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# PRECISE CALIBRATION OF SURFACES OF LARGE RADIO REFLECTORS BY MEANS OF ANALYTICAL PHOTOGRAMMETRIC TRIANGULATION

## RESEARCH AND ANALYSIS TECHNICAL REPORT NO. 10

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# CONTENTS

## 1. INTRODUCTION

Each year of the past decade has witnessed the development and construction of increasingly sophisticated antenna systems for radio astronomy and for the tracking and telemetry of space vehicles. The trend towards utilization of higher and higher frequencies together with the need for improved gain have led to tighter and tighter design tolerances for reflecting surfaces. It has accordingly become increasingly difficult to verify conformance with specifications. Today, the problem of sufficiently precise determination of surface conformity has assumed critical importance with some of the newer reflectors in operation and is of major concern to the using agencies of many of the reflectors currently under design or construction. In view of this, Instrument Corporation of Florida has sponsored an in-house program to determine the feasibility of a new photogrammetric approach to the problem.

## 2. OPTOMECHANICAL PHOTOGRAMMETRIC TRIANGULATION

Ockert 1959 [1] was apparently the first to apply photogrammetry to the problem. He employed conventional analog techniques of photogrammetry in which the relative orientation of a pair of stereophotographs is first recovered instrumentally in an optomechanical analog computer (the Wild A-7 Autograph, in this case) whereupon selected points in the stereopair are triangulated, again instrumentally, to produce three dimensional coordinates of a model of the object photographed. The process is essentially the same as that employed for mapping from aerial photography. From a pair of photographs of a 40 foot reflector taken by a camera of 165 mm focal length, Ockert was able to obtain estimated accuracies in triangulated coordinates of about 10 mm rms, or of about 1 part in 1200 of the diameter of the reflector. He expressed the opinion that this could be improved to perhaps 3 mm rms (or to 1 part in 4000 of the diameter) through the following measures: (1) use of a wide-angle camera with a larger format to allow larger scale photography, and (2) use of special control located off the antenna to decrease model errors caused by relative orientation. After looking into the problem, we have concluded that the optomechanical or analog method, pushed to its limits, cannot reasonably be expected to produce accuracies in triangulation of better than 1 part in 10,000 of the diameter of the reflector. Apart from purely instrumental limitations, the most serious drawback to analog triangulation is geometrical. In order to permit stereo perception, the axes of the cameras should be nominally parallel with each other and nominally perpendicular to the baseline. In practice this restriction leads to fairly acute angles of intersection (usually under 45°) of the triangulating rays, whereas with a pair of stations an angle of intersection of 120° is required for maximum over-all accuracy of triangulation for a given distance from baseline to object.

## 3. ANALYTICAL PHOTOGRAMMETRIC TRIANGULATION

Although an accuracy of the order of 1: 10,000 of the diameter of the reflector is entirely adequate for some antennas, it ranges from marginal to completely inadequate for others. If we consider an adequate measuring tolerance to be one third the magnitude of the tolerance of construction, the most severe requirement for a large reflector (over 50 feet in diameter) of which we are presently aware translates to about 1:75000 for rms measuring tolerance. This is about tenfold beyond what can reasonably be expected from optomechanical stereotriangulation. From such considerations we have concluded that if the more difficult requirements are to be met at all through photogrammetry, they are to be met only through the use of highly sophisticated techniques of analytical triangulation. Among the advantages of analytical triangulation are the following:

a. Highly convergent photography from an optimal geometrical configuration can be utilized, (geometrical restrictions of stereo restitution equipment need not be considered).

b. Plates can be measured individually on a simple monoscopic comparator of extremely high accuracy; accuracies of 2 to 3 microns can be realized for the plate coordinates of ideal images. (This is from 3 to 5 times better than the results obtainable from first order stereoplotters).

c. Errors in relative orientation can be reduced to insignificance through a least squares adjustment in which a large number of relative control points are carried (in the optomechanical process of recovering relative orientation only five relative control points, the minimum number required for a solution, can be utilized; consequently the reconstructed relative orientation suffers the full effect of the observational errors of these points).

d. If required for higher accuracies, photographs from several different exposure stations can be carried through a common least squares triangulation (in optomechanical solutions only stereopairs can be used in triangulation).

e. Measuring residuals from the least squares solution provide material for judging the efficacy of the adjustment and for determining the possible presence of systematic errors. Thus least squares residuals, properly interpreted, are invaluable for quality assessment; (residuals are not obtained from analog triangulation).

The photogrammetric theory employed in our study for the least squares adjustment and error propagation of analytical photogrammetric triangulation was adopted directly from the general photogrammetric adjustment derived by the writer in an earlier paper [2]. Inasmuch as the mathematics of the adjustment are rather involved, they will not be reproduced here; those interested in full mathematical details may obtain a copy of reference [2] by writing the author. Suffice it to say that the solution considers in full generality the problem of multistation analytical stereotriangulation with no restrictions being placed on either the number, the orientation or the placement of cameras or on the distribution or type of control. Since in the present application the only control points available are relative control points (or pass points) on the antenna, the primary photogrammetric triangulation is most readily performed in a coordinate system associated with any one of the cameras. Thus the X, Y, Z coordinates of the center of projection of one of the cameras may arbitrarily be taken as (0, 0, 0), the Z axis may be defined to coincide with the optical axis of this camera, and the X and Y axes may be defined to coincide with the $\bar{x}$ and $\bar{y}$

axes of the plate coordinate system. The positions and orientations of the remaining cameras relative to the camera defining the coordinate system can be reconstructed by making use of the fact that the relative positions and orientations must be such that a set of rays from any one camera will intersect the corresponding set of rays from any other camera. Inasmuch as the property of intersection is independent of scale, the baseline joining any two of the stations may arbitrarily be taken as unit length. In a two station problem it turns out that the simultaneous intersection of five pairs of rays is a necessary and sufficient condition for establishing the three rotations and three translations (the translations being constrained to the surface of a unit sphere) which define the location and orientation of the second camera relative to the first. A solution based on the minimum of five relative control points would suffer the full effect of measuring errors. However, if say ten times this number were carried in a least squares adjustment, errors in the resulting relative orientation would be reduced to insignificance. The error in triangulation of individual points would then be primarily attributable to the plate measuring errors of the images of the points themselves rather than to imperfections in the reconstructed relative orientations of the cameras.

## 4. SOME RELATIVE MERITS OF LONG AND SHORT FOCAL LENGTH CAMERAS

We were particularly interested in the applicability of long focal length cameras to the problem at hand, first because of our possession of a pair of 1000 mm f/5 cameras of ICF manufacture and second because of our feeling that a combination of subtle factors would prevent practical attainment of the requisite accuracies with wide angle cameras conventionally used in mapping. In the ensuing discussion we shall consider the format size to be fixed to 7.5 inches square. Then a camera of 85 mm focal length may be regarded as ultra wide angle (90°x90° field) one of 115 mm focal length as wide angle (76°x76°) and one of 210 mm (46°x46°) as normal angle. For our purposes we shall consider "long focal length" photogrammetric cameras to range from 600 mm (17°x17° field) to 1500 mm (7°5x7°5) in focal length. A focal length much in excess of 1500 mm becomes undesirable in the present application because of the unwieldy size of the camera (for the specified format), and because of the increasing significance of atmospheric refraction and of atmospherically induced image degradation as the camera is moved further and further from the subject.

Among the factors which assume increasing importance with decreasing focal length (for a fixed format) are the influence of deviations from flatness of the photographic emulsion, the influence of errors in the calibrated elements of interior orientation of the camera, the variation of radial distortion with object distance, the influence of small tilts of the plate, the appreciable variation in radial and tangential resolution across the photographic format, and the possibility of significant dependence of radial distortion on image size as a consequence of the increasing complexity of the image patch with increasing angular departure from the axial ray (the so-called "magnitude effect" considered by astronomers). For a fixed format all of these factors are of appreciably less practical consequence with a long focal length camera than with a short focal length camera. For instance, an error as large as two millimeters in each coordinate of the principal point (i.e., intersection of camera axis with image plane) have a completely insignificant effect on the reconstruction of the bundle of rays from a camera of 1000 mm focal length and 10°x10° field of view (7.5 inch square format). On the other hand, a two millimeter error in the principal point of a camera of 115 mm focal length and 76°x76° field of view (again, a 7.5 inch square format) would produce a very serious deformation in the photogrammetric reconstruction. Similarly with the 1000 mm camera departures of the surface of the photographic emulsion of as much as ± 20 microns from a best fitting plane can be tolerated, whereas with the 115 mm camera, such departures should not exceed ± 2 microns.

Since plate measuring accuracies are essentially independent of focal length, there is (from a strictly geometrical viewpoint) little basis for preferring a long focal length camera over a considerably shorter focal length camera of the same format, or vice versa. Although greater angular accuracies are obtainable from the long focal length camera, this advantage is almost precisely offset by the greater distance at which the camera must necessarily be placed from the subject in order to obtain full coverage. Accordingly, a preference for the use of a long focal length camera must be based chiefly on nongeometrical considerations, such as those considered above. It should be emphasized that this statement applies only to analytical triangulation; with analog triangulation, wherein the axes of the cameras are nominally parallel, wide angle cameras of a given format are generally to be preferred over long focal length cameras, for in so-called "normal" photography the wide angle camera enjoys such a degree of geometrical superiority that its relative shortcomings as outlined above become a secondary importance.

To summarize, we agree with Ockert's [1] point of view on the desirability of using wider angle cameras to obtain larger scale photography insofar as opto-mechanical stereotriangulation is concerned. On the other hand, we feel that when the analytical approach is used in conjunction with optimal, highly convergent photography, cameras having focal lengths several (from 3 to 10) times greater than those of conventional mapping cameras show greater promise of attaining accuracies in triangulation of 1 part in 50,000 or better of the diameter of the object photographed.

## 5. RESULTS OF NUMERICAL STUDIES

In order to determine whether or not an experimental program of antenna calibration would be warranted, a theoretical and numerical study was performed

