PHYSICS 831 PLAN

1. TOPICS:

A. Review of basic relations on fields, potentials, sources
(Revision of undergraduate material and familiarity with notation)

B. Radiation from single charges
Lienard-Wiechert potentials, fields of moving charges, velocity fields, ionization by fast particles, virtual photons, radiation from accelerated particles at low velocities, Larmor's Formula, Thomson cross-section, damping and radiation reaction, forced oscillators, Rayleigh scattering, directivity and polarization of Thomson scattering, radiation at large velocities (Lienard's Formula).

C. Radiation from ensembles
Multipole expansion, dipole approximation, magnetic dipole and electric quadrupole radiation, radiation from identical particles.

D. Radiation from antennas
Formulation of antenna problems, Schelkunoff Vector, linear antennas, radiation resistance, measures of directivity, loop antenna

E. Scalar diffraction theory
Fields as sources of radiation, Kirchhoff formulation, Fresnel and Fraunhofer diffraction, angular and spatial spectra of radiation, array theory, applications of Fourier Transform relationships in image processing, vector theory (brief).

F. Propagation in material media
Free electron theory, plasma effects in metals and gases, anisotropic media, magnetoionic theory and Faraday effect.

G. Other topics if time permits (usually doesn't).

2. BOOKS:

No book exactly parallels the course. Those using electromagnetics in their research, or taking 832* later, may wish to purchase Jackson, "Classical Electrodynamics" (Wiley). It's a good reference book but not so great to learn the subject from. On Reserve for 831* in the Physics Library are:

Jackson - Classical Electrodynamics
Marion - Classical Electromagnetic Radiation
Panofsky and Phillips - Classical Electricity and Magnetism
Papas - Theory of Electromagnetic Wave Propagation
Ramo, Whinnery and van Duzer - Fields and Waves in Communications Electronics

These will all be referred to from time to time in class. The last is an undergraduate engineering book, strong on antennas, etc. You may well be able to get by without a text by using these Reserve books in the Library.

A.H.Bridle Sept 82

Units in Electromagnetism (see Appendix in Jackson)

We introduce arbitrary constants in two force laws:

Electrostatic force between charges $\quad \underset{\sim}{F_e} = \dfrac{k_e Q_1 Q_2 \underset{\sim}{R}}{R^3}$

Magnetostatic force between currents $\quad \underset{\sim}{F_m} = k_m I_1 I_2 \oint\oint_{12} \dfrac{d\underset{\sim}{l}_1 \times (d\underset{\sim}{l}_2 \times \underset{\sim}{R})}{R^3}$

Dimensionally $\quad [F] = [k_e]\dfrac{[Q^2]}{[L^2]} = [k_m]\dfrac{[Q^2]}{[T^2]}$

$$\frac{[k_e]}{[k_m]} \ \underline{\text{fixed}} = \frac{[L]^2}{[T]^2}, \quad \underline{\underline{\text{observe}}} = c^2$$

RMKS $\quad$ Write $k_e = \dfrac{1}{4\pi\epsilon_0} \qquad k_m = \dfrac{\mu_0}{4\pi}$

Suppresses 4π factors in Maxwell's Eqns.

Keep engineering unit of current (Ampere)

$\mu_0 = 4\pi \times 10^{-7}\ \text{nt}/A^2$

$\rightarrow \epsilon_0 = \dfrac{10^7}{4\pi c^2}$ farad/m

Disadvantage $\rightarrow \underset{\sim}{E}$ and $\underset{\sim}{B}$ different dimensions $\begin{pmatrix}\text{volts/m} \\ \text{volt·sec/m}^2\end{pmatrix}$

Gaussian $\quad$ Write $k_e = 1, \quad k_m = 1/c^2 \quad$ ($\underline{\text{Choose}}$ $[Q] = [ML^3T^{-2}]$)

Use c.g.s. units throughout

Merit $\rightarrow \underset{\sim}{E}$ and $\underset{\sim}{B}$ same dimensions

$\underset{\sim}{F} = q\left(\underset{\sim}{E} + \dfrac{v}{c} \times \underset{\sim}{B}\right) \quad$ looks like a Lorentz-transformation term.

