Calculations on the Parabolic Surface.

June 20, 1937

30' dia mirror

Length of Parabolic Curve $a = 20$, $4a = 80$

$s = \log^{-1} 3.56634$ $4a^2 = 1600$

$2s = 2.3026 \cdot 4a^2 = 3684.24$ $p = \log_{10} 2a = 1.60206$

x	$\sqrt{x^2+4a^2}$	$\log_{10}(x+\sqrt{x^2+4a^2})$	$x\sqrt{x^2+4a^2}$
		f	u
1	40.012	1.61291	40.012
2	40.050	1.62377	80.100
3	40.113	1.63461	120.339
4	40.199	1.64541	160.796
5	40.311	1.65620	201.555
6	40.447	1.66696	242.682
7	40.608	1.67768	284.256
8	40.792	1.68835	326.336
9	41.000	1.69897	369.000
10	41.235	1.70957	412.35
11	41.485	1.72004	456.34
12	41.761	1.73047	501.13
13	42.060	1.74084	546.78
14	42.380	1.75113	593.32
15	42.720	1.76133	640.80
16	43.080	1.77144	689.28
17	43.463	1.78149	738.87

x	$q-p$	$n \cdot s$	$\frac{t}{\bar{x}+u}$	$\frac{v}{4a}$ = burst	Differences −	Differences +
1	.01085	39.174	79.956	1.000		−
2	.02171	79.983	160.083	2.001		.001
3	.03255	119.92	240.26	3.002		.001
4	.04335	159.71	320.51	4.005		.003
5	.05414	199.46	401.02	5.011		.006
6	.06490	239.08	481.76	6.022		.009
7	.07562	278.60	562.83	7.035		.013
8	.08629	317.91	644.25	8.053		.018
9	.09691	357.03	726.03	9.076		.023
10	.10751	396.09	808.44	10.104		.028
11	.11798	434.66	891.00	11.138		.034
12	.12841	473.09	974.22	12.178		.040
13	.13878	511.29	1058.07	13.225		.047
14	.14907	549.20	1142.52	14.280		.055
15	.15927	586.81	1227.61	15.343		.063
16	.16938	624.03	1313.31	16.416		.073
17	.17943	661.04	1399.91	17.499		.083

30' dia minor

Ordinates in a Parabola

$y = \frac{x^2}{4a}$ where $a = 20$

X	Y
1	.0125
2	.0500
3	.1125
4	.2000
5	.3125
6	.4500
7	.6125
8	.8000
9	1.0125
10	1.2500
11	1.5125
12	1.8000
13	2.1125
14	2.4500
15	2.8125
16	3.2000
7.25 =	.65703
15.67	3.070

30' dia. Mirror

Outside Segment

Use 36 segments of 10° each = 2θ

θ = 5°

sin θ = = log⁻¹ 8.94030-10

cos θ = = log⁻¹ 9.99834-10

tan θ = = log⁻¹ 8.94195-10

(1) $a = (c+e)\sin\theta$

6th (2) $b = d\sin\theta = .090552 = \log^{-1} 8.95690$

2nd (3) $x = 3.833 - y = 2.7983 = \log^{-1} .44690$

5th (4) $f = 10 - a = 9.66462$

(5) $f + 2g = 10/\cos 2\theta$

3rd (6) $d = y/\cos\theta = 1.03895 = \log^{-1} .01660$

9th (7) $e = 3.833/\cos\theta - c = 1.1243$

7th (8) $g = a - b = .244828$

(9) $x = y + 10\tan 2\theta$

from (3) & (9)