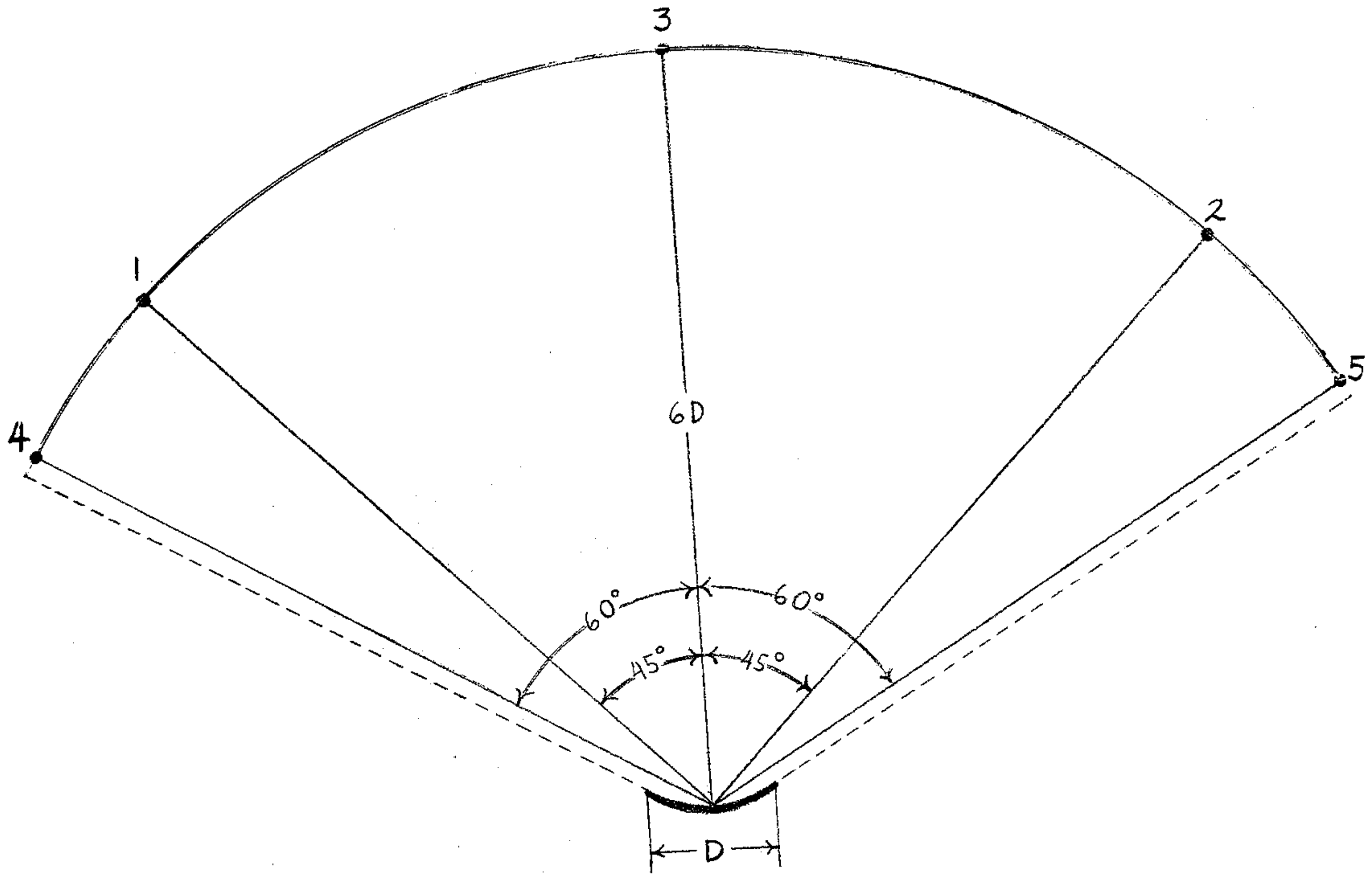

Figure 1. Illustrating configurations of ground-based cameras considered in numerical error study summarized in Table 1.

to ascertain precisely what accuracies could reasonably be expected from the analytical approach. In the first phase of the study a hypothetical configuration of ground-based cameras of 1000 mm focal length and 10°x10° field was assumed to observe a paraboloid of revolution of diameter D. The geometry of the configuration is illustrated in Figure 1. In order to obtain full coverage at maximum scale the postulated cameras were located on a circle of radius 6D from the reflector. Two, three, four and five camera solutions were investigated. Stations 1 and 2 subtending an angle of 90° from the center of the reflector were employed in the two station triangulation, for this pair provides about the strongest geometry possible without serious foreshortening of the limbs of the reflector. (Stations 4 and 5 would produce greater accuracies of triangulation but could provide adequate coverage of only about two thirds of the surface of the reflector).

The results of the error propagation for a point at the center of the reflector are presented in Table 1. The standard deviations of the plate measurements were taken as three microns, an accuracy routinely attained in the reduction of stellar plates, and the relative orientations and scale were regarded as error free (the implications of this are considered later). The standard deviations $\sigma_x$, $\sigma_y$, $\sigma_z$ of the triangulated coordinates are referred to an antenna coordinate system with origin at the vertex of the reflector, with z axis coincident with the axis of the reflector, and with x and y axes being horizontal and vertical, respectively, when the axis of the reflector is at a zero degree angle of elevation.

| Stations | $\sigma_x/D$ | $\sigma_y/D$ | $\sigma_z/D$ |
|---|---|---|---|
| 1, 2 | 1:57,100 | 1:77,900 | 1:55,600 |
| 1, 2, 3 | 1:78,900 | 1:93,800 | 1:55,600 |
| 1, 2, 3, 4 or 5 | 1:82,200 | 1:109,100 | 1:70,600 |
| 1, 2, 3, 4, 5 | 1:88,200 | 1:122,500 | 1:87,000 |

Table 1. Theoretical relative standard deviations of coordinates of point triangulated at vertex of reflector of diameter D by various combinations of ground-based stations depicted in Figure 1 (3 micron plate measuring accuracy assumed).

Table 1 shows that within the framework of the stated assumptions, standard deviations of better than 1 part in 50,000 of the diameter of the reflector are attainable for all three coordinates. Indeed, when all five stations are employed an average standard deviation of almost 1 part in 100,000 is attainable.

The foregoing results are, of course, strictly hypothetical; they represent the potentialities of the approach and their practical attainment remains to be demonstrated. Nonetheless, the findings were regarded as more than adequate to warrant proceeding further with the investigation. Had they been marginal, there would have been no justification for going beyond this initial stage of the study.

## 6. EFFECTS OF ATMOSPHERIC REFRACTION

An assumption implicit in the numerical study was that of a perfect central projection. In reality atmospheric refraction and lens distortion may prevent practical attainment of a perfect central projection unless appropriate corrections are applied. In order to gain some insight into the possible magnitude of atmospheric refraction for fairly short ranges at low elevation angles, a typical atmospheric refractive profile was employed in a ray tracing reduction to determine the refractive angles of three points at heights of 30, 60 and 90 feet on a hypothetical 60 foot reflector oriented horizontally. The observing station was assumed to have been at a ground distance of 360 feet from the reflector. The points were thus at elevation angles of approximately $4\overset{\circ}{.}8$, $9\overset{\circ}{.}6$ and $14\overset{\circ}{.}8$ respectively. Approximately 40 points equally spaced in height along the refractive profile were employed in each numerical integration of the ray tracing integral developed in reference [3]. In order to obtain sufficient numerical significance, a double precision version of the ray tracing program was programmed for the IBM 1620 computer. The refractive angles for the three points turned out to be $0\overset{''}{.}4885$, $0\overset{''}{.}4851$, $0\overset{''}{.}4813$ arc seconds respectively. Inasmuch as the differential refraction between extreme points was less than $0\overset{''}{.}01$ arc seconds, we concluded that there was no need to apply refractive corrections in this case. Indeed, on the basis of additional ray tracing simulations we find it difficult to envision any practical problem of antenna calibration wherein the differential refraction of rays within a projective bundle of 10° spread would be as much as 0".5 arc seconds. Because it is the relative or differential refraction (rather than the absolute refraction) which matters in analytical stereotriangulation, only in unusual situations will there ever be a need for considering refraction when the projective bundle is as narrow as 10° and ranges are limited to a few thousand feet.

Anomalistic refraction, sometimes called shimmer, is a rapidly fluctuating, random effect caused by atmospheric turbulence. Ground shimmer on hot sunny days can amount to several seconds of arc rms when apertures of one or two inches are used (image motion decreases with increasing aperture). Because shimmer is a random refractive effect, it poses no serious difficulties in static calibrations with ground-based cameras, for powerful neutral density filters can be employed to prolong the exposures to several seconds thereby effectively averaging out the influence of shimmer; the only residual effect is an enlargement or slight blurring of images and this is of no practical consequence when well defined targets of high contrast are employed.

## 7. CALIBRATION OF DISTORTION

In view of the foregoing we consider normal refraction to be of no practical consequence with narrow projective bundles as long as ranges are under a few thousand feet, and we feel that difficulties with shimmer can be surmounted through implementation of appropriate observational techniques. On the other hand, the effects of lens distortion can be most serious if not properly taken into account. Radial distortion is normally calibrated at infinity focus. Accuracies of ± 2 microns rms or better for the distortion function are not difficult to obtain from a rigorous stellar calibration (see, for example, reference [4]). However, optical ray tracing theory tells us that Gaussian radial distortion is a function of object distance (as is the focal length). Thus when the focal plane is set for a sensibly finite object distance, it is necessary to employ the distortion function appropriate to that distance. Actually, it is sufficient if the distortion is calibrated for two distinct focal settings, for then the distortion for any other setting can be computed from theory. Thus, even though the distortion function for infinity setting may be known from a stellar calibration, the problem remains of calibrating the camera for at least one, and preferably two, (the second to serve as a check), well spaced, finite focal settings. Inasmuch as one micron in the image plane of a 1000 mm camera corresponds to 0.″2 arc seconds, the construction of a target range of sufficient accuracy and stability for calibrating distortion of long focal length cameras at finite focal settings poses enormous difficulties.

In view of this we have developed a new method for calibration of distortion which is both powerful and practical. It involves photographing a set of plumb lines arrayed in the desired object plane and exploits the fact that, in the absence of distortion, the central projection of a straight line is itself a straight line. Systematic deviations of the images of plumb lines from straight lines thus provide a measure

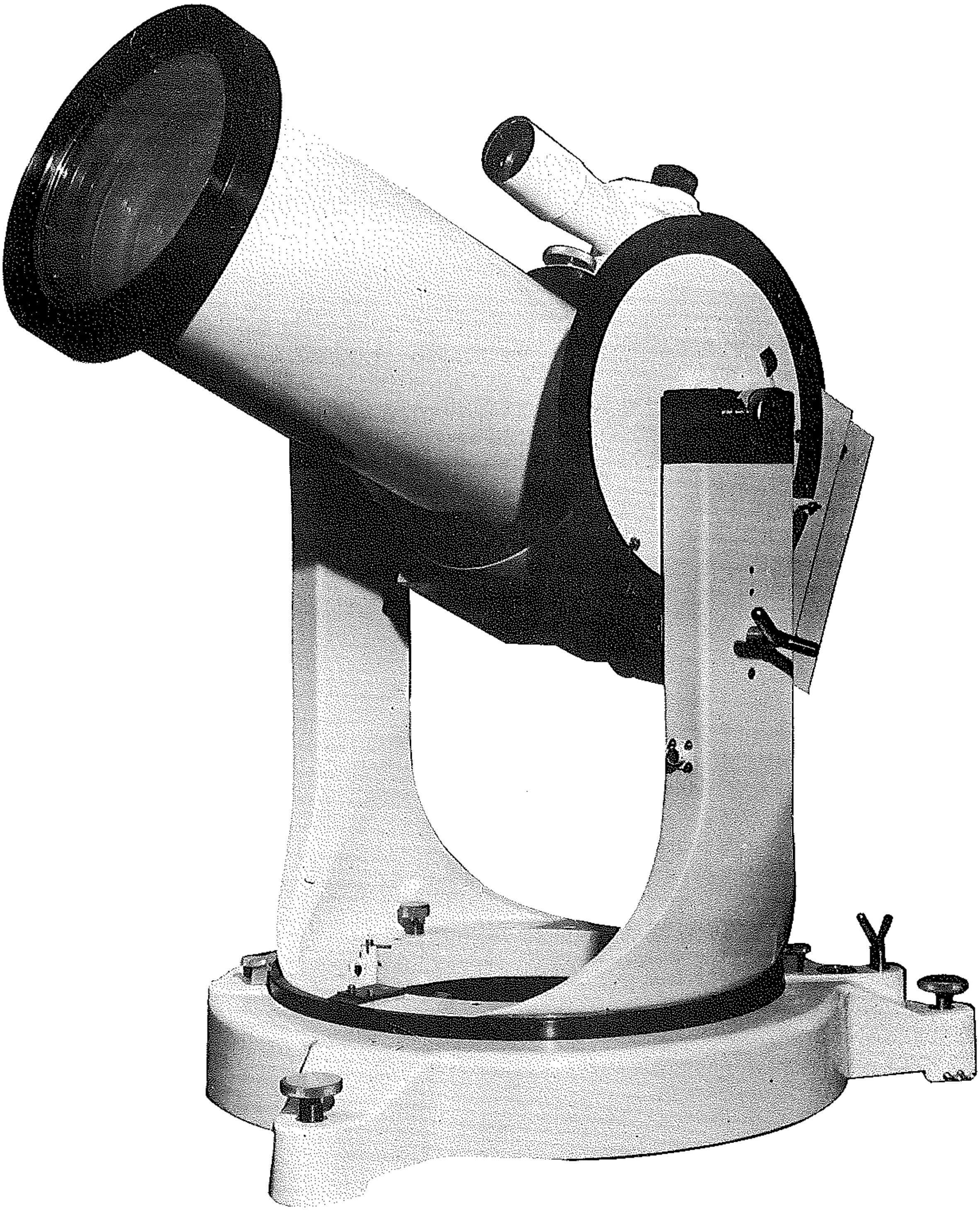

Figure 2. The ICF PC-1000 Camera, a 1000 mm f/5 camera originally developed and manufactured by Instrument Corporation of Florida for missile and satellite tracking. Over twenty have been produced. The camera has been modified for use in antenna calibration.

of distortion when properly reduced. The derivation of the method is the subject of a forthcoming paper. It is sufficient for present purposes to point out that it provides a completely adequate solution to the problems of calibrating both radial and tangential distortion of long focal length cameras at finite focal settings.