Jackson gives tables for RMKS / Gaussian conversion

$\left.\begin{array}{l}\text{Most practical applications use RMKS} \\ \text{theoretical texts use Gaussian}\end{array}\right\} \ !$

Classical Nonrelativistic Development of Field Equations

1) Static Fields from the Force Laws. (Free Space only, at first)

Historically, it was found that bodies could be "electrified" by friction, heat, chemical action, etc. so that static electrified bodies exert forces on one another at a distance. Then found from experiments in air that could assign attribute of CHARGE, q, to each "electrified" body so that forces exerted on and by that body are both proportional to q.

Also find inverse square law
central force
linear superposition

This situation summed up in Coulomb's Law:

$$\underset{\sim}{F}_i = q_i \sum_j \frac{q_j \underset{\sim}{R}_{ij}}{4\pi\epsilon_0 R_{ij}^3} \qquad \underset{\sim}{R}_{ij} = \underset{\sim}{r}_i - \underset{\sim}{r}_j$$

Hence define $\underset{\sim}{E}(\underset{\sim}{r}_i) = \lim_{q_i \to 0} \left(\frac{\underset{\sim}{F}_i}{q_i}\right)$ to be continuous vector field.

$$\underset{\sim}{E}(\underset{\sim}{r}_i) = \lim_{q_i \to 0} \sum \frac{q_j \underset{\sim}{R}_{ij}}{4\pi\epsilon_0 R_{ij}^3}$$

This seems straightforward enough until you recognise that the intermediate step really involves considering a test body, volume V, charge density ρ, momentum $\underset{\sim}{p} = m\underset{\sim}{v}$ with $\underset{\sim}{v}$ small, for which

$$\underset{\sim}{E} = \lim_{\substack{v \to 0 \\ \tau \to 0 \\ \rho V \to 0 \\ V \to 0}} \left\{ \frac{\underset{\sim}{p}(t+\tau) - \underset{\sim}{p}(t)}{\rho V \tau} \right\}$$

$v \to 0$ is necessary to exclude "magnetic effects", $\tau \to 0$ and $V \to 0$ are necessary f[illegible] imuity, and $\rho V \to 0$ is necessary to avoid self-force trouble

In fact we know that we cannot take limit $V \to 0$ and still measure ρ, also we cannot take limit $\rho V \to 0$ because of charge quantisation. This is what makes the theory "classical" — at its very basis it ignores both quantisation and the uncertainty principle.

Normally when developing the theory we shrug this off and say that we will not consider the theory to represent the actual microscopic level of electrical phenomena, but to represent a "suitable" spatial and temporal average over many microscopic systems. These averages are supposed to be taken over macroscopically infinitesimal volumes and durations which nevertheless smooth out real quantum discreteness and fluctuations. Yet time and again we will return to discussion of individual particles and use of "macroscopic" electromagnetic concepts in descriptions of microscopic systems. It is not obvious that the classical theory should be so robust, and the physicist should not take its seeming robustness for granted.

For example, the problems of a classical finite electron are nontrivial:

How does the E-field of a finite charge act on itself?

Is there microscopic saturation of the E-field to limit self-energies of real particles?

Does linear superposition hold at all ranges and all field strengths?

- for a thorough discussion of the classical problems associated with descriptions of particles, see ROHRLICH, "Classical Charged Particles".

If we add obvious quantum effects, things get rapidly worse

— uncertainties in localisation, charge distribution

— can a localised electron be nearly at rest, so that electric and magnetic effects are properly separable?

— is superposition safe to assume at high energies

- e.g.

$$\begin{array}{ccccc} h\nu \rightsquigarrow & \nearrow e^+ & \searrow & \rightsquigarrow h\nu \\ h\nu \rightsquigarrow & \searrow e^- & \nearrow & \rightsquigarrow h\nu \end{array}$$

emergent wave fields are not necessarily linear superposition of entrant wave fields.

