The Genesis of SIS Mixers – The Legacy of John Tucker in Radio Astronomy

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