## 8. CALIBRATION OF A 60 FOOT REFLECTOR

Encouraged by the favorable outcome of our preliminary studies, we decided to proceed with an experimental program designed to test the practicality of our approach. We requested and received from the Air Force Missile Test Center permission to photograph the 60 foot telemetry antenna at the Mercury Control Center, Cape Canaveral. One of our PC-1000 Ballistic Cameras (Figure 2) was modified so that it could be critically focused for 400 feet. In order to obtain adequate depth of field at this distance it was necessary to stop the lens down to f/20. We decided to conduct the test at night with the dish illuminated by floodlights in order to avoid any possible difficulties with ground shimmer. Three camera sites were established on a circle approximately 400 feet from the ground projection of the vertex of the antenna. In order to avoid serious foreshortening of the images of the limbs of the reflector, the intersection angle of the camera axes of the outer pair of cameras was held to about 60° (we have since learned that this angle can usually be increased to about 90° without causing undue foreshortening).

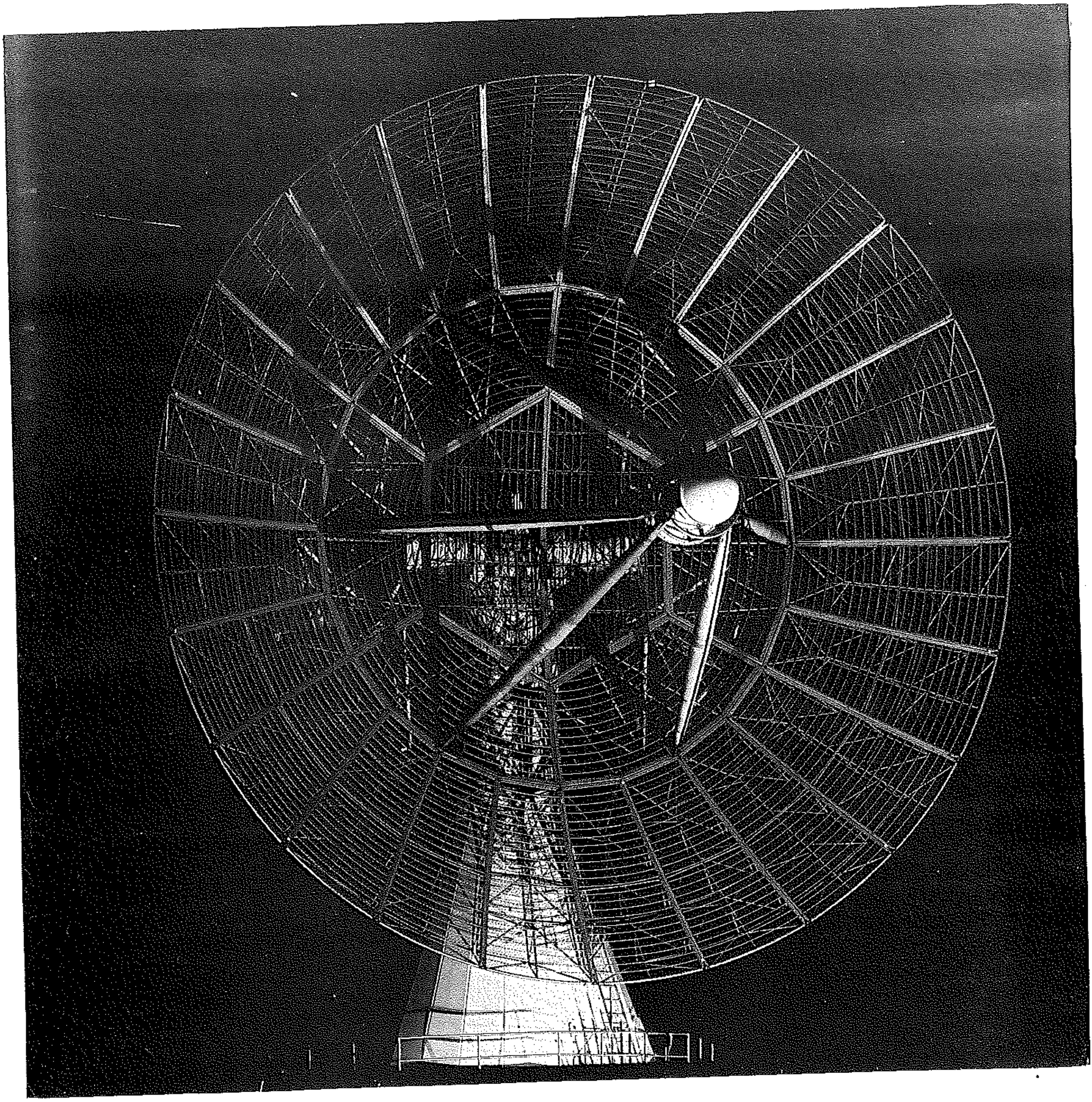

Figure 3. Print of PC-1000 plate of 60 foot TLM-18 antenna at Mercury Control, Cape Canaveral. View from left hand station approximately 400 feet from dish.

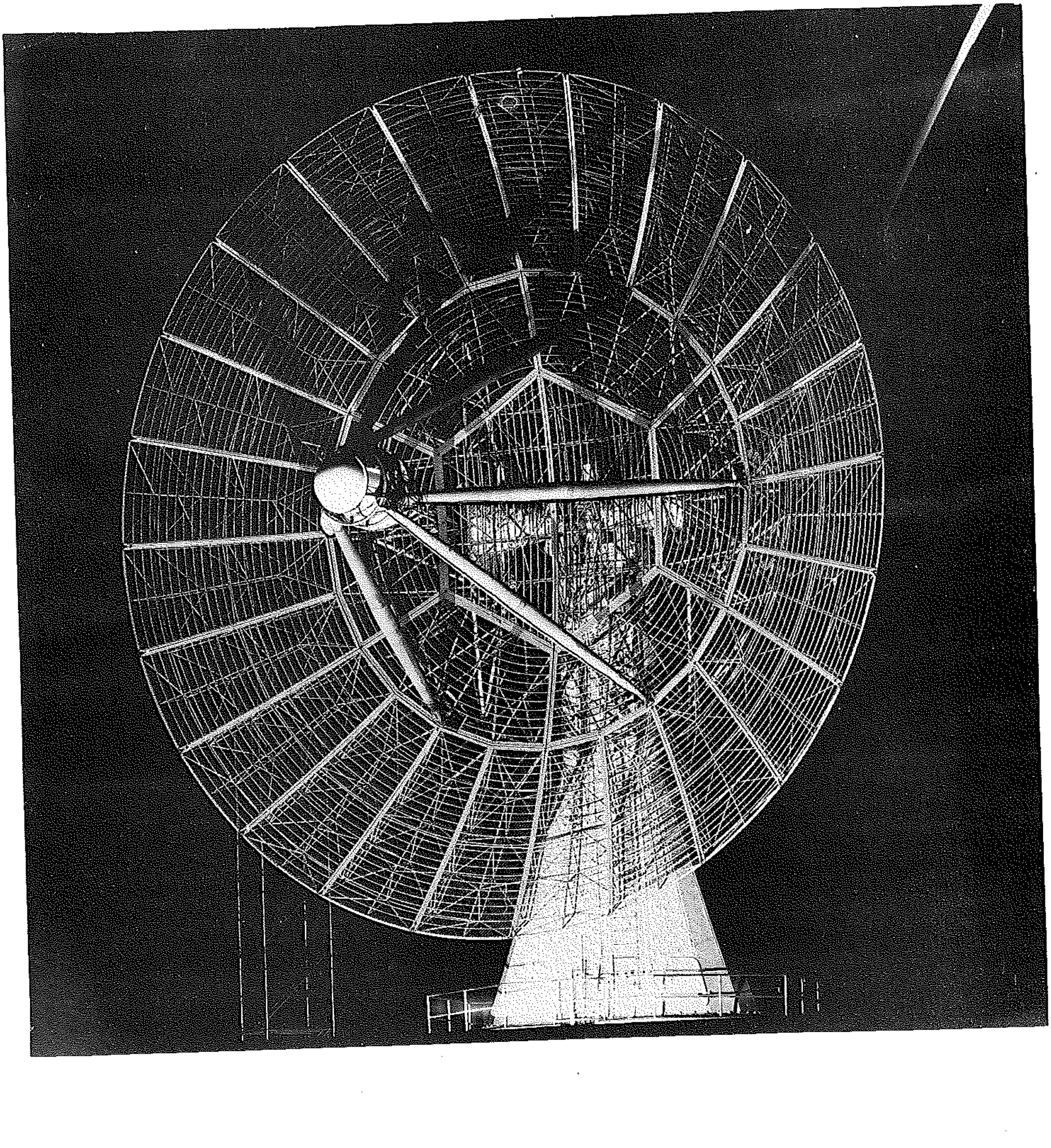

Figure 4. Print of PC-1000 plate of 60 foot TLM-18 antenna at Mercury Control, Cape Canaveral. View from right hand station approximately 400 feet from dish.

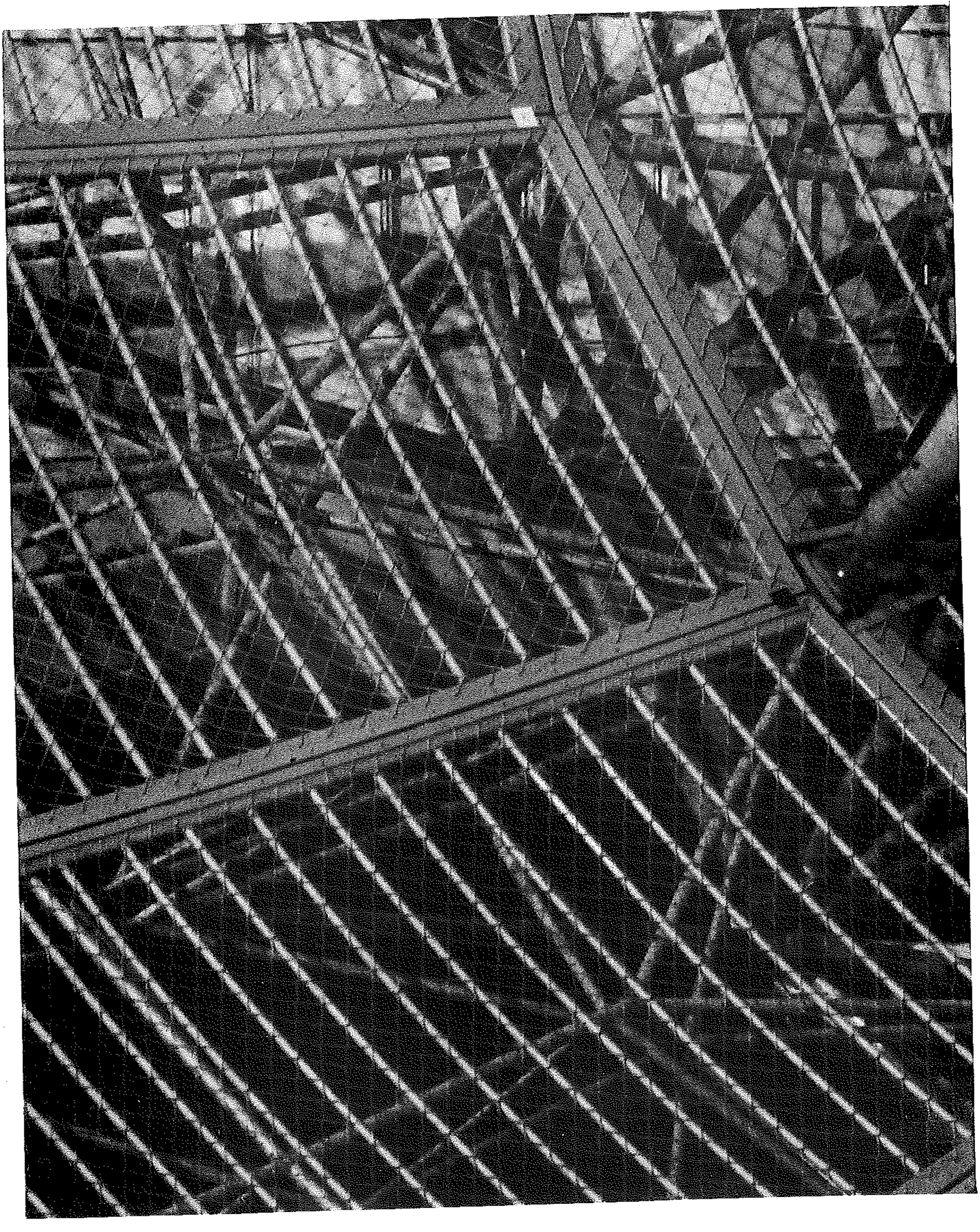

Figure 5. Detail of Figure 4 showing two different kinds of targets as well as details of structure and indulations of mesh about back supports.

