THE FLOATING SPHERE ANTENNA

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INTRODUCTION

As a mount for a large parabolic dish, the floating sphere was considered by the CSIRO group in Australia when preliminary studies for the 210-foot were being made. The idea was not followed far because more conventional designs were clearly satisfactory for reflector sizes up to about 300 feet. The following simple note sketches some possible advantages and disadvantages of the floating sphere mount for much larger dishes.

GENERAL DESCRIPTION

Figure 1 shows the general idea. The dish is supported within a sphere which is partly rigid and partly completed by a radome. The sphere floats on water. The motion of the sphere may be controlled and its position indicated in a variety of ways. The general properties of such a design can be shown by a few simple "order of magnitude" calculations. For these, a sphere of 100 meters radius, which is about right to hold a 600-foot dish, will serve as an example.

SIMPLE PROPERTIES OF A SPHERICAL SHELL

For simplicity, consider a uniform complete shell of radius R and thickness t.

(a) How deep does it sink in water?

Let the density of the sphere material be ρ kgrms/m^3. For water $\rho_w = 10^3$; for steel $\rho \doteqdot 7.8 \times 10^3$; for concrete $\rho \doteqdot 2.7 \times 10^3$.

$$\text{Mass of sphere} = 4\pi R^2 t\rho \text{ kgrms} \qquad (1)$$

Let the depth of immersion be D meters.

$$\text{Mass of displaced water} = \pi D^2(R-D/3)\rho_w \text{ kgrms.} \qquad (2)$$

Equate (1) and (2) to give D.

Roughly neglecting D compared with 3R we get

$$D = (4Rt\rho/\rho_w)^{1/2} \qquad (3)$$

For R = 100 t = 0.1 ρ/ρ_w = 7.8 D = 17.7 meters. A more accurate solution of the cubic equation for D gives a value of D = 18.2 meters.

(b) What is the pressure of the sphere in water?

The maximum pressure on the spherical shell is that arising from the hydrostatic pressure at depth D. From (3) approximately

$$\text{Pressure} = (4Rt\rho\rho_w)^{1/2}\, g \qquad (4)$$

where the units are Newtons m^{-2}. In engineering units of lbs/sq. inch at 18.2 meters (60 feet) of water the pressure is about 26 lbs/sq. inch.

(c) How thick should the sphere be?

To illustrate the stresses in the sphere due to its own weight, imagine it first cut into two halves by a horizontal plane. The gravity forces across this plane are

$$\text{Force} = 2\pi R^2 t\rho g \qquad (5)$$

The area of the surface of the plane is $2\pi Rt$ so that the force per unit area is

$$\text{Force/unit area} = R\rho g \tag{6}$$

{Note that this stress is independent of the shell thickness.}

Using practical units, with R = 100 meters (3940 inches) in (6), we get

Aluminum:	ρ = 0.1 lbs/cubic inch	Stress = 394 lbs/sq. inch
Concrete:	ρ = 0.1 lbs/cubic inch	Stress = 394 lbs/sq. inch
Steel:	ρ = 0.28 lbs/cubic inch	Stress = 1100 lbs/sq. inch

These stresses are very small for the materials in question. However, consider the stresses in the steel at the water-line. It is floating with D submerged. For the sphere at the water surface we have

$$\text{Radius of surface section} = (2RD-D^2)^{1/2} \tag{7}$$

The stresses at this section may be estimated by taking the total weight above this section and the area of the section.

$$\text{Total weight} = (4\pi R^2-2\pi RD)t\rho \tag{8}$$

$$\text{Area of section} = 2\pi(2RD-D^2)^{1/2} \tag{9}$$

$$\text{Stress} = \frac{(2R^2-RD)\rho}{(2RD-D^2)^{1/2}} = R\rho\sqrt{\frac{2R-D}{D}} \tag{10}$$

Again, in lbs/sq. inch for steel, approximately, using D = 18.2 meters (718 inches)

$$\text{Stress} = 3.8 \times 10^3 \text{ lbs/sq. inch} \tag{11}$$

As before, the stress is independent of t and is still a reasonable value.

It is clear that dead-load stresses in the shell are reasonable, but they do not lead to a choice of shell thickness when treated in this elementary way. The choice of shell thickness and material is obviously going to be determined by a proper structural analysis which considers

(a) extra loading imposed on the shell by the dish and its supports,

(b) wind loads,

(c) permissible deflections of the shell, and

(d) economics of fabrication and erection.

(d) What sky cover can be achieved with this design?

Fairly simple geometry shows that, for a sphere of 100 meters radius, carrying a 600-foot dish and floating to a depth of 18.2 meters, the open edge of the hole in the sphere touches the water when the zenith angle of the telescope is 77.6°. Thus it seems reasonable to hope that the design could reach the zenith angle requirement of 72° (which reaches below the galactic center at the NRAO).

FURTHER DESIGN SUGGESTIONS

(a) Location of feed.

The shell is the primary support for the dish and the feed system. It seems fairly obvious to set the focal length so that the feed is either in the plane of the shell aperture (i) or at the center of the sphere (ii). In our case this gives focal lengths of

(i) $f = 2d + 300^2/4f$

Hence $f = 333$ feet $f/D = 0.56$

(ii) $f = d + 300^2/4f$

Hence $f = 230$ feet $f/D = 0.38$

A further alternative would be to use a Cassegrain system with the feed at either (i) or (ii).

(b) Deflection studies.

The concept may fail because the shell deflections are too great. However, it seems likely that the deflection pattern of the shell may be superior to that of a dish mounted on conventional bearings. The dish itself should be made as light as possible to give maximum stiffness under its own weight. Perhaps such novel structural materials as foam plastic might be considered.

Alternatively, the dish surface may be adjusted by an "open-loop" servo system. E. G. Bowen suggests (letter to JWF) air pressure differentials from the back to the front of the dish. A very few pounds per square foot could compensate for gravity loads on a light surface. Possible division of the surface into cells with independently adjustable pressure is worth thought.

None of these schemes can be considered quantitatively until the deflection pattern of the shell is known. This is thus a very important part of the study.

(c) Drive and position indicator systems.

Drives might be friction wheels (with say 95%) of the sphere weight resting on water and 5% on the wheels), racks, cables or gravity by driving two heavy trucks around the inside of the sphere. In the early stages of the study any reasonable system may be considered. The problems will lie in determining what the dynamic behaviour of the telescope will be. Some thought has to be given to the choice of the drive axes--for example, should a polar drive be attempted?

The system will have a large moment of inertia. About a diameter, a spherical shell of mass M has a M of I of 2/3 MR^2. For our sphere, made of steel 10 cm thick, this is 6.5 x 10^{11} MKS units. However, to give an angular acceleration (in the absence of friction) of 0.05 degrees/sec^2 requires equal forces at the ends of a diameter of the sphere of 325 tons weight. This shows that the drive problems may be a limitation to the design of the system.

For position indicating, gravity sensors and stabilized gyroscopes are obvious. The gyro drift could be corrected by an optical pointing device when needed.

CONCLUSIONS

The main problem areas, at first sight, appear to be:

(a) What are the deflections and the deflection pattern of the spherical shell?

(b) How can the sphere be driven?