\cos(\tau + \delta)$
  - $E_\eta = E_b \sin(\tau + \delta) = (E_r - E_l)\cos(\tau + \delta - \pi/2)$
  - $E_r = ½ (E_a + E_b)$
  - $E_l = ½ (E_a - E_b)$
- Free to choose the orthogonal basis for polarization:
  - a monochromatic wave can be expressed as the superposition of two orthogonal linearly polarized waves
  - equally well described as superposition of two orthogonal circularly polarized waves!

The Poincare Sphere

- Treat 2ψ and 2χ as longitude and latitude on sphere of radius S_0





Stokes parameters

- Spherical coordinates: radius I, axes Q, U, V
  - $S_0 = I = E_a^2 + E_b^2$
  - $S_1 = Q = S_0 \cos 2\chi \cos 2\psi$
  - $S_2 = U = S_0 \cos 2\chi \sin 2\psi$
  - $S_3 = V = S_0 \sin 2\chi$
- Only 3 independent parameters:
  - $S_0^2 = S_1^2 + S_2^2 + S_3^2$
  - $I^2 = Q^2 + U^2 + V^2$
- Stokes parameters I,Q,U,V
  - form complete description of wave polarization
  - NOTE: above true for monochromatic wave!
  - for non-monochromatic sources: partial polarization
    - $I^2 > Q^2 + U^2 + V^2$

Stokes parameters and ellipse

- Spherical coordinates: radius I, axes Q, U, V
  - $S_0 = I = E_a^2 + E_b^2$
  - $S_1 = Q = S_0 \cos 2\chi \cos 2\psi$
  - $S_2 = U = S_0 \cos 2\chi \sin 2\psi$
  - $S_3 = V = S_0 \sin 2\chi$
- In terms of the polarization ellipse:
  - $S_0 = I = E_1^2 + E_2^2$
  - $S_1 = Q = E_1^2 - E_2^2$
  - $S_2 = U = 2 E_1 E_2 \cos \delta$
  - $S_3 = V = 2 E_1 E_2 \sin \delta$
  - Note: this is equivalent to what linearly polarized optics see!

Stokes parameters special cases

- Linear Polarization
  - $S_0 = I = E^2 = S$
  - $S_1 = Q = I \cos 2\psi$
  - $S_2 = U = I \sin 2\psi$
  - $S_3 = V = 0$

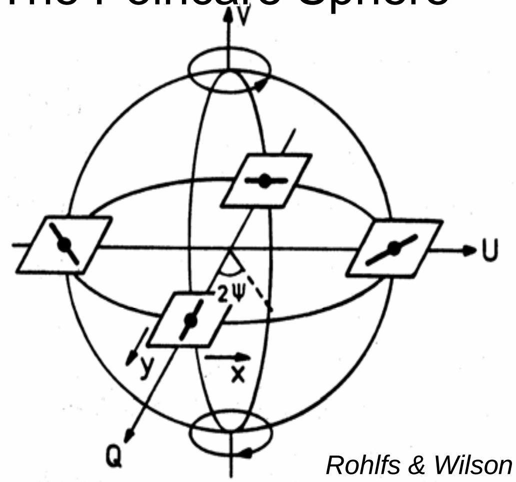
Note: cycle in 180°

- Circular Polarization
  - $S_0 = I = S$
  - $S_1 = Q = 0$
  - $S_2 = U = 0$
  - $S_3 = V = S$ (RCP) or $-S$ (LCP)

Stokes Parameters & coordinates

- Stokes parameters (parital polarization):
  - intensity I (Poynting flux) $I^2 = E_1{}^2 + E_2{}^2$
  - linear polarization Q,U $(m\ I)^2 = Q^2 + U^2$
  - circular polarization V $(v\ I)^2 = V^2$
- Coordinate system dependence:
  - I independent
  - V depends on choice of "handedness"
    - V > 0 for RCP
    - V = 0 for CMB (no "handedness" in Standard Cosmology!)
  - Q,U depend on choice of "North" (plus handedness)
    - Q "points" North, U 45 toward East
    - EVPA $\Phi = ½ \tan^{-1} (U/Q)$ (North through East)
  - Note: because of coordinate system dependence, Q and U not useful for CMB statistics in homogeneous and isotropic Universe!