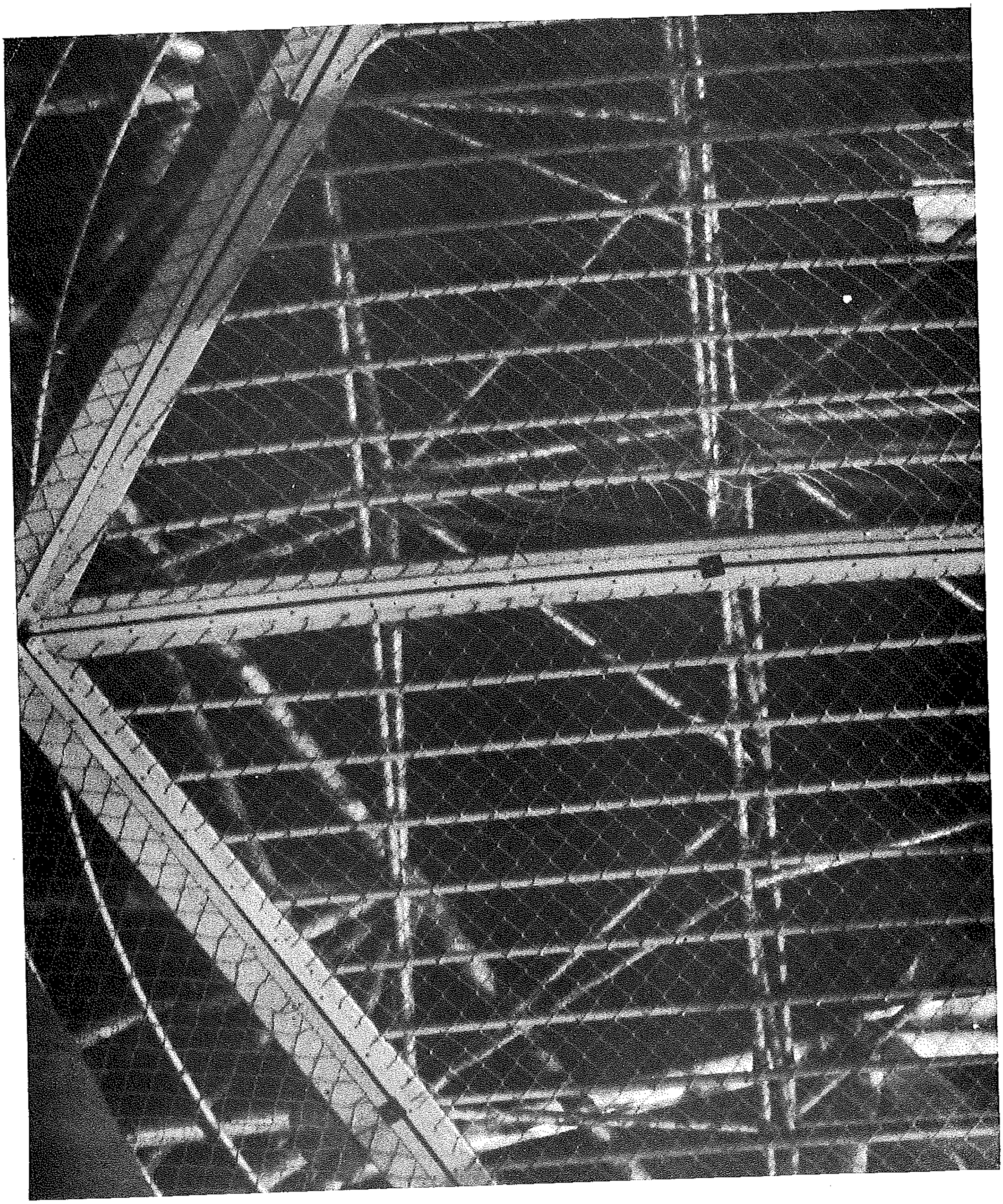

Figure 6. Detail of Figure 4 showing three targets and further details of structure and mesh. Note torn mesh near central target.

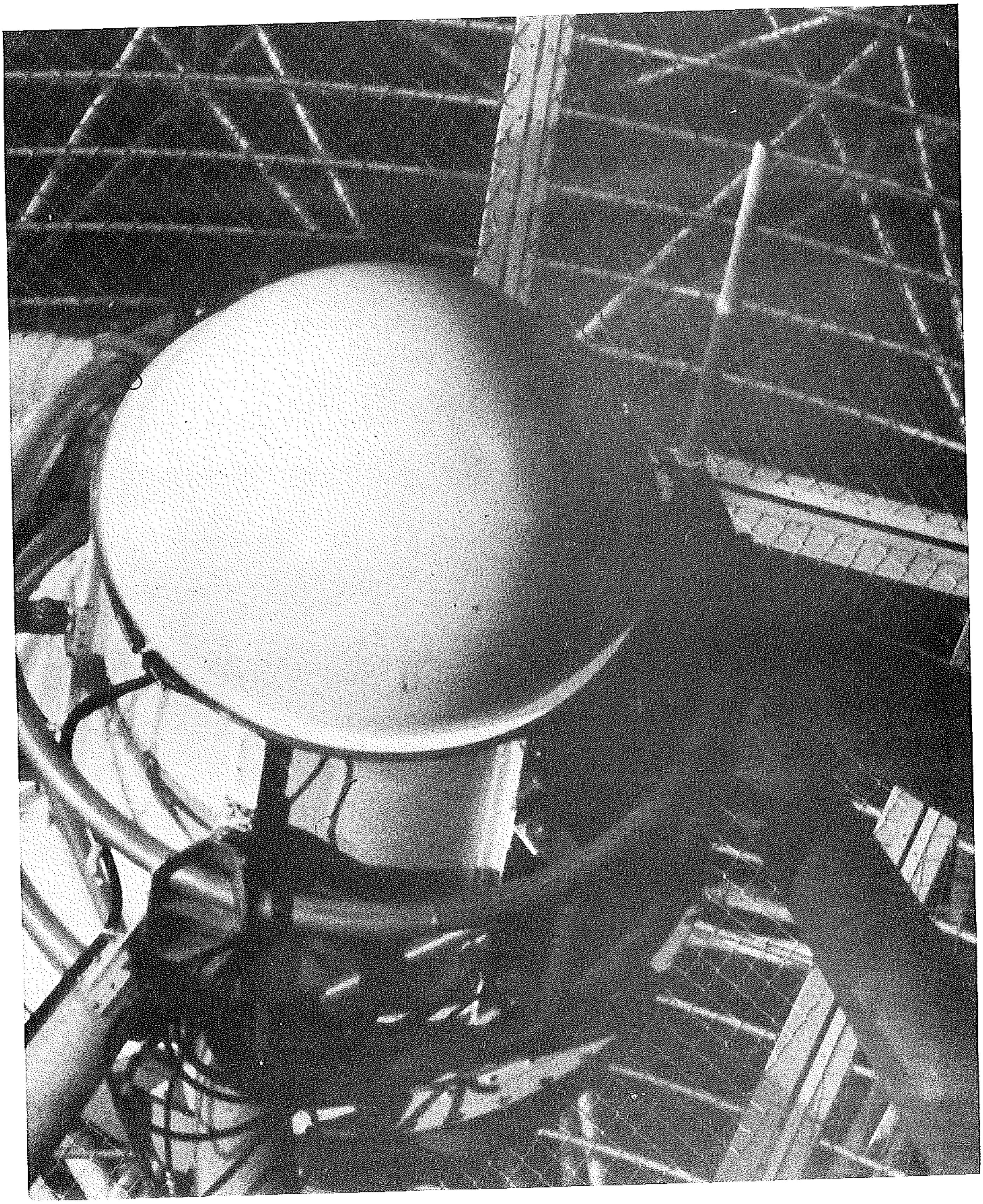

Figure 7. Detail of Figure 4 defining structure of feed assembly.

In order to permit precise scaling of the photogrammetric model a calibrated 50 foot invar tape was stretched across the dish under proper tension. Our intention was to photograph a pair of targets affixed to the tape precisely over the zero and fifty foot divisions. Similar targets, about 60 in number, were evenly distributed over the surface of the antenna. Two types of targets were used, the one consisting of a solid white circle of 6mm diameter against a 50x50mm black background and the other consisting of a solid black circle of 6mm diameter against a 50x50 white background. The targets were fabricated from one mil Mylar having an adhesive backing.

The reflector, illumed by three floodlamps, was exposed for 10 seconds on ultraflat photographic plates coated with 103-F emulsion. The same camera was employed at all three stations, the reflector remaining in a fixed horizontal position between exposures. Figures 3 and 4 are contact prints of the plates from the left and right hand stations. The photographs were extremely sharp, revealing an unexpected wealth of detail. Direct enlargements of selected areas of the plates are reproduced in Figures 5, 6 and 7.

A total of 138 points were identified and correlated on the plates taken from the extreme right and left hand camera stations. Thirty of these points were targets (those with the solid white circle in the center) and the remaining 108 were images of rivet heads securing the mesh to the panels. The plates were