1st (10) $y = \frac{1}{2}(3.833 - 10\tan 2\theta) = 1.0350 = \log^{-1} .01494$

sin 2θ = = log⁻¹

cos 2θ = = log⁻¹

tan 2θ = .17633 = log⁻¹

from (1) & (7) = log⁻¹ 9.52553-10

4th (11) $a/\sin\theta = 3.833/\cos\theta$, $a = 3.833\tan\theta = .33538$

8th (12) $c = d + 2f\sin\theta = 2.72365 = \log^{-1} .43515$

Trial sheet

Outside radius $r = \frac{36c}{2\pi} = \log^{-1} 1.19327 = 15.605$

$r_1 = .605(1.073) + 15.343 = 15.992$

$r_2 = 15.992 - 9.665 = 6.327$

Inside radius $= \frac{(6.327 - 6.022)}{1.013} + 6 = 6.30109$

arc at this radius $= \frac{6.30109 \cdot 2\pi}{36} = \log^{-1} 1.0995 = .04120$

clearance $= 1.0995 - 1.0390 = .0605' = .726''$

Assume $x = 2'9\frac{1}{4}''$, $y = 1'\frac{3}{4}''$

$c = 2.69615$ $\quad d = 1.06645$

$= \log .43075$ $\quad r = 15'5\frac{3}{8}''$

Outside radius $r = \frac{36c}{2\pi} = 15.448 = \log^{-1} 1.18887$

$r_1 = .448(1.073) + 15.343 = 15.824$

$r_2 = 15.824 - 9.665 = 6.159$

Inside radius $= \frac{(6.159 - 6.022)}{1.013} + 6 = 6.13524'$

$= 6'1\frac{5}{8}''$

arc at this radius $= \frac{6.13524 \cdot 2\pi}{36} = \log^{-1} .02971 = 1.0706$

clearance $= 1.0706 - 1.0665 = .0041' = .049''$

~~Assume x = 2' 8 7/8" , y = 1' 1 1/8"~~

~~$C = \log^{-1}.42569 = 2.66493$; $d = 1.09770$~~

~~Outside radius $r = \frac{360}{2\pi} = \log^{-1}.18281 = 15.269$~~

~~$r_1 = .269(1.073) + 15.393 = 15.632$~~

~~$r_2 = 15.632 - 9.665 = 5.967$~~

~~Inside radius $= \frac{(5.967 - 5.011)}{1.009} + 5 = 5.947475$~~

~~arc at this radius $= \frac{5.9475 \cdot 2\pi}{36} = \log^{-1} 1.01620 = 1.0380$~~

~~Clearance $= 1.0380 - 1.0350 = .003' = .036"$~~

Cutter width $= (c+d)/2 = 1.8813'$

center radius $r_c = 15.824 - \frac{9.665}{2} = 10.991$

curvature radius $= \frac{(10.991 - 10.104)}{1.034} + 10 = 10.857$

arc at this radius $= \frac{10.857 \cdot 2\pi}{36} = 1.8949 = \log^{-1}.27759$

Clearance $= 1.8949 - 1.8813 = .0136' = .163" = 5/32"$ approx

these values are less 1" initial overlap.

Inside Segment

Use 9 segments of 40° each

$r_2 = 6.159' = 6' 1\ 29/32''$

Outside radius = 6.13524'

angle = $\frac{6.135}{6.159} \times \frac{360}{9} = 39.844° = 39° 50.6' = \theta$

$(8 - 4 \cot\theta)/2 = 1.6182' = 1' 7\ 7/16'' = 19\ 7/16''$

$\cos\theta = .768$

$\cot\theta = 1.1984 = \log^{-1}.07860$

$\sin\theta = .64069 = \log^{-1} 9.8067/-10$

for a 1" end overlap increase r_2 to 6.242' = $\log^{-1}.79532$
$= 6' 2\ 7/8''$

$r_2 \sin\theta = \log^{-1}.60203 = 3.9997'$

$(4.0000 - 3.9997) = .0003' = .004''$ approx (insufficient

Advance plate 1" on sheet to obtain 1" edge overlap
This will cause a 1" triangle to be cut off one corner.

make r_3 inside radius 6"

$2 - \cos 39° 50' = 1.232\ r_2$ length req'd.
$= 7.690'$ which is satisfactory for 8.0' sheet.
Outside radius = 6.13524 + .083 = 6.21824
$= 6' 2\ 5/8''$