The Poincare Sphere

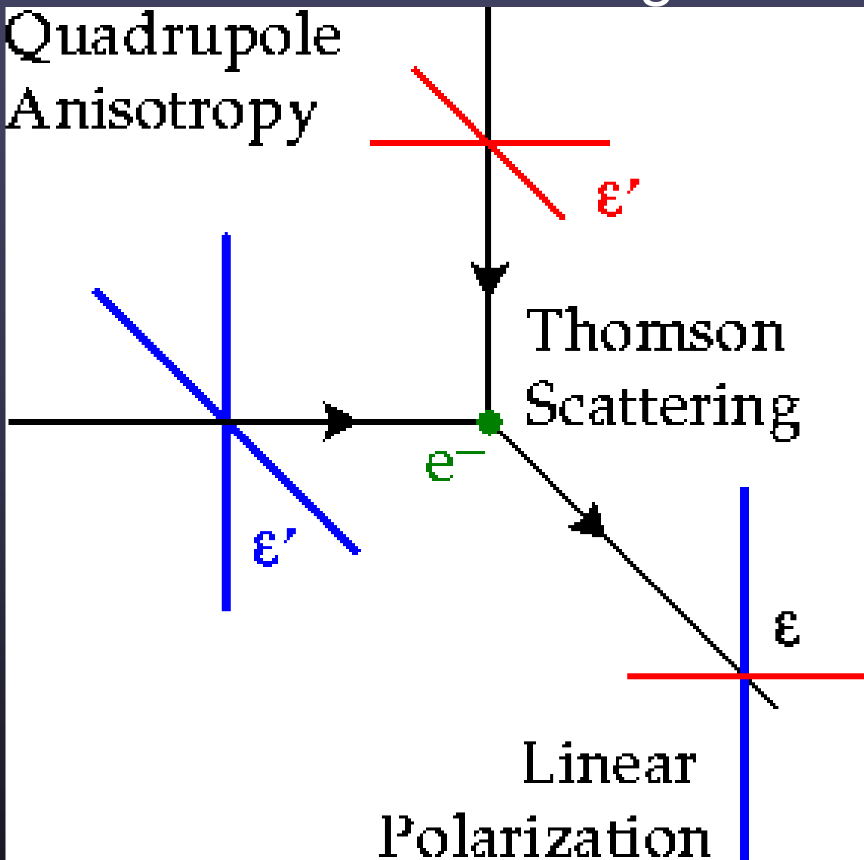


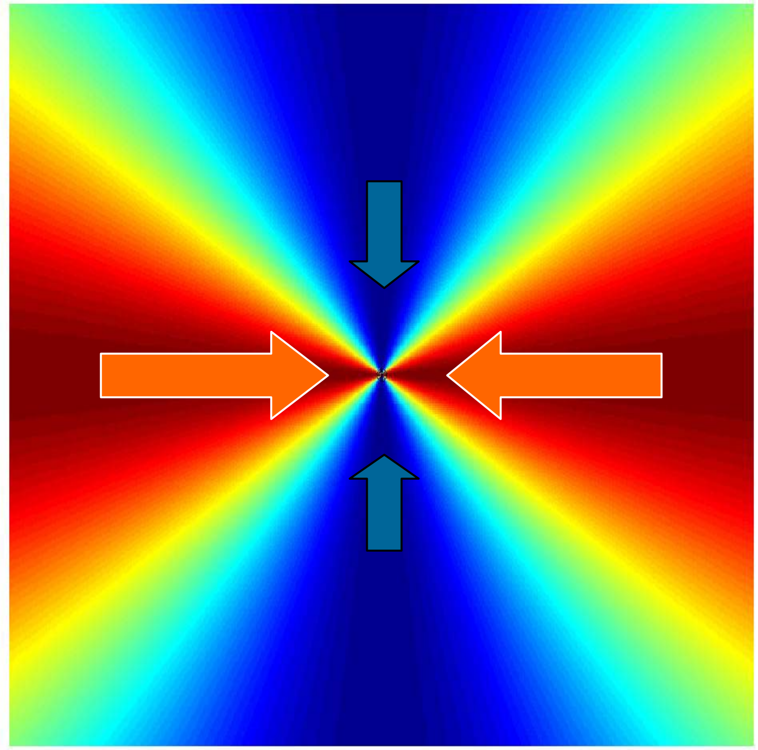
Rohlfs & Wilson

CMB Polarization

The CMB is Polarized

Thomson scattering:





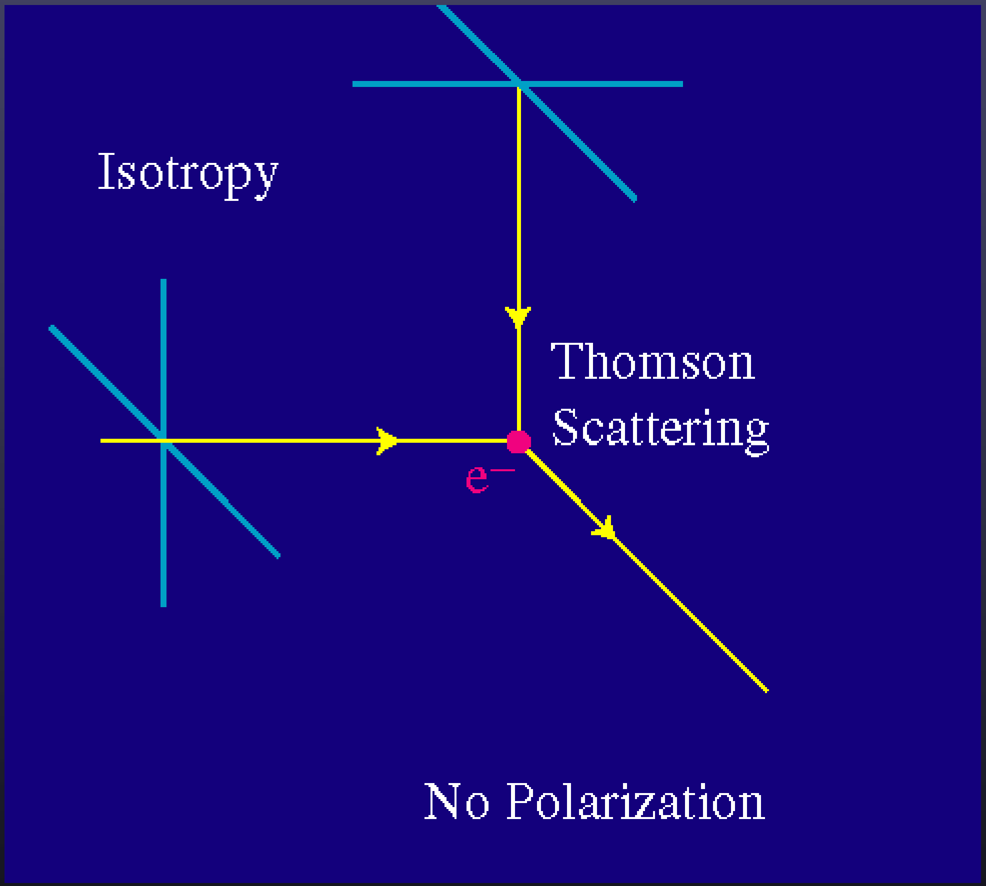
polarization transverse to line-of-sight transmitted on scattering!