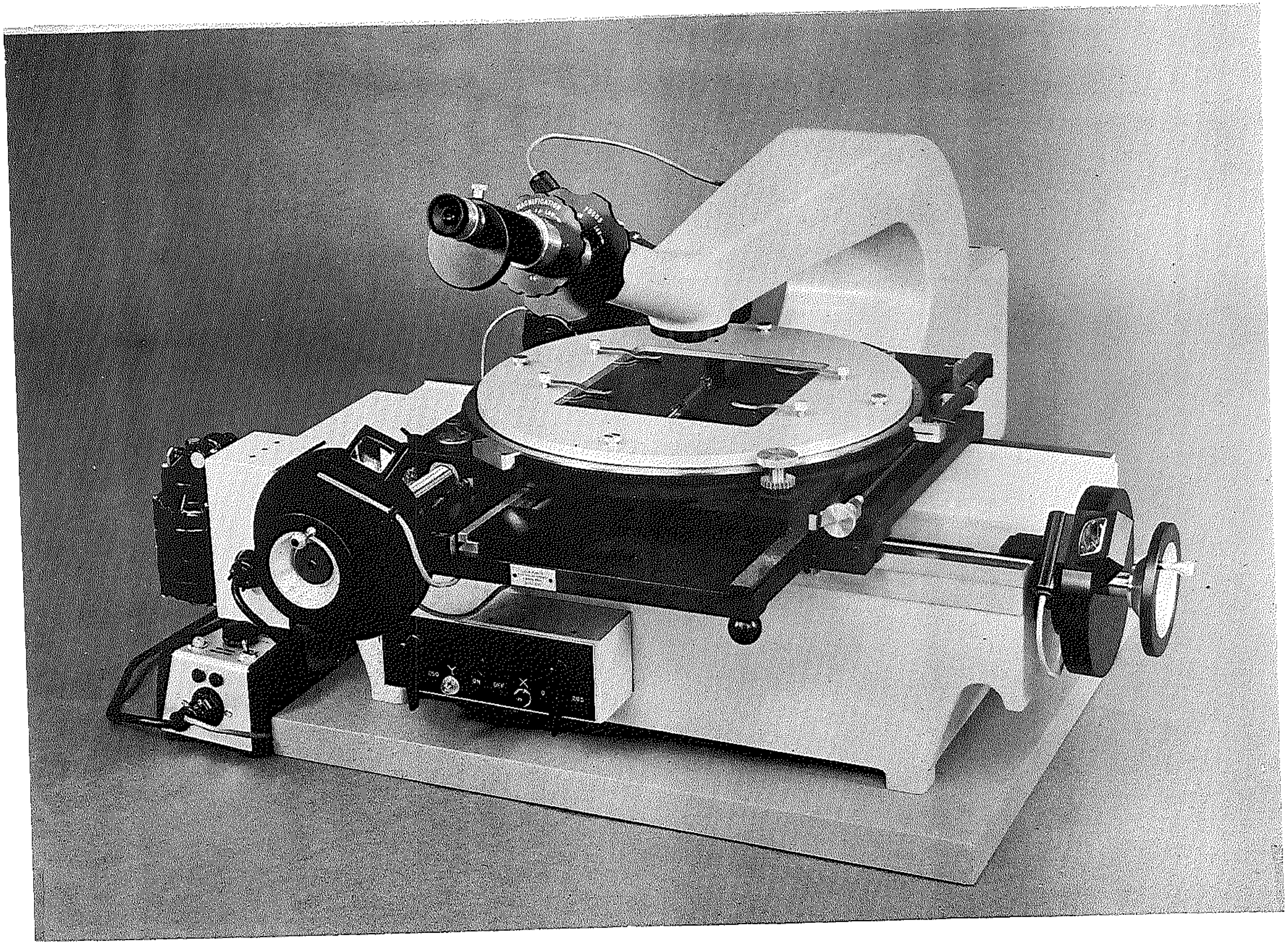

Figure 8. Mann comparator employed to measure plates for antenna calibration. Well-defined photographic images can be measured on this instrument to accuracies of up to two microns rms. Stability of photographic emulsion is main obstacle to consistent attainment of higher accuracies.

measured by a skilled operator on a calibrated Mann 422 C/D comparator (Figure 8). Normal plate measuring accuracy on the Mann comparator ranges from 2 to 4 microns rms for well defined circular images, with 3 microns rms being fairly typical. We are confident that systematic errors attributable to the comparator never exceed 2 microns after application of calibrated corrections.

Images of the selected targets were found to range in quality from excellent to fair, the variation being attributable to exposure. Those targets near the lower periphery of the dish were almost perfectly exposed, whereas those near the upper periphery were rather underexposed. This indicates that more careful attention must be directed in the future towards achieving an even illumination of the dish. Images of the rivet heads ranged in quality from good to rather marginal, again because of exposure. The targets on the Invar tape did not register at all, because a ten knot wind caused the tape to swivel during the 10 second exposures. This constitutes another minor problem for which corrective measures must be taken in any future undertakings. All things considered, we were quite pleased with the over-all quality of the plates obtained in this first venture.

## 9. RESULTS OF THE ANALYTICAL STEREOTRIANGULATION

The measured plate coordinates, properly corrected for comparator errors and lens distortion, were processed through the rigorous, least squares analytical stereotriangulation. Although observations of a minimum of five pairs of rays would have been sufficient for a unique solution, the entire set of observations for 138 pairs of rays was processed through the least squares adjustment in order that errors in the resulting relative orientation would be reduced to practical insignificance. Six rays were dropped from the final solution because their residuals were excessively large (the cause was later traced to mismatched pairs of rivet heads). Three iterations were necessary before the adjustment converged to a sufficiently stable solution. The root mean square error of the residuals from the final solution was 5.5 microns. The residual vectors were random from point to point; no systematic patterns were evident. Thus from the standpoint of internal consistency the adjustment was completely successful. The rms closure of 5.5 microns in plate coordinates is equivalent to an angular closure of 1″.1 arc seconds.

As might be expected, we found that the residuals for images of targets tended to be distinctly smaller than those for images of rivet heads. Accordingly the adjustment was repeated, carrying only observations of the 30 targets. This time the rms error of the least squares residuals turned out to be 4.0 microns which is equivalent to an angular closure of 0″.8 arc seconds. Because of the variable quality of the target images, the mean error of 4.0 microns was considered to be consistent with normal plate measuring accuracies. The randomness and character of the target residuals indicates that with images of uniformily high quality a mean error of 2 to 3 microns is likely to be attainable. Thus we feel that the hypothetical accuracies quoted in Table 1 constitute reasonable goals. The present results are themselves adequate to demonstrate that an rms closure of 1″.0 arc seconds (or 5 microns on the plate) is a conservative claim for the internal consistency of the method when suitable targets are observed by the PC-1000 camera.

## 10. TRANSFORMATION TO ANTENNA COORDINATE SYSTEM

The X, Y, Z coordinates resulting from the analytical stereotriangulation constitute coordinates of a model of the antenna in an arbitrarily defined camera coordinate system. The model can be properly scaled if the true distance between two points in the model is known, or alternatively if the true distance between the cameras is known. In the present case targets over the end divisions of an Invar tape stretched across the antenna were to have provided the precise scale. However, as pointed out earlier, the targets on the tape did not record because of wind. It was thus necessary to employ the distance between the exposure stations for scaling. Since this had been measured only crudely (to about a foot or so), the scaling of the model is no better than about 1 part in 400. In this regard one should appreciate that an accurate survey of the camera stations is normally unnecessary for analytical stereotriangulation. Rough estimates of the relative positions of the stations and orientations of the cameras suffice to initiate the adjustment; these are refined in an iterative process associated with the adjustment. In the ensuing discussion of accuracies the error in scaling will be ignored, for it is the relative dimensions of the surface with respect to a paraboloid which is of principal concern.

Since the antenna engineer is interested primarily in the departure of the set of triangulated points from a "best fitting" paraboloid of revolution, the reduction must proceed beyond the photogrammetric triangulation. Let us assume for the moment that the reflector is indeed a true paraboloid of revolution, defined in the antenna coordinate system by the equation

(1) $x^2 + y^2 = 4fz$

where f is the focal length of the paraboloid. This may also be written

(2) $x^2 + y^2 + z^2 = 4fz - z^2$,

or in matrix notation

(3) $\xi^T \xi = 4fz - z^2$,

where

(4) $\xi = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

and the superscript T denotes transposition.

Let $X_o$, $Y_o$, $Z_o$ denote the coordinates of the vertex of the paraboloid in the camera coordinate system and let $\alpha$ and $\omega$ denote the azimuth and elevation of the axis of the paraboloid defined in the same manner as the azimuth and elevation angles of the camera axes. Then the relation between the x, y, z and X, Y, Z systems may be written

$$(5)\quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\cos\alpha & \sin\alpha & 0 \\ -\sin\alpha\sin\omega & -\cos\alpha\sin\omega & \cos\omega \\ \sin\alpha\cos\omega & \cos\alpha\cos\omega & \sin\omega \end{bmatrix} \begin{bmatrix} X - X_o \\ Y - Y_o \\ Z - Z_o \end{bmatrix}$$

or, equivalently,

$$(6)\quad \xi = R\,\eta\,.$$

Substitution of this into (3) gives

$$(7)\quad \eta^T R^T R \eta = 4fz - z^2.$$

Inasmuch as $R^T R = I$ (since R is orthogonal) and since

$$(8)\quad z = D(X-X_o) + E(Y-Y_o) + F(Z-Z_o),$$

where D, E, F represent the elements in the third row of R, equation (7) can be reduced to

$$(9)\quad (X-X_o)^2 + (Y-Y_o)^2 + (Z-Z_o)^2 = 4f\,[D(X-X_o) + E(Y-Y_o) + F(Z-Z_o)] - [D(X-X_o) + E(Y-Y_o) + F(Z-Z_o)]^2.$$

This represents the equation of the paraboloid in the camera coordinate system. The equation involves a total of six unknowns: three translations $X_o$, $Y_o$, $Z_o$ defining the position of the vertex of the paraboloid; two rotations $\alpha$, $\omega$ defining the direction of the axis of the paraboloid (only two rotations are involved in the transformation, for the paraboloid is a surface of revolution); and the focal length f of the paraboloid.

In the absence of errors of any kind it is clear that the X, Y, Z coordinates of six distinct points on the paraboloid would generate six equations of the form (9) which could be solved for the six parameters of the transformation. In reality, of course, the X, Y, Z coordinates resulting from the stereotriangulation are subject to random errors. Moreover, even if the coordinates were perfect, the fact remains that the points do not actually lie on the surface of a true paraboloid. The matter of central interest to the antenna engineer is the nature of the departures of the actual surface from a "best fitting" paraboloid of revolution. The best fitting paraboloid may be defined as that for which the sum of the squares of the distances from the surface of the observed points is minimum. We have adopted a somewhat different criterion. The best fitting surface is defined as that which minimizes the quadratic form

$$(10)\ s = \sum_{i=1}^{n} v_i^t \Lambda_i^{-1} v_i \quad ,$$

wherein

$$(11)\ v_i = \begin{bmatrix} v_{x_i} \\ v_{y_i} \\ v_{z_i} \end{bmatrix} = \begin{bmatrix} X_i - \hat{X}_i \\ Y_i - \hat{Y}_i \\ Z_i - \hat{Z}_i \end{bmatrix} \quad ,$$

in which $\hat{X}_i$, $\hat{Y}_i$, $\hat{Z}_i$ denote the point on the surface of the paraboloid corresponding to the observed point $X_i$, $Y_i$, $Z_i$ and $\Lambda_i$ is the covariance matrix of

$X_i$, $Y_i$, $Z_i$ (this matrix is computed in the analytical stereotriangulation). In the event that $\Lambda_i$ is a multiple of the unit matrix this criterion is equivalent to that mentioned first. In general it has the advantage of properly weighting the different coordinates according to their relative accuracies and also takes into account the correlations existing between the coordinates. If the actual surface were indeed a true paraboloid, so that the departures were attributable solely to observational errors, the resulting adjustment would lead to parameters of the transformation having the smallest possible variance. Thus under such an assumption the resulting paraboloid would differ from the true paraboloid to smallest degree possible (put slightly differently, no other paraboloid computed from the given set of observations would agree more closely with the true paraboloid). Inasmuch as the distribution of the actual departures attributable to imperfections in construction is unknown in advance, it is a practical necessity to throw all of the adjustment onto the observations.

The detailed derivation of the minimum variance adjustment transforming the camera coordinates into antenna coordinates will not be given here inasmuch as it is a straightforward application of the general adjustment derived in Appendix A of reference [3]. The adjustment has been programmed on our IBM 1620 Computer. The output of the routine provides not only the parameters of the transformation and their covariance matrix but also the x, y, z components of the departures of the triangulated points from the best fitting paraboloid. The perpendicular distance from each point to the best fitting paraboloid is also computed.

The minimum variance transformation from camera coordinate system to antenna coordinate system was applied to all 132 points resulting from the analytical stereotriangulation. The perpendicular departures from the best fitting paraboloid are plotted in Figure 9. The rms perpendicular departure turned out to be 0.29 inches. This is 8.2 times greater than the rms departure attributable solely to errors in triangulation. Thus the departures are statistically significant and must be considered to be real. The standard deviations of triangulated target points are typically

$$\sigma_x = 0.0012 \text{ ft},$$
$$\sigma_y = 0.0010 \text{ ft},$$
$$\sigma_z = 0.0023 \text{ ft}.$$

Since the diameter of the dish is 60 feet, these are equivalent to proportional accuracies of

$$\frac{\sigma_x}{60} \cong \frac{1}{50{,}000} \quad ,$$

$$\frac{\sigma_y}{60} \cong \frac{1}{60{,}000} \quad ,$$

$$\frac{\sigma_z}{60} \cong \frac{1}{26{,}000} \quad .$$

The relatively low proportional accuracy of z is attributable to geometry; had the cameras been so placed that the intersection angle of their axes were nominally 90° instead of 60° and had the 4 micron rms closure of the stereotriangulation been maintained, the ratio $\sigma_z/60$ would have been reduced to 1:42,000.