Normally say we will avoid all of this by restricting ourselves to

1) describing phenomena on scales $\gg$ de Broglie wavelengths of particles
2) restricting frequencies so that $h\nu \ll m_0 c^2$ for particles
3) phenomena involving such numbers of particles that we can effectively treat charge as a <u>continuous fluid</u> whose charge density

$$\rho = \sum_V q_i \, \delta(\mathbf{r} - \mathbf{r}_i) \qquad (\mathbf{r} \text{ within } V)$$

is derived from a finite but small volume average around $\mathbf{r}$. Mobility of the charged fluid then sets up a current density

$$\mathbf{J} = \sum_V q_i \mathbf{v}_i \, \delta(\mathbf{r} - \mathbf{r}_i)$$

and conservation of charge requires

$$\boxed{\nabla \cdot \mathbf{J} + \dot{\rho} = 0}$$

(We will return to the constraints on the averaging process later)

Then it is best to assert that our experimental experience with charged-particle assemblies shows that the general force density experienced by a moving charge distribution that does not radiate significantly is, in the rest-frame of the laboratory observer,

$$\boxed{\underset{\sim}{f} = \rho \underset{\sim}{E} + \underset{\sim}{J} \times \underset{\sim}{B}}$$

as if the force on an individual charged particle were

$$\underset{\sim}{F} = q(\underset{\sim}{E} + \underset{\sim}{v} \times \underset{\sim}{B})$$ even though we can't really show this is true.

i.e. we assert that a) charge does not appear to depend on velocity (if it did, atoms could not preserve overall electrical neutrality, as observed, and "neutral" atoms or molecules could be deflected)

b) the total force has a velocity-independent part that we associate with $\underset{\sim}{E}$ and a velocity-proportional part that we associate with $\underset{\sim}{B}$.

Assertion (b) is really a result of failure to build a properly relativistic theory right from the start, and to describe all phenomena in the rest-frame of the observer. In a properly relativistic theory, the only forces on charges are electrostatic forces in the charges' own rest-frames. The velocity-dependent component comes from the transformation of the force from the particle's rest-frame to the observer's frame.

We then appeal to experiment again to suggest how to relate $\underset{\sim}{E}$ and $\underset{\sim}{B}$ to their sources.

With the "continuous-fluid" averaging procedure, our experience of Coulomb's law from static charge distributions becomes summarised as:

$$\boxed{\underset{\sim}{E}(\underset{\sim}{r}_f) = \iiint_V \frac{\rho(\underset{\sim}{r}_s)\,\underset{\sim}{R}}{4\pi\epsilon_0 R^3}\,dV\ , \quad \underset{\sim}{R} = \underset{\sim}{r}_f - \underset{\sim}{r}_s}$$

where $\underset{\sim}{r}_f$ is vector coordinate of a field point at which $\underset{\sim}{E}$ is to be evaluated

and $\underset{\sim}{r}_s$ is vector coordinate of a source point at which ρ is known

and V is a volume including <u>all</u> $\underset{\sim}{r}_s$ for which $\rho(\underset{\sim}{r}_s) \neq 0$.

The analogous statement about magnetic forces comes experimentally from the Biot-Savart Law, which is demonstrated from observations with steady currents in complete circuits:

$$\boxed{\underset{\sim}{B}(\underset{\sim}{r}_f) = \mu_0 \iiint_V \frac{\underset{\sim}{J}(\underset{\sim}{r}_s) \times \underset{\sim}{R}}{4\pi R^3}\,dV}$$

It is an <u>assumption</u> of classical nonrelativistic electromagnetism that this expression holds for current <u>elements</u> (incomplete current circuits) and/(later) for accelerated charges. The need for this assumption disappears in a fully relativistic treatment (e.g. ROSSER, "Classical Electromagnetism via Relativity").