Above figure courtesy Hu & White 1997

more radiation from horizontal (hot) than vertical (cold)

➔ outgoing polarization net vertical

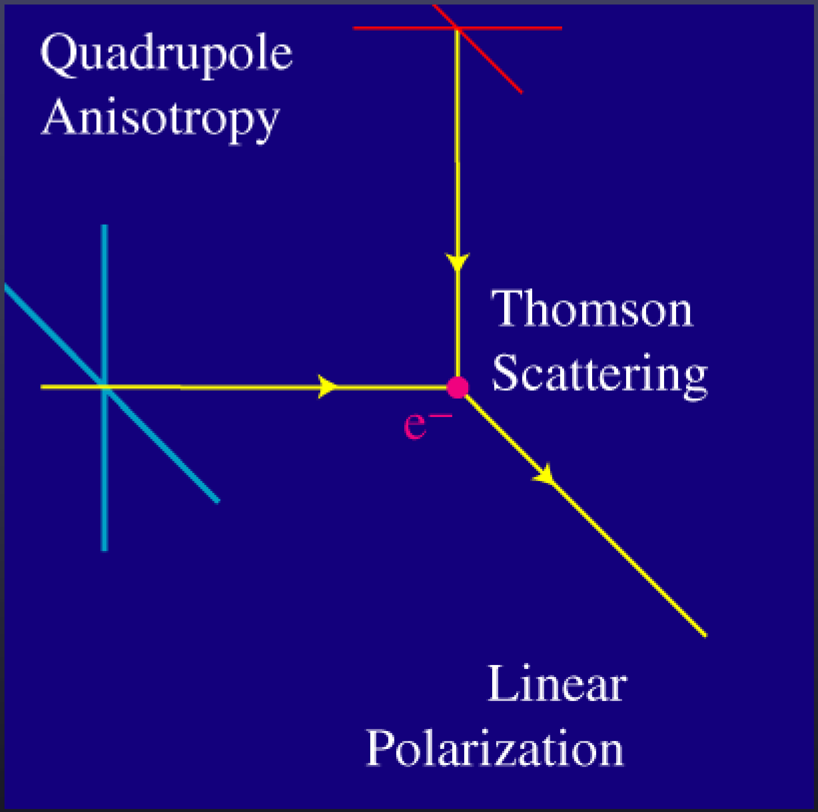
Isotropic (monopolar) Scattering



➔ NO net linear polarization

animations from Wayne Hu

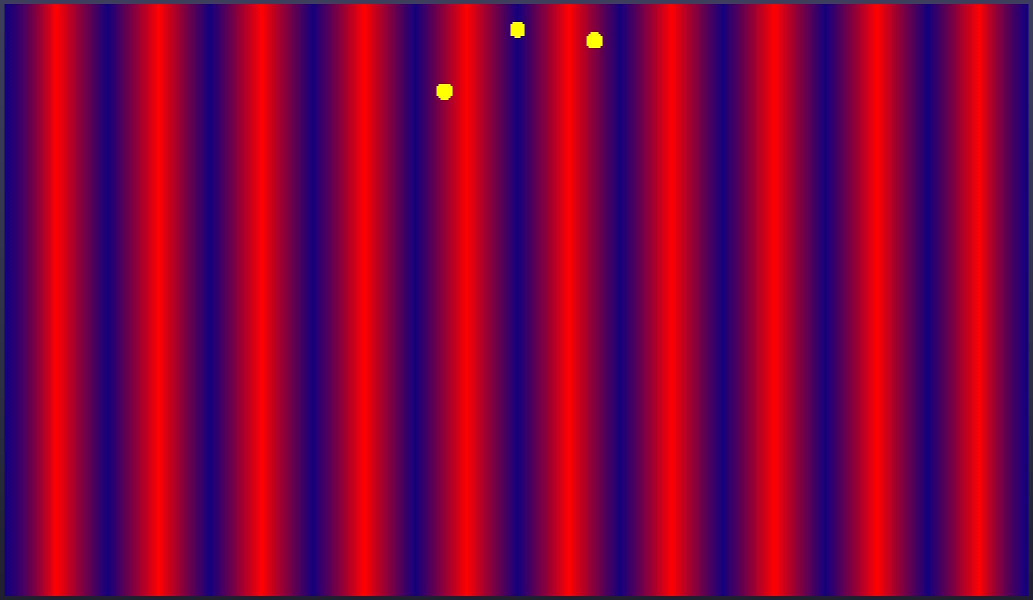
Quadrupolar Scattering



➔ net linear polarization

Animations from Wayne Hu

Quadrupole and plane wave

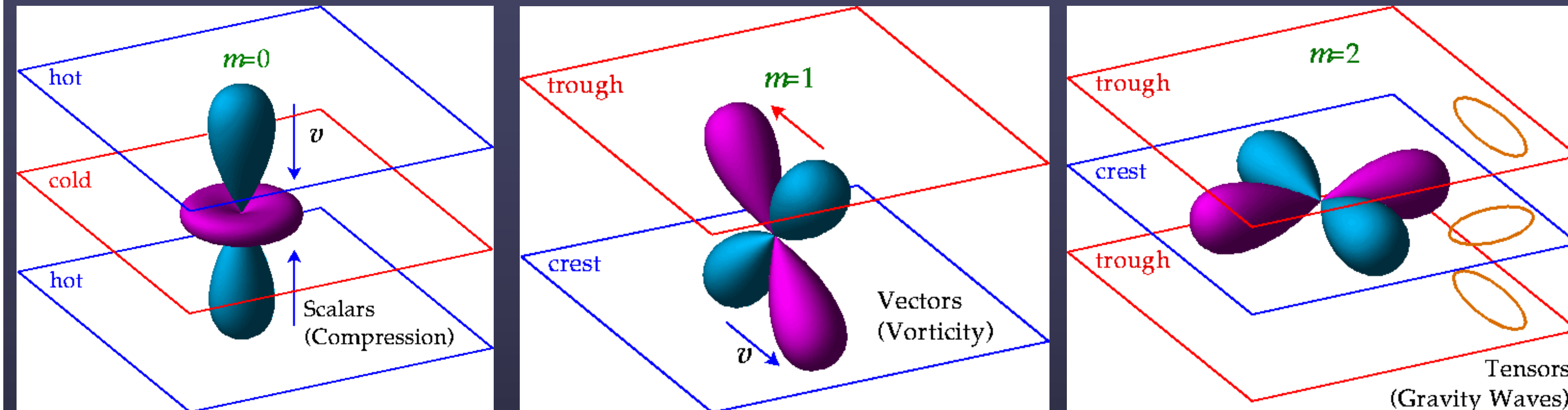