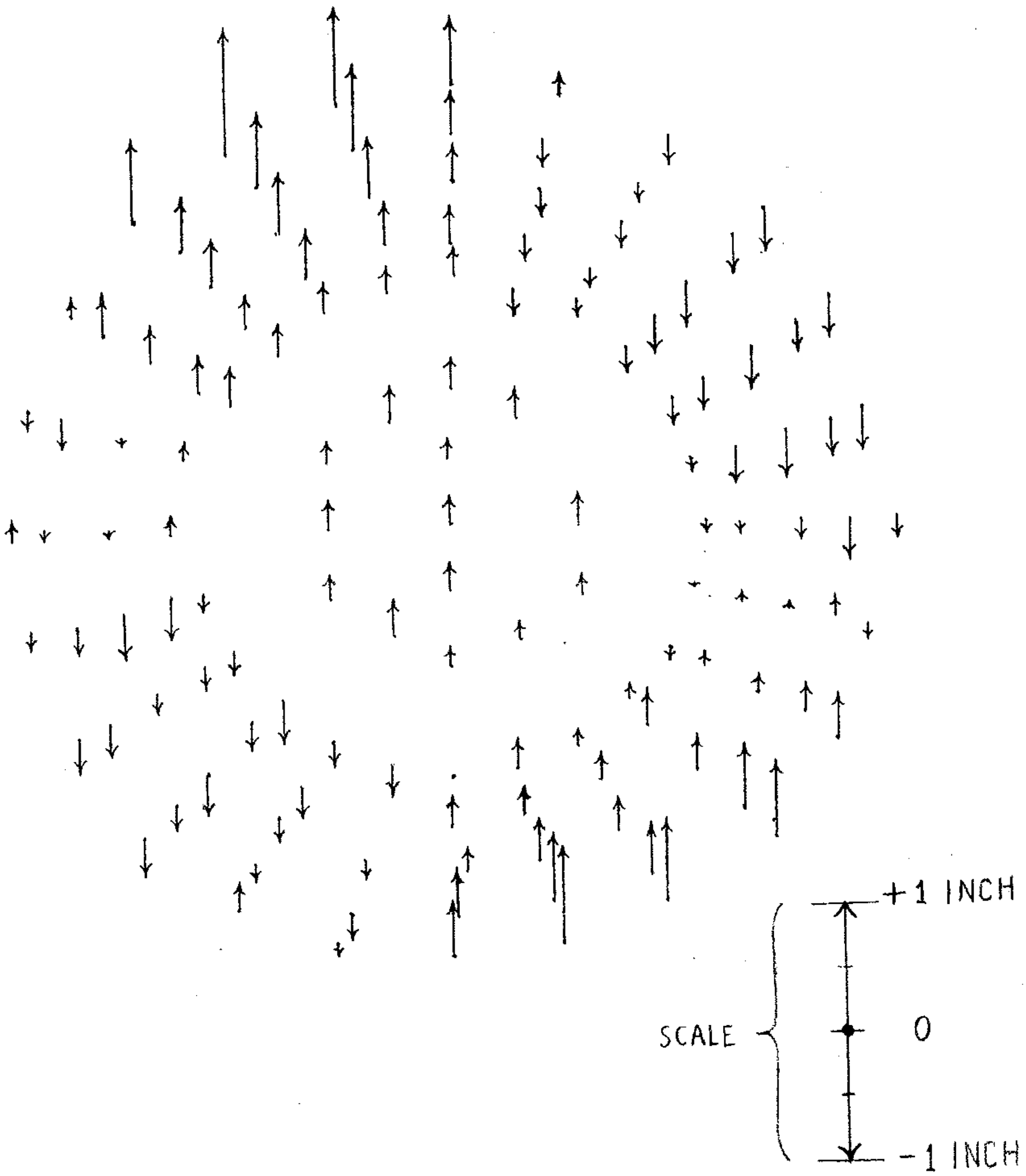

Figure 9. Plot of perpendicular departures of photogrammetrically triangulated points on TLM-18 reflector from best fitting paraboloid of revolution. Positive departures indicated by upward arrows, negative departures by downward arrows.

The departures plotted in Figure 9 displayed such systematic characteristics that it was feasible to employ them to construct a three dimensional model of the error surface. A photograph of the resulting model with isoerror contours inscribed at intervals of 0.01 feet is reproduced in Figure 10. The contours of the error surface are superimposed on a photograph of the reflector in Figure 11. We would hasten to note that the error surface is strictly valid only for the panels supporting the mesh, for no points on the mesh itself were measured. The same is true of the rms error of 0.29 inches quoted above. The mesh on some of the panels was observed to be in rather poor condition. Consequently it is likely that the rms departure would have increased significantly had a sizeable sample of points on the mesh been included in the adjustment. It should thus be appreciated that the error surface of Figure 11 depicts only the broad departures of the structure from a perfect paraboloid. A considerably larger sample would have been required to define the indulations of the mesh.

## 11. ANALYSIS OF RESULTS

The results of our limited test program are sufficient to demonstrate to our satisfaction that analytical stereotriangulation using photography from long focal length, narrow angle cameras is both practical and capable of impressively high accuracies when properly executed. The procedures we employed were adapted from the ballistic camera technology which has been refined

Figure 10. Three dimensional model of error surface of TLM-18 at Mercury Control. Isoerror contours inscribed at 0.01 ft intervals.

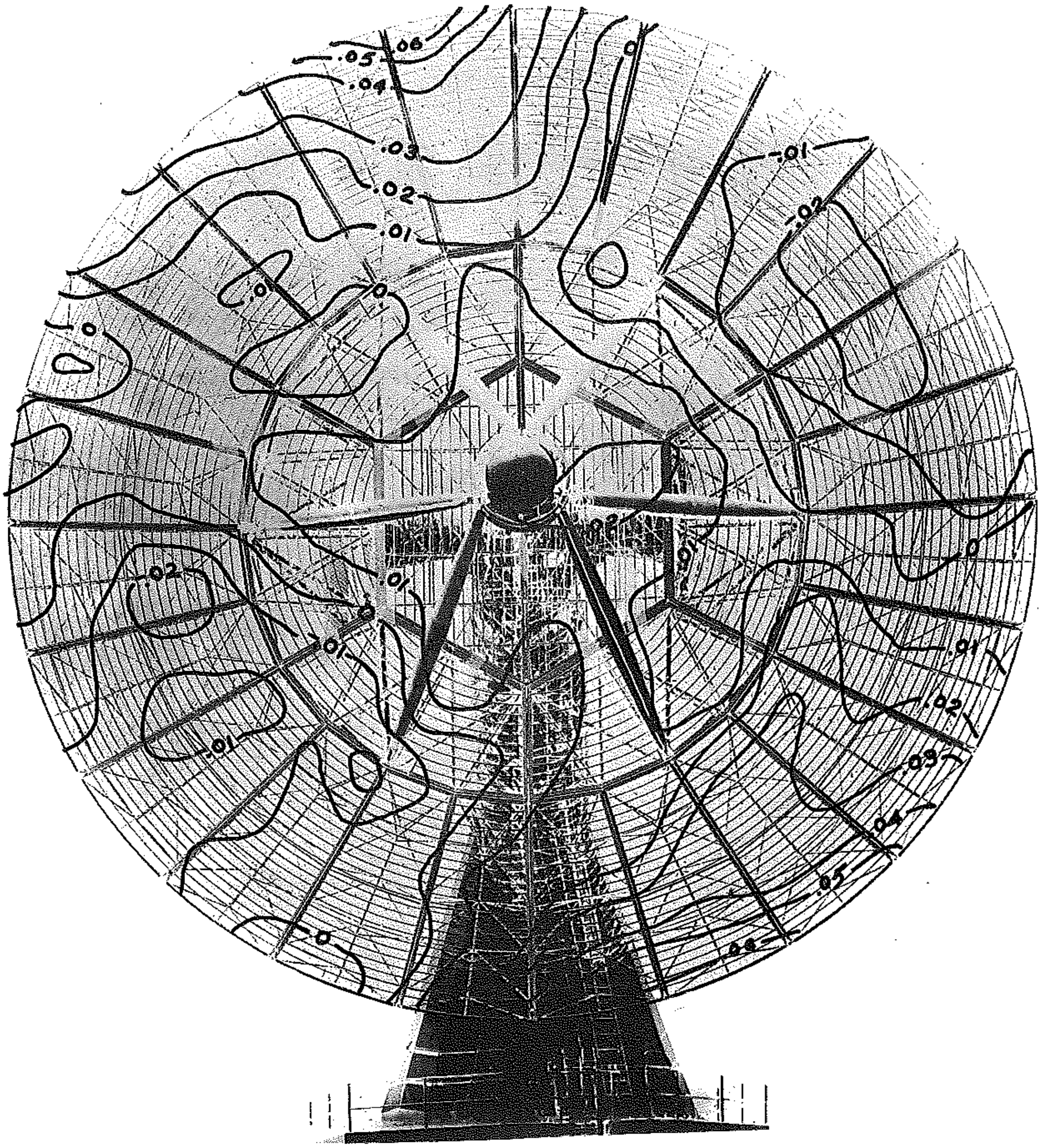

Figure 11. Contours of error surface superimposed on photograph of TLM-18. Contour interval is 0.01 ft.

over the years (primarily at the Ballistic Research Laboratories, the Atlantic Missile Range, and Instrument Corporation of Florida) to a state of perfection unrivaled in metric photography. The key to success in this discipline is strict adherence to highly evolved techniques of observation, calibration and mensuration coupled with the rigorous exploitation of observational redundancy. The importance of an accurate and comprehensive calibration of both camera and comparator cannot be stressed too highly, for only then do the closures of the triangulation provide a trustworthy indication of absolute accuracies. While randomness of residuals of triangulation is necessary to demonstrate that the photogrammetric model is sensibly unaffected by systematic errors, randomness alone is not sufficient to guarantee freedom from systematic errors. For instance, the degree of symmetry involved in the analytical stereotriangulation of points on a typical reflector is such that a rather sizeable systematic error in corrections applied for radial distortion of the lens will have only a slight effect on the randomness of the closures (hence the importance of the dependence of distortion on object distance). Therefore, only when all possible systematic errors of significance to the reconstruction of the photogrammetric bundle are adequately understood and corrected can one have well-founded confidence in closure as an indication of absolute accuracy. The challenge of any process of mensuration is one of suppressing systematic errors to insignificance relative to random errors (or one of obtaining accuracy compatible with precision). We believe that this challenge has been successfully met in our calibration of the Mercury Control Antenna and that it can be met in similar calibrations of other antennas. If we are correct in this belief, accuracies of at least 1 part in 50,000 of the diameter of the antenna have been demonstrated for two coordinates (x and y) and accuracies of 1 part in 26,000 have been demonstrated for the z coordinate. Since only a moderate improvement in geometry would have sufficed to improve accuracies of the z coordinate to better than 1:42,000, we feel safe in asserting that a two station analytical stereotriangulation employing ground-based PC-1000 cameras is capable of accuracies of triangulation of at least 1 part in 40,000 of the diameter of the reflector. This is based on the demonstrated rms closure of 4 microns in the measured plate coordinates of target images. Since the best of the target images were comparable to excellent stellar images, we see no reason why the accuracies of 2 to 3 microns attained in the reduction of stellar plates cannot be achieved with optimally exposed targets. This, however, remains to be demonstrated.