The Biot-Savart law relates $\underset{\sim}{B}$ to its $\underset{\sim}{J}$ sources analogously to the relation of $\underset{\sim}{E}$ to its ρ sources through Coulomb's Law. Note the different symmetries of the fields:

$\underset{\sim}{E}$, $\underset{\sim}{J}$ are changed in sign if we invert the co-ordinate system

$\underset{\sim}{B}$, ρ are unchanged ....

$\underset{\sim}{B}$, $\underset{\sim}{J}$ are changed in sign under time reversal

$\underset{\sim}{E}$, ρ are unchanged ....——

The Biot-Savart Law also leads us to the form of the magnetic (vector) potential, as the following argument shows:

Noting that $\underset{\sim}{\nabla}_f\left(\frac{1}{R}\right) = -\frac{\underset{\sim}{R}}{R^3}$, we can write the Biot-Savart Law

$$\underset{\sim}{B}(\underset{\sim}{r}_f) = \frac{\mu_0}{4\pi}\iiint \underset{\sim}{J}(\underset{\sim}{r}_s) \times -\underset{\sim}{\nabla}_f\left(\frac{1}{R}\right) dV$$

$$= \frac{\mu_0}{4\pi}\iiint \underset{\sim}{\nabla}_f\left(\frac{1}{R}\right) \times \underset{\sim}{J}(\underset{\sim}{r}_s)\, dV$$

Now use vector identity $\underset{\sim}{\nabla}\times(\psi\underset{\sim}{v}) = (\underset{\sim}{\nabla}\psi)\times\underset{\sim}{v} + \psi(\underset{\sim}{\nabla}\times\underset{\sim}{v})$

i.e. $(\underset{\sim}{\nabla}\psi)\times\underset{\sim}{v} = \underset{\sim}{\nabla}\times(\psi\underset{\sim}{v}) - \psi(\underset{\sim}{\nabla}\times\underset{\sim}{v})$

Then $\underset{\sim}{\nabla}_f\left(\frac{1}{R}\right)\times\underset{\sim}{J}(\underset{\sim}{r}_s) = \underset{\sim}{\nabla}_f\times\left(\frac{\underset{\sim}{J}(\underset{\sim}{r}_s)}{R}\right) - \frac{1}{R}\underbrace{\underset{\sim}{\nabla}_f\times\underset{\sim}{J}(\underset{\sim}{r}_s)}_{=0 \text{ identically}}$

so $$\underset{\sim}{B}(\underset{\sim}{r}_f) = \frac{\mu_0}{4\pi}\iiint \underset{\sim}{\nabla}_f\times\left(\frac{\underset{\sim}{J}(\underset{\sim}{r}_s)}{R}\right) dV$$

$$= \underset{\sim}{\nabla}_f\times\frac{\mu_0}{4\pi}\iiint\frac{\underset{\sim}{J}(\underset{\sim}{r}_s)}{R}\,dV$$

because $\underset{\sim}{\nabla}_f\times$ and $\iiint$ over $\underset{\sim}{r}_s$ commute

i.e. $\boxed{\underset{\sim}{B}(\underset{\sim}{r}_f) = \underset{\sim}{\nabla}_f\times\underset{\sim}{A}(\underset{\sim}{r}_f)}$

where $\underset{\sim}{A}(\underset{\sim}{r}_f)$ is a vector potential related to sources $\underset{\sim}{J}(\underset{\sim}{r}_s)$ via

$$\boxed{\underset{\sim}{A}(\underset{\sim}{r}_f) = \frac{\mu_0}{4\pi}\iiint\frac{\underset{\sim}{J}(\underset{\sim}{r}_s)}{R}\,dV}$$

These relations are vector analogues of :-

$$\boxed{\begin{aligned}\underset{\sim}{E}(\underset{\sim}{r}_f) &= -\underset{\sim}{\nabla}_f\,\phi(\underset{\sim}{r}_f)\\ \phi(\underset{\sim}{r}_f) &= \frac{1}{4\pi\epsilon_0}\iiint\frac{\rho(\underset{\sim}{r}_s)}{R}\,dV\end{aligned}}$$