➔ <u>net linear polarization</u>

is oriented along cold axis of quadrupole

Animations from Wayne Hu

The local quadrupole at scattering



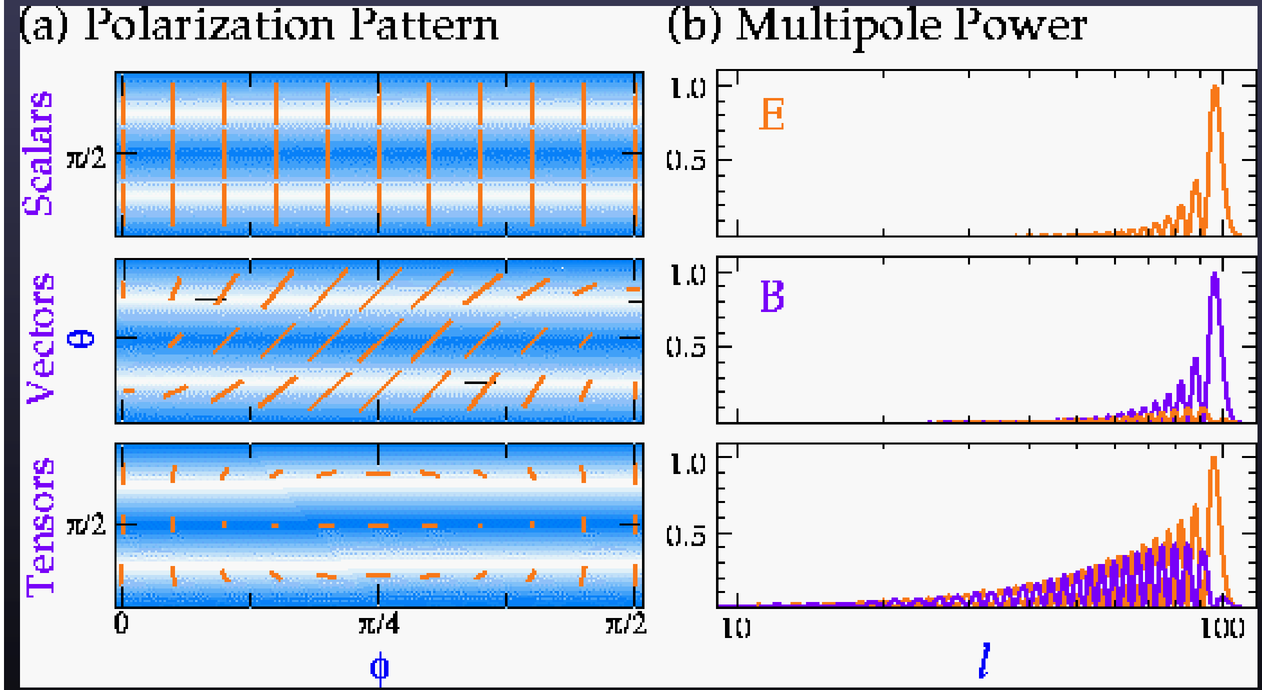
Scalar modes | Vector modes | Tensor modes

- Quadrupole (ℓ=2) has 8 components: m=0, m=±1, m=±2
- Density perturbations (from potential fluctuations H) produce scalar modes
- Vector modes indicate vorticity, and can be produced by defects (e.g. cosmic strings)
- Tensor modes are produced by gravity waves