## 12. UTILIZATION OF AERIAL PHOTOGRAPHY FOR CALIBRATION

Inasmuch as the precise locations of the exposure stations need not be known for analytical stereotriangulation, there is no reason why airborne cameras cannot be employed. The primary technical problem is one of obtaining sufficiently sharp photography to permit accurate measurements of the relative positions of target images. Because successful aerial photography would permit calibration of antennas in any desired orientation, we set out to determine the feasibility of long focal length photography from a helicopter. To accomplish this a pattern simulating a 85 foot reflector was outlined on the ground and nine stakes, eight on the perimeter and one at the center, were set out with targets attached. The positions of the outer targets relative to the central target were surveyed with a Wild T-2 Theodolite, the distances relative to the central target were taped, and the relative heights of the targets were determined by levelling. For a check on closure, the distances between successive targets around the perimeter were also taped. A PC-1000 camera mounted on three Aeroflex cable vibration isolators was installed in a Bell J-2 helicopter (Figure 12). Four photographs were made of the target array from an altitude of 360 feet. Since the elevation angle of the camera was depressed to -45°, the slant range to the central target was approximately 500 feet. A strong wind of 30 knots in gusts made hovering extremely difficult and induced considerable vibration in the helicopter. An exposure of 1/400 second at f/36 was used for all photographs.

Despite the unfavorable observational conditions, one of the photographs (Figure 13) was judged to be of good quality, two were fair and one was poor. Figure 14 shows an enlargement of one of the target stakes of Figure 13. On all plates the targets were measurable, an rms repeatibility of

Figure 12. Modified PC-1000 mounted on vibration isolators aboard Bell J-2 helicopter employed in tests simulating aerial photography of 85 foot reflector.

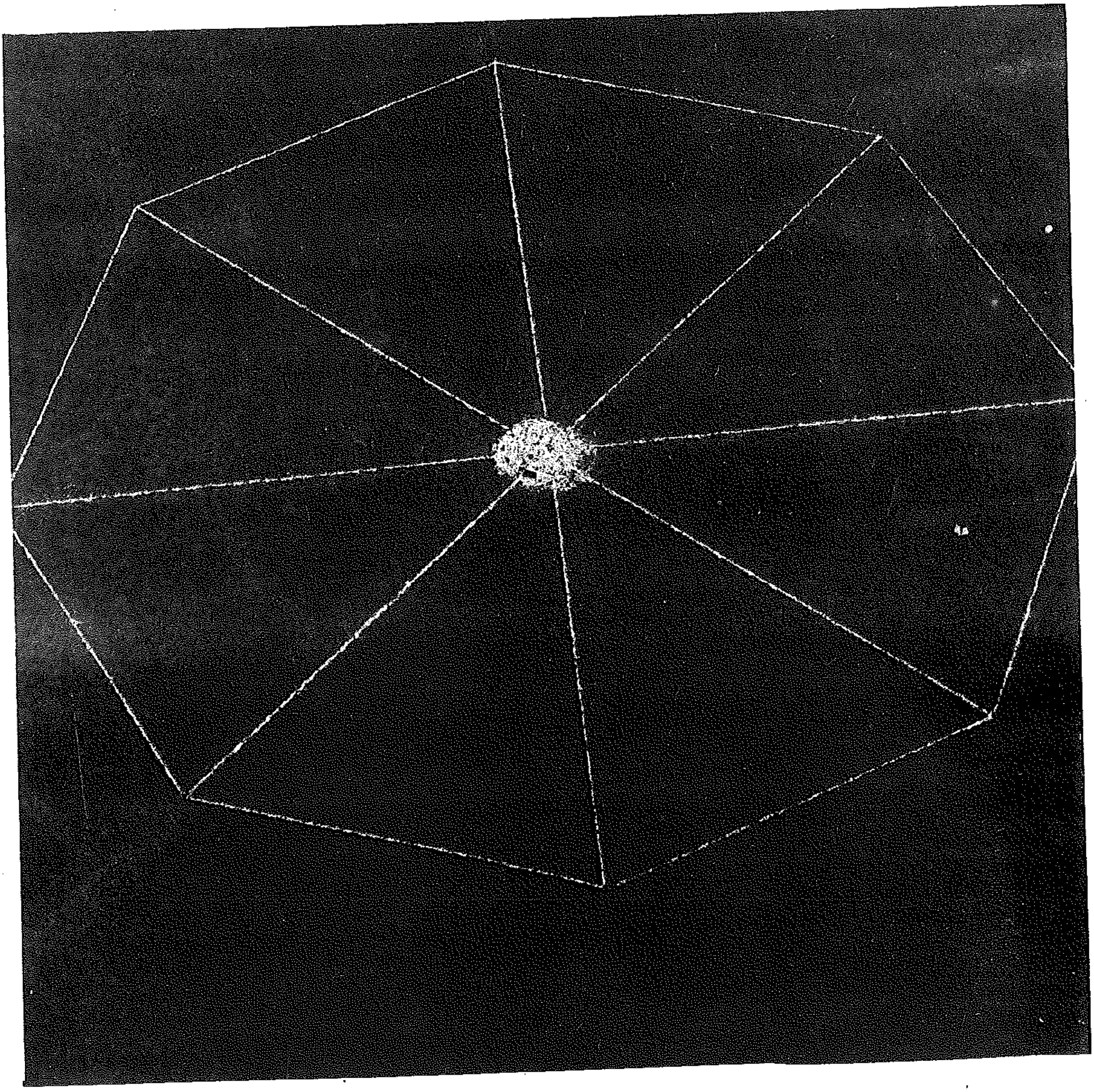

Figure 13. Print of PC-1000 plate of simulated 85 foot dish photographed from helicopter at 500 foot slant range. Targets are affixed to stakes at center and at corners of octogonal pattern.

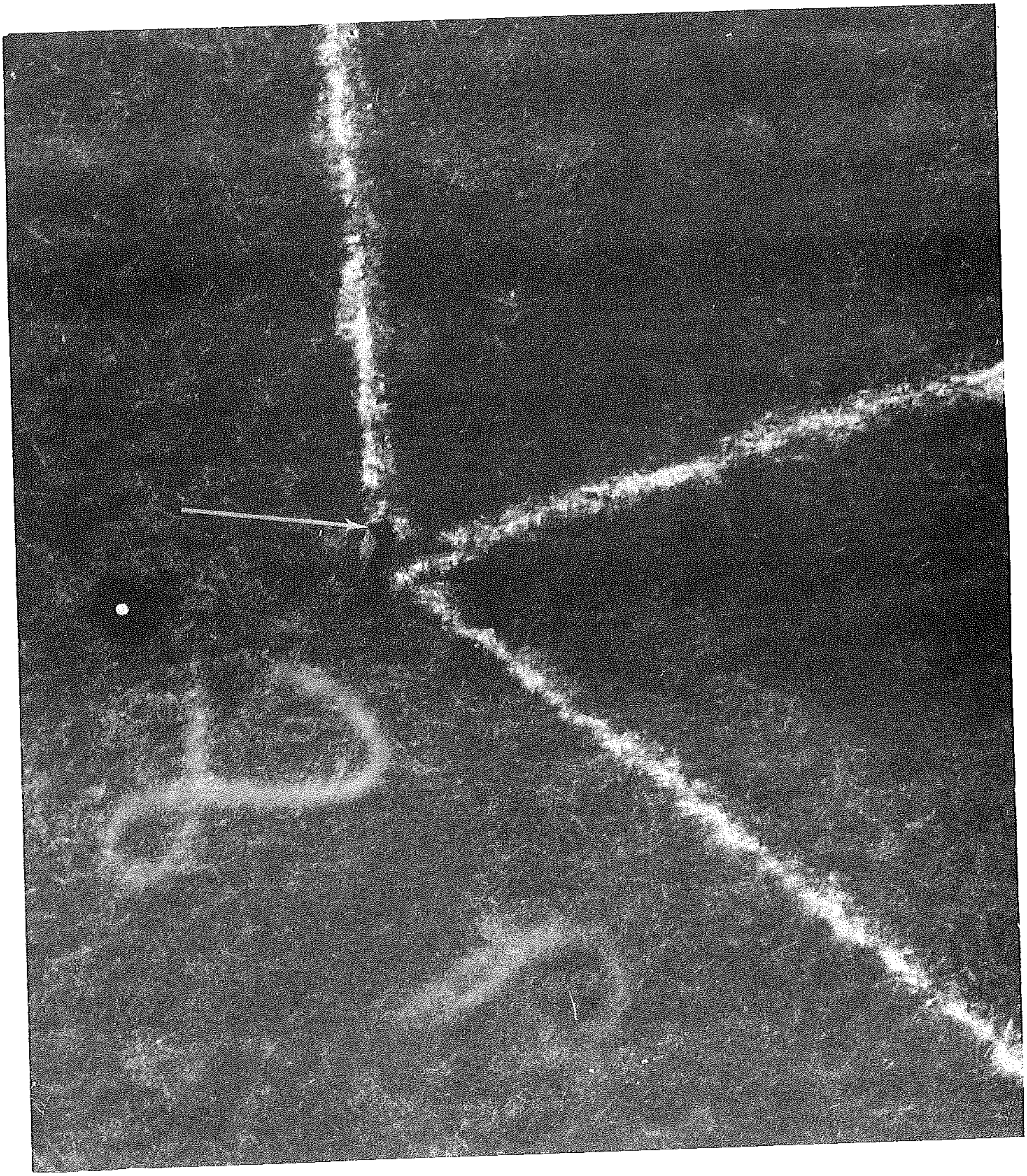

Figure 14. Enlargement of area of plate printed in Figure 14 showing target on stake (arrow). Actual diameter of target dot is 6mm; diameter of image on plate is 50 microns. Head of eight penny finishing nail in center of larger target lying on ground to left is clearly discernible on original plate.

setting of better than 5 microns being attained on the best of the plates. The survey of the ground targets was made in order to establish a more meaningful check on the metric characteristics of the helicopter plates. Accordingly, a least squares resection employing seven of the nine targets (two fell outside the plate format) was performed with measurements of the plate reproduced in Figure 13. The root mean square error of the plate measuring residuals turned out to be 5.5 microns. However, since the ground survey was accurate to 0.002 ft. rms and since this is equivalent to 4 microns rms on the plate of the PC-1000 at 500 feet, it follows that the rms plate measuring accuracy must actually be closer to 4 microns, for $\sqrt{4^2 + 4^2} \cong 5.6$. This result indicates that a long focal length camera aboard a helicopter can produce metrically acceptable photography of objects which are only a few hundred focal lengths away. It also indicates that the local turbulence caused by the blades of the helicopter probably does not constitute a serious problem.

The fact that acceptable photography was obtained from a helicopter under rather adverse conditions has convinced us that accurate stereotriangulation of points on antennas is possible with aerial photography. With favorable flying weather and with more experience on the part of helicopter and camera operators in working closely together, it seems likely that long focal length aerial photographs of excellent quality can consistently be achieved at the relatively close ranges required for antenna calibration. Though limited to a few hours, our helicopter tests also taught us valuable lessons in maneuvering the helicopter into proper position and indicated the desirability of an auxiliary viewfinder for the helicopter operator.

## 13. ACCURACIES TO BE EXPECTED FROM OPTIMAL THREE STATION SOLUTION

The feasibility of three dimensional positioning of exposure stations by means of helicopters makes a three station analytical stereotriangulation particularly attractive. It can be shown that if the three exposure stations were equidistant from the vertex of the reflectors and were such that axes of the cameras nominally intersected near the vertex at right angles, the standard deviations $\sigma_x$, $\sigma_y$, $\sigma_z$ of the triangulated points would be essentially equal ($\sigma_x = \sigma_y = \sigma_z$) and the triangulated coordinates would be essentially independent ($\sigma_{xy} = \sigma_{yz} = \sigma_{zx} = 0$). More specifically if the three cameras were of 1000mm focal length and were located six diameters (6D) from the reflector, and if 5 micron rms plate measuring accuracy were achieved, one could expect accuracies of

$$\frac{\sigma_x}{D} = \frac{\sigma_y}{D} = \frac{\sigma_z}{D} = \frac{1}{48,000}$$

from the three station analytical stereotriangulation. With three micron plate measuring accuracy, the proportional accuracies would be increased to 1 part in 80,000. Accordingly, theoretical inducements to the accomplishment of successful aerial operations cannot be said to be lacking.

## 14. POSSIBILITY OF DYNAMIC CALIBRATIONS

Analytical stereotriangulation can be employed for dynamic calibration of antennas as well as static calibration. The primary operational difference is that the antenna must be observed simultaneously by two or more different cameras at suitable locations; as far as data reduction is concerned there are no differences between static and dynamic calibrations. Simultaneity of exposures is best obtained by the use of a flashbulb or electronic flash lamp to illuminate the reflector at the desired instant (perhaps the instant of maximum acceleration). An exposure of a fraction of a millisecond is possible with electronic flash lamps. Inasmuch as a flash producing 10 lumen seconds per square foot of the dish will yield an adequate exposure of 103F emulsion with the camera lens stopped down to f/20, a single 500 watt second flash lamp having an efficiency of 40 lumens per watt would adequately illumine an 85 foot dish if mounted just below the feed in a properly oriented 150° reflector. A dynamic test would best be conducted at night, for this would obviate the need for accurately synchronized shutters and would minimize difficulties with atmospheric shimmer. When shimmer is insignificant, we see no reason why a properly executed dynamic calibration should be any less accurate than a static calibration. In principle, dynamic tests could even be conducted with airborne cameras, although this would entail the simultaneous use and coordination of two or more helicopters (or other suitable vehicles), an operation which might prove to be rather difficult.

## 15. THE NOISE TRACE

In both aerial tests and dynamic tests the shortness of the exposures is such that the possible effects of shimmer induced by turbulence are naturally of some concern. In static tests this is no problem because exposures can be prolonged to average out the influence of shimmer. While we presently have no extensive information on the probable magnitude of errors induced by shimmer, we have given the matter considerable thought and have come up with a partial solution to the problem (if indeed it really is a problem). The solution does nothing to eliminate the effects of shimmer, but it does make possible the accurate estimation of the rms magnitude of image displacements induced by shimmer. The solution involves the use of a steel tape on which a sizeable number of target dots are placed at uniform intervals. A suitable tape for use in conjunction with an 85 foot dish would be about 75 feet in length, about 1/4 inch wide and would have circular targets 1/4 inch in diameter affixed at one foot intervals. A uniformity of spacing of ±0.002 inch both longitudinally and laterally would not be difficult to achieve if the targets were affixed instrumentally with the aid of a special jig attached to the table of a comparator or milling machine. The tape would be stretched across the dish under about 20 pounds of tension so that only a slight sag would be present. It follows that the centers of the images of the targets on the tape should ideally lie on a perfectly smooth curve. A significant degree of shimmer would displace the images in a random fashion, thereby increasing the dispersion of the images about a smooth curve. Thus a dispersion significantly greater than the 2 to 3 microns attributable to normal plate measuring errors would indicate the presence of a sensible degree of shimmer. The dispersion of the plate coordinates about a smooth curve could be estimated either by the variate difference method or else by fitting least squares polynomials to the x and y plate measurements with the target number serving as the independent variable. The standard deviations resulting from such an analysis would reflect the combined effect of all sources of random error (setting errors, emulsion instability, shimmer, etc.) and thus could be used to characterize the accuracies of the plate coordinates of similar targets affixed to the reflector. Because this concept serves the same function as the "noise trace" on stellar plates (Appendix A, Reference [5]), we shall also refer to the series of images of targets on the tape as the "noise trace".

To test the concept of the noise trace as applied to antenna calibration, we prepared a 25 foot tape on which targets were mounted every 3 inches throughout the length of the tape. The targets were mounted by hand and were found through spot checks to have a uniformity of spacing of about ±0.015 inches longitudinally and about ±0.005 inches laterally. The tape was stretched horizontally between a pair of supports about four feet above the ground and was photographed by a PC-1000 located 500 feet away. The exposure was 1/100 second at f/45 through a K-2 filter. Figure 15 is an enlargement of the section of the plate containing the noise trace. The measured plate coordinates of the 100 images of the noise trace were subjected to a variate difference analysis. The resulting estimated standard deviations were $\sigma_x = 3.6$ microns, $\sigma_y = 2.6$ microns. The greater standard deviation of x can be explained by the longitudinal errors in the placement of the targets; these alone are equivalent to 2.5 microns rms in terms of plate coordinates. Because the standard deviations of the coordinates of the targets were judged to be wholly consistent with normal plate measuring accuracies for well defined points, we concluded that shimmer was insignificant in this case. It should be pointed out that the photograph was made late in the afternoon on a cool day (60°F) across a grassy field; all of these conditions are normally favorable to good seeing. Thus while the results do serve to demonstrate that shimmer can be entirely negligible under favorable conditions, they do not guarantee that this will always be the case. Hence we feel it important that the noise trace be incorporated routinely into the reduction of each individual plate. In view of this and in view of the success of our preliminary experiment with

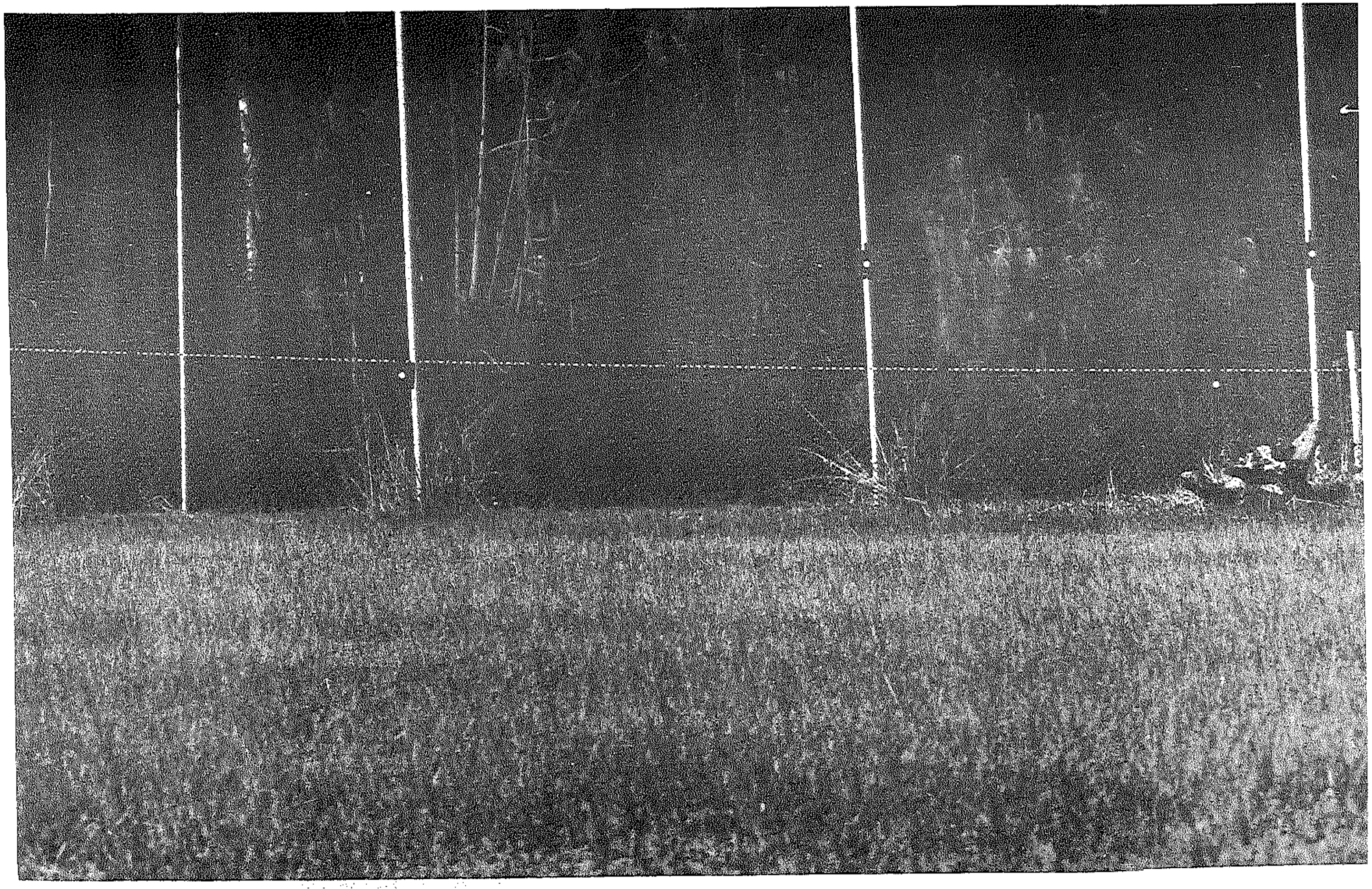

Figure 15. Enlargement (5x) of portion of PC-1000 plate showing experimental "noise trace" generated by series of 9 mm circular targets mounted 3 inches apart throughout length of 25 foot steel tape. Camera was 500 feet away. Feasibility of "noise trace" technique was established through analysis of plate measurements.

the noise trace, we are preparing another tape on which the targets will be placed with considerably greater precision. This tape will be employed in further experimentation as well as in possible future calibrations of antennas.

Since the relative distances between targets on the tape used to generate the noise trace are precisely known, the noise trace can also be exploited as a check on the accuracies of the analytical stereotriangulation. This entails merely carrying a number of the points of the noise trace through the triangulation and comparing the distances between triangulated points with the known distances on the tape. Such a check on accuracies would be far more meaningful and convincing to a nonphotogrammetrist than the closures of the adjustment or the character of the measuring residuals, for these require a measure of expert interpretation to be meaningful. Thus the noise trace is valuable not only for its primary purpose of accurately estimating the standard deviations of the plate coordinates but also for the purpose of providing a built-in check on the accuracies of the photogrammetric triangulation.

## 16. POTENTIALITIES OF CALIBRATION BY ANALYTICAL STEREOTRIANGULATION

While emphasis has been placed on the triangulation of points on the surface of the reflector, it should be mentioned that points (preferably targets) off the reflector can be triangulated as well. Of particular interest would be points on the feed so that "sag" of the feed structure might be determined. For certain purposes it might also be desirable to triangulate some points located on the stationary part of the structure. On the reflector itself it may be worthwhile to employ a high density of targets in special areas in order to obtain particularly detailed error contours — for instance, where the struts supporting the feed intersect the reflector. Inasmuch as the vertex of the best fitting paraboloid is determined mathematically (no assumptions are made concerning the location of the vertex), a target placed at the precise point which is regarded as the vertex (the vertex of construction) would permit the error in the position of the vertex to be determined. These examples are cited to impart some appreciation of the flexibility and capabilities of the photogrammetric method. Undoubtedly an experienced antenna engineer could pose additional special problems of interest which might be amenable to a photogrammetric solution.

Precisely how far it is practical to push the accuracies of calibration by analytical stereotriangulation remains to be determined. As far as plate measuring accuracies are concerned, a standard deviation of about 2 microns may be regarded as the practical limit within today's technology (though perhaps significant improvement beyond this could be accomplished through measurement of diffraction images generated by a suitable wire diffraction grating mounted in front of the lens). If a standard deviation of 2 microns were attained, accuracies of calibration of the order of 1 part in 120,000 of the diameter of the reflector would result from an optimal three station triangulation. To obtain significantly higher accuracies it would be necessary to photograph the reflector by sections. If the need arose and if developmental resources were adequate, a twelve plate analytical stereotriangulation employing sectional photographs could be formulated to produce an accuracy of perhaps 1 part in 300,000. Somewhere around this figure is probably the practical limit of the method if only from an economical standpoint.

## 17. CONCLUSIONS

The method of antenna calibration described in this report is the result of a successful, year-long, in-house program of analysis, engineering, computer programming, and experimentation intended to develop a photogrammetric method for the precise mensuration of large structures. We believe ours is the first successful application of long focal length, narrow-angle cameras to "close-in" terrestrial photogrammetry. From a photogrammetrist's point of view this is perhaps the result of greatest significance. To the antenna engineer, the significant matter is the existence of an extremely versatile method of calibration which is capable of accuracies of upwards of 1 part in 40,000 of the diameter of the antenna. Now that the fundamentals of measuring techniques, instrumentation, data reduction, and data acquisition have been worked out, our continuing efforts will be directed towards further refining the method until its practical limitations are approached. In its present stage of development the method is sufficiently advanced to permit Instrument Corporation of Florida to offer its services of structural calibration to interested agencies.

## 18. REFERENCES

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