From these we can derive differential relationships for the static (time-stationary) fields arising from steady currents and static charges:

$\mathbf{E} = -\nabla\varphi$ implies $\nabla \times \mathbf{E} = -\nabla \times \nabla\varphi \equiv 0$ identically

$\mathbf{B} = \nabla \times \mathbf{A}$ " $\nabla \cdot \mathbf{B} = \nabla \cdot (\nabla \times \mathbf{A}) \equiv 0$ "

$\mathbf{E} = \iiint_V \frac{\rho(\mathbf{r}_s)\mathbf{R}}{4\pi\epsilon_0 R^3} dV$ implies $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$

$\mathbf{B} = \iiint_V \frac{\mu_0 \mathbf{J}(\mathbf{r}_s) \times \mathbf{R}}{4\pi\epsilon_0 R^3} dV \rightarrow \nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ (provided $\nabla \cdot \mathbf{J} = 0$)

Note the symmetry:

	$\nabla\cdot$	$\nabla\times$
$\mathbf{E}$	I	H
$\mathbf{B}$	H	I

(I - inhomogeneous
H - homogeneous)

The validity of the homogeneous (H) equations amounts to the statement that the only known sources of macroscopic $\mathbf{B}$-fields are current loops, i.e. there are no macroscopic free magnetic monopoles (and hence no monopole currents to give static-field E-configurations with nonvanishing curl).

2) Time-Variable Fields

The generalisation from these static-field equations to general equations involving time-variable fields and accelerated charges is on a much less secure experimental basis in a nonrelativistic theory.

① We assume that the $\nabla\cdot$ equations apply equally well to time-variable fields associated with variable currents in incomplete circuits, and thus with accelerated charges.

i.e. we assert that $\underset{\sim}{\nabla}.\underset{\sim}{E} = \rho/\epsilon_0$ are <u>always</u> true in free space.
$\underset{\sim}{\nabla}.\underset{\sim}{B} = 0$

The modification to the $\underset{\sim}{\nabla} \times \underset{\sim}{E}$ equation is usually justified by appeal to experiments involving the appearance of emfs in circuits in which there are changes in flux linkages — e.g. in <u>moving</u> circuits (the dynamo effect) or in fixed circuits threaded by variable magnetic flux (the transformer effect). It is important to realise that Maxwell's $\underset{\sim}{\nabla} \times \underset{\sim}{E}$ equation goes well beyond these experiments and <u>asserts</u> that there is an underlying VACUUM effect that has nothing to do with the media involved in the dynamo or transformer phenomena —

② Assert that: $\boxed{\underset{\sim}{\nabla} \times \underset{\sim}{E} + \dot{\underset{\sim}{B}} = 0}$, noting that this

is a free-space equation whose implications are only <u>directly</u> verified by experiment in situations such as the betatron accelerator. In this sense it may be misleading still to call it "Faraday's Law of Induction." It is invoked for example to explain the production of low-energy cosmic-ray particles as a result of the collapse of magnetised interstellar gas clouds under their own gravity — possibly the closest to an all-vacuum example of its applicability. Again, a properly relativistic theory makes it clear why this generalisation is correct, and the evidence for relativity's correctness is indirect evidence for the validity of this law.

③ Note that $\underset{\sim}{\nabla} \times \underset{\sim}{B} = \mu_0 \underset{\sim}{J}$ cannot be generally true because $\underset{\sim}{\nabla}.(\underset{\sim}{\nabla} \times \underset{\sim}{B}) \equiv 0$
but $\underset{\sim}{\nabla}.\underset{\sim}{J}$ can $\neq 0$

This is rescued via the continuity eqn. $\underset{\sim}{\nabla}.\underset{\sim}{J} + \dot{\rho} = 0$