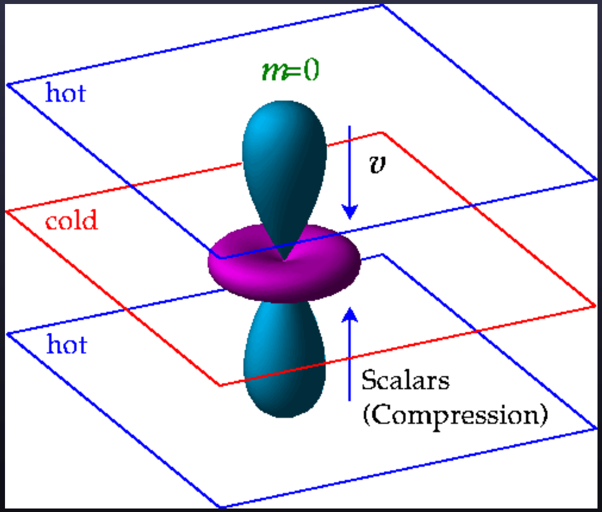
Courtesy Hu & White– *http://background.uchicago.edu*

E-modes from Scalars

- Linear polarization "vectors" for plane waves
  - E (even parity = aligned 0° or 90° to k-vector)
    - from scalar density fluctuations ➔ predominant in standard model!



Density waves produce "E-mode" polarization:

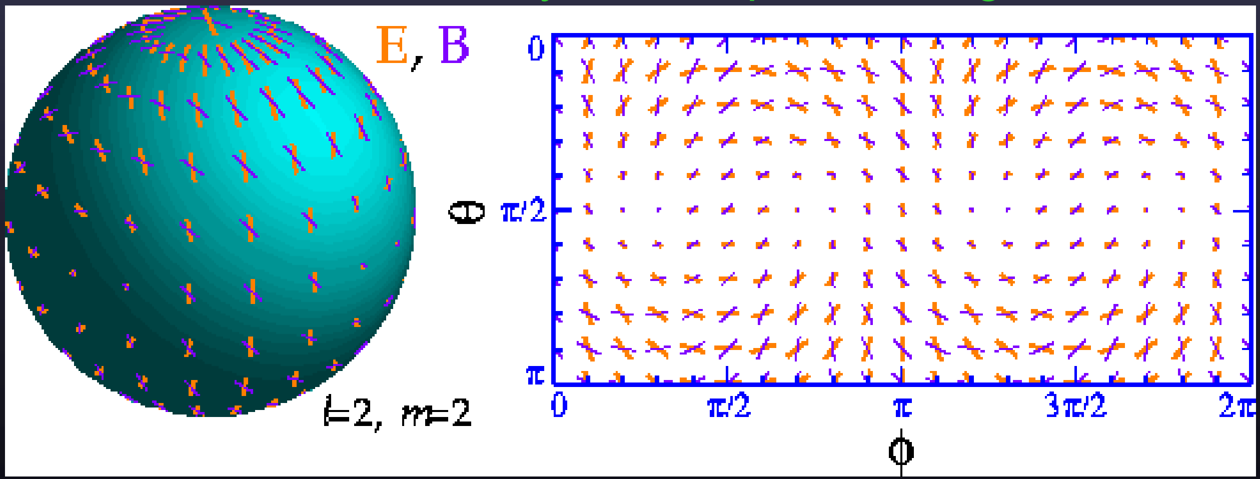


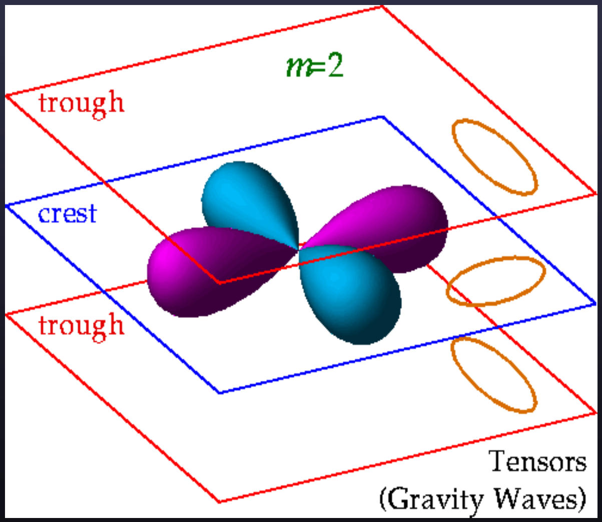
Figures courtesy Hu & White 1997

E & B modes from Tensors

- Linear polarization "vectors" for plane waves
  - E (even parity = aligned 0° or 90° to k-vector)
    - from scalar density fluctuations ➔ predominant in standard model!
  - B (odd parity = at ±45° to k-vector)
    - vector perturbations (vorticity or defects) produce only **B**-modes
    - tensors (gravity waves) produce both **E** & **B**
    - lensing makes **B** modes from **E** modes
    - also secondary anisotropies & foregrounds

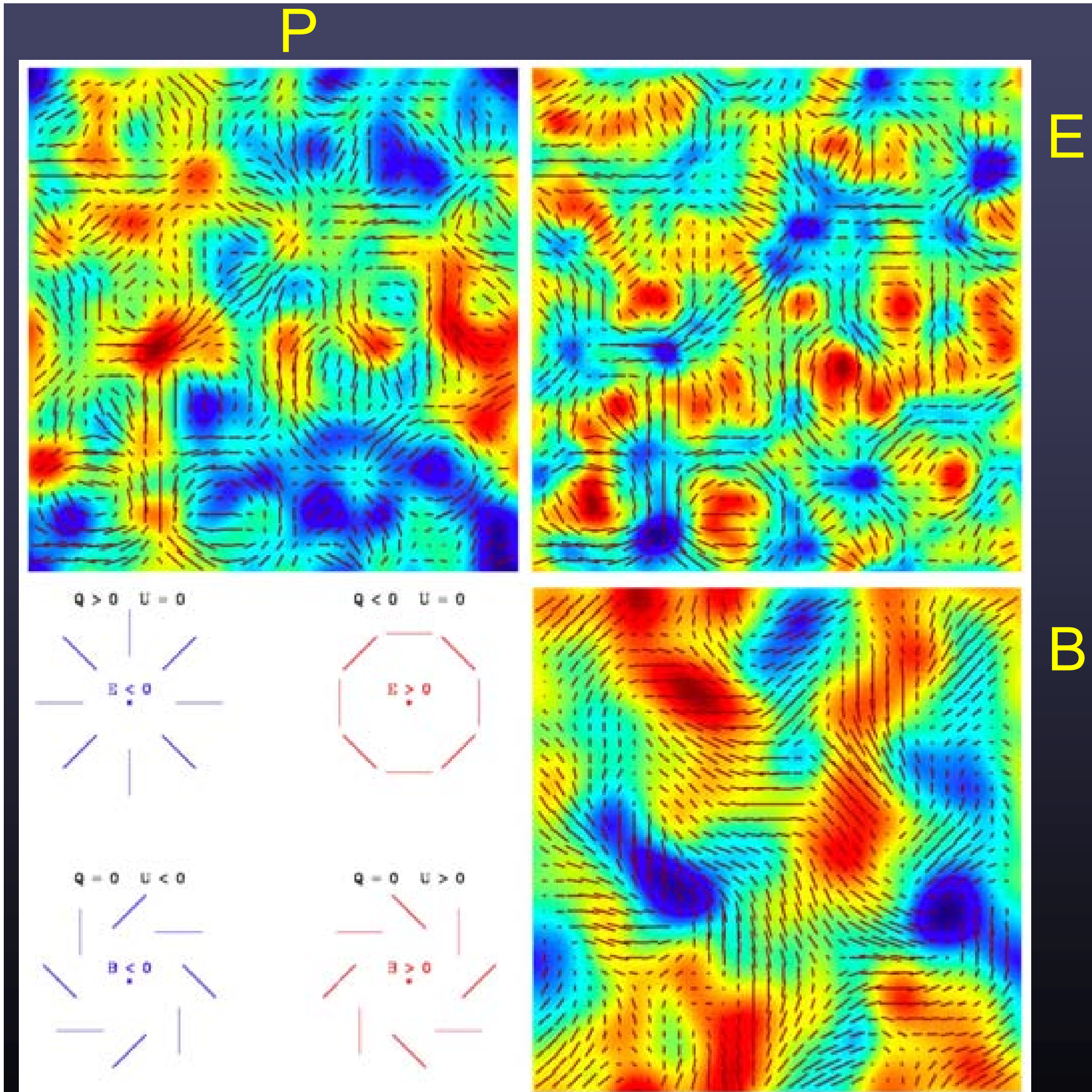
Gravity waves produce "B-mode" polarization:





Figures courtesy Hu & White 1997

E & B modes on the sky



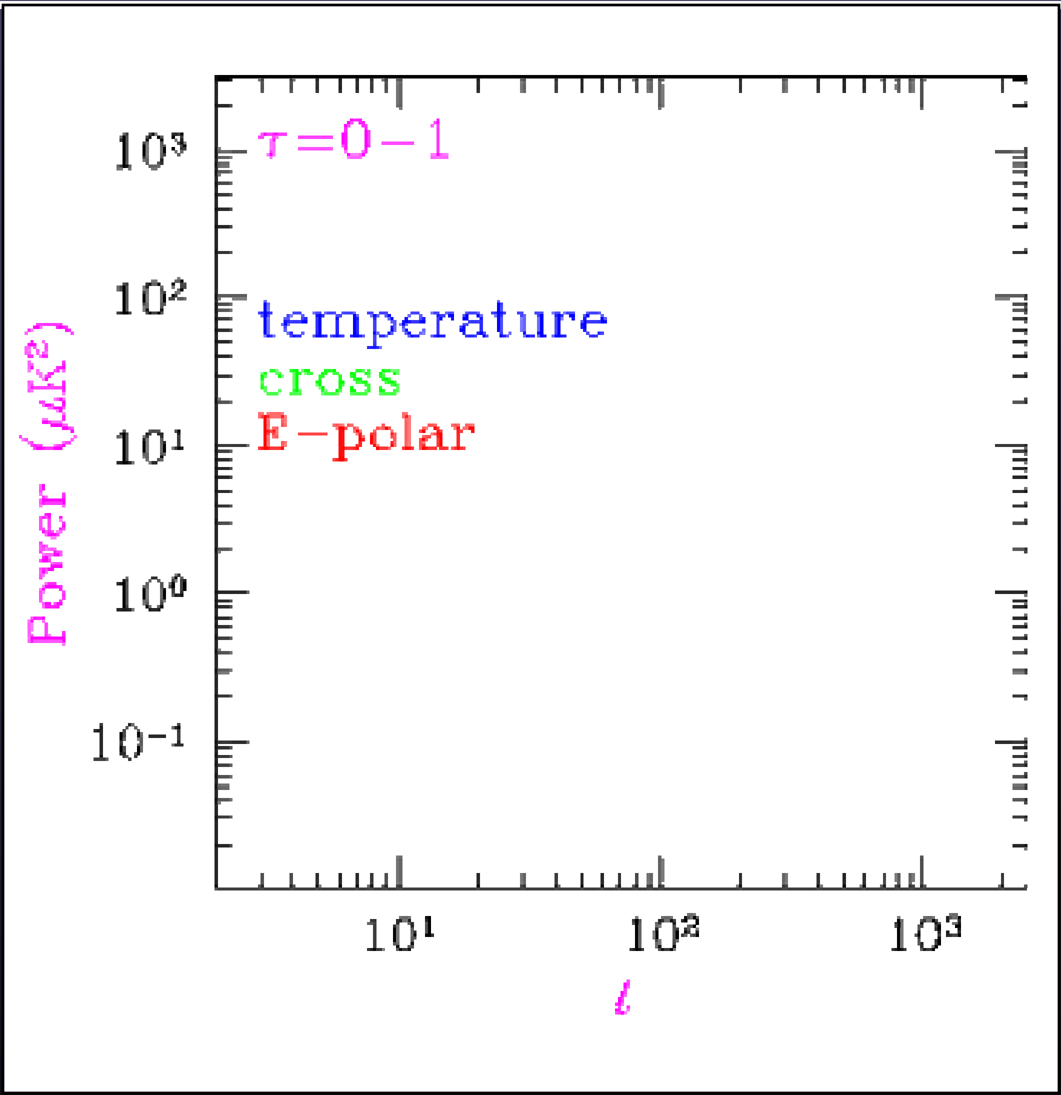
Plane waves add up or interfere on the sky to make “hot” and “cold” spots.

E-mode polarization vectors line up as radial “hedgehog” or tangential patterns.

B-mode polarization vectors “pinwheel” or curl around peaks/holes.

> Maxima in E/B are not P maxima – but measure non-local coherences in P
>
> E & B live in wave-space!

Reionization and Polarization



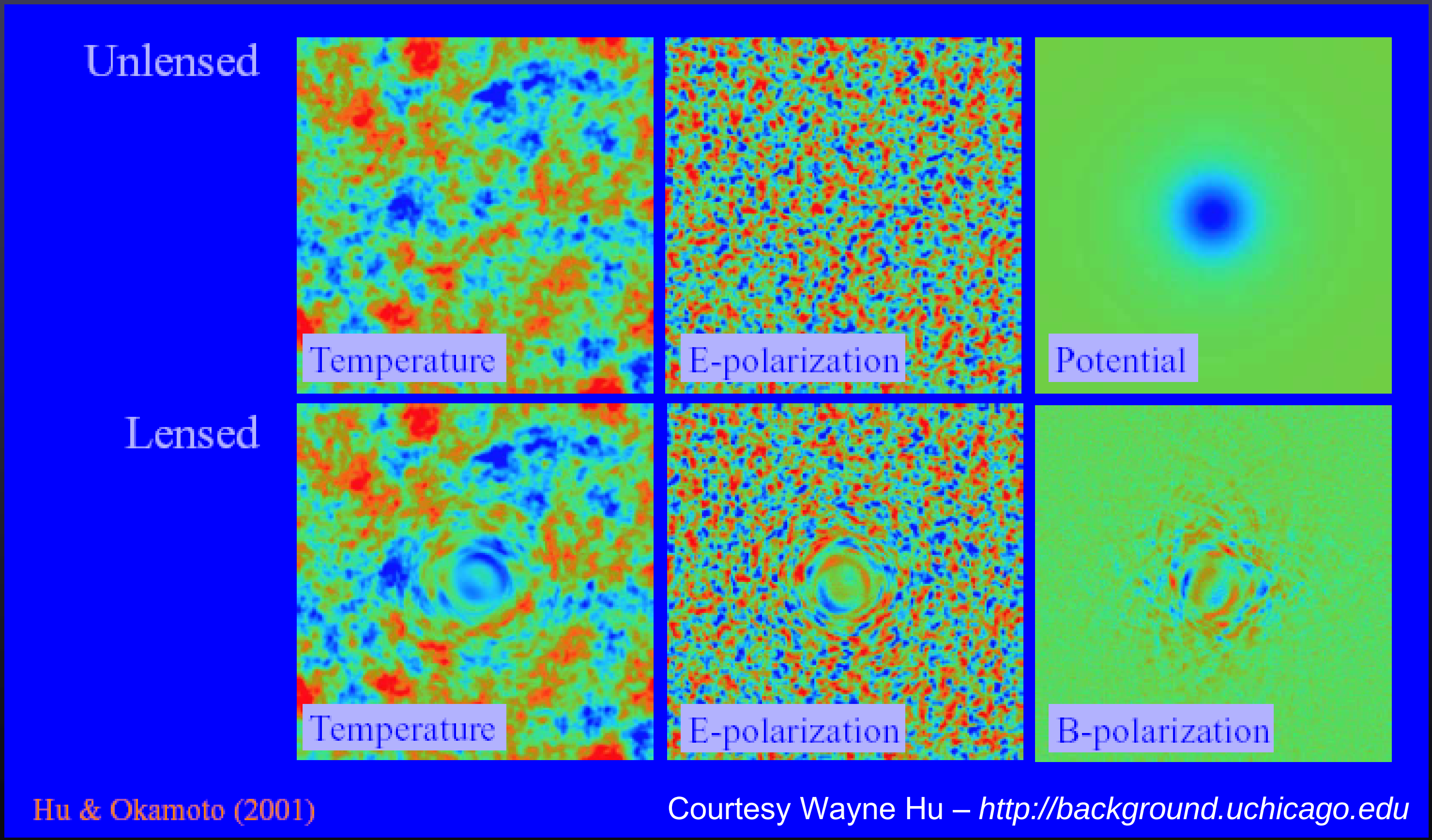
Late reionization reprocesses CMB photons

- Supression of primary temperature anisotropies
  - as exp(-τ)
  - degenerate with amplitude and tilt of spectrum
- Enhancement of polarization
  - low ℓ modes E & B increased
- Second-order conversion of T into secondary anisotropy
  - not shown here
  - velocity modulated effects
  - high ℓ modes

Courtesy Wayne Hu – *http://background.uchicago.edu*

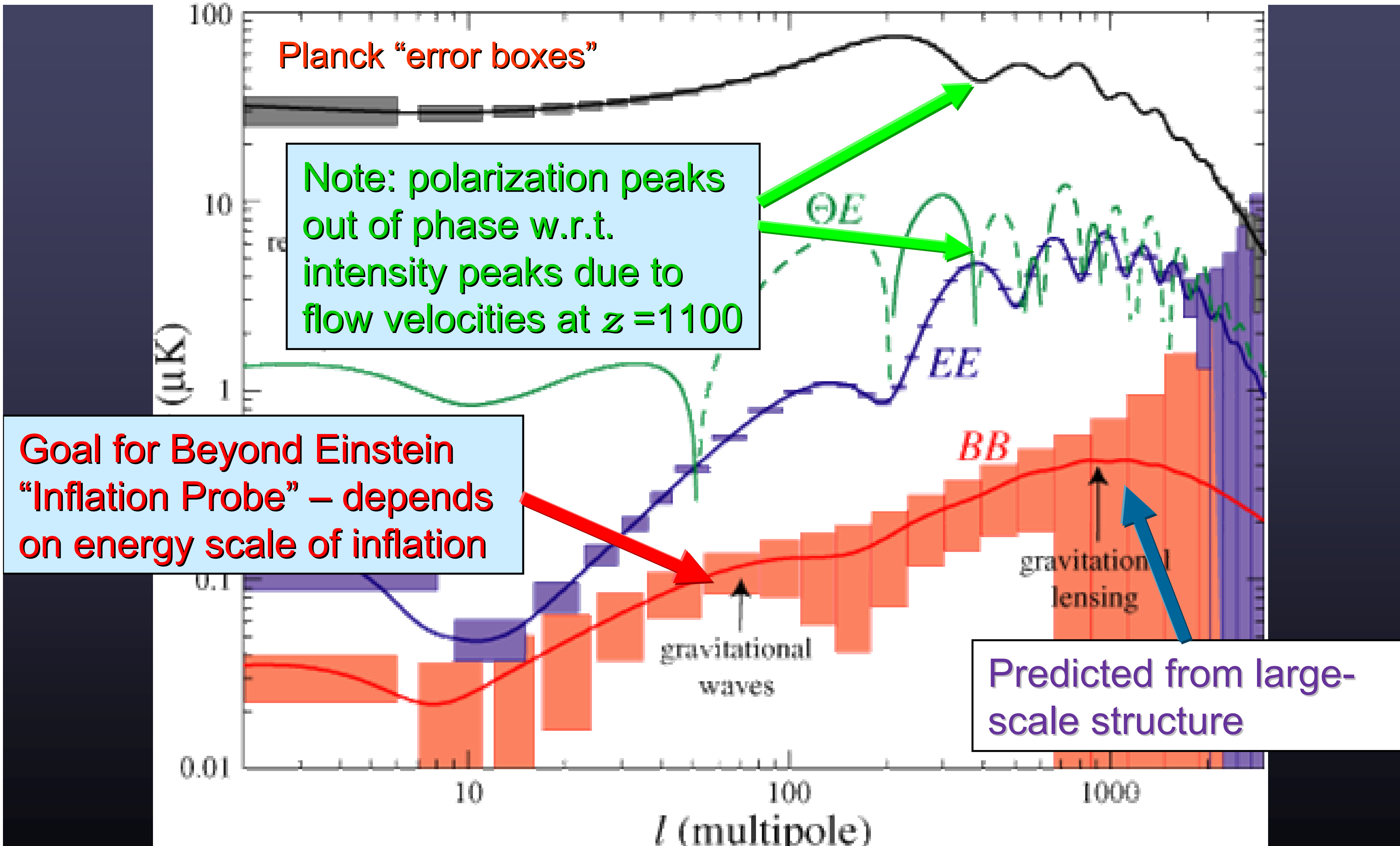
Lensing and Polarization

- Distorts the background temperature and polarization
- Converts E to B polarization
- Can reconstruct from T,E,B on arcminute scales
- Can probe clusters



Courtesy Wayne Hu – *http://background.uchicago.edu*

Planck: Predicted Power Spectrum



Hu & Dodelson *ARAA 2002*

CMB Checklist

Polarization predictions from inflation-inspired models:

- CMB is polarized
  - acoustic peaks in E-mode spectrum from velocity perturbations
  - E-mode peaks 90° out-of-phase for adiabatic perturbations
  - vanishing small-scale B-modes
  - reionization enhanced low ℓ polarization
- gravity waves from inflation
  - B-modes from gravity wave tensor fluctuations
  - very nearly scale invariant with extremely small red tilt ($n \approx 0.98$)
  - decay within horizon ($\ell \approx 100$)
  - tensor/scalar ratio r from energy scale of inflation ~ $(E_{inf}/10^{16}\ \mathrm{GeV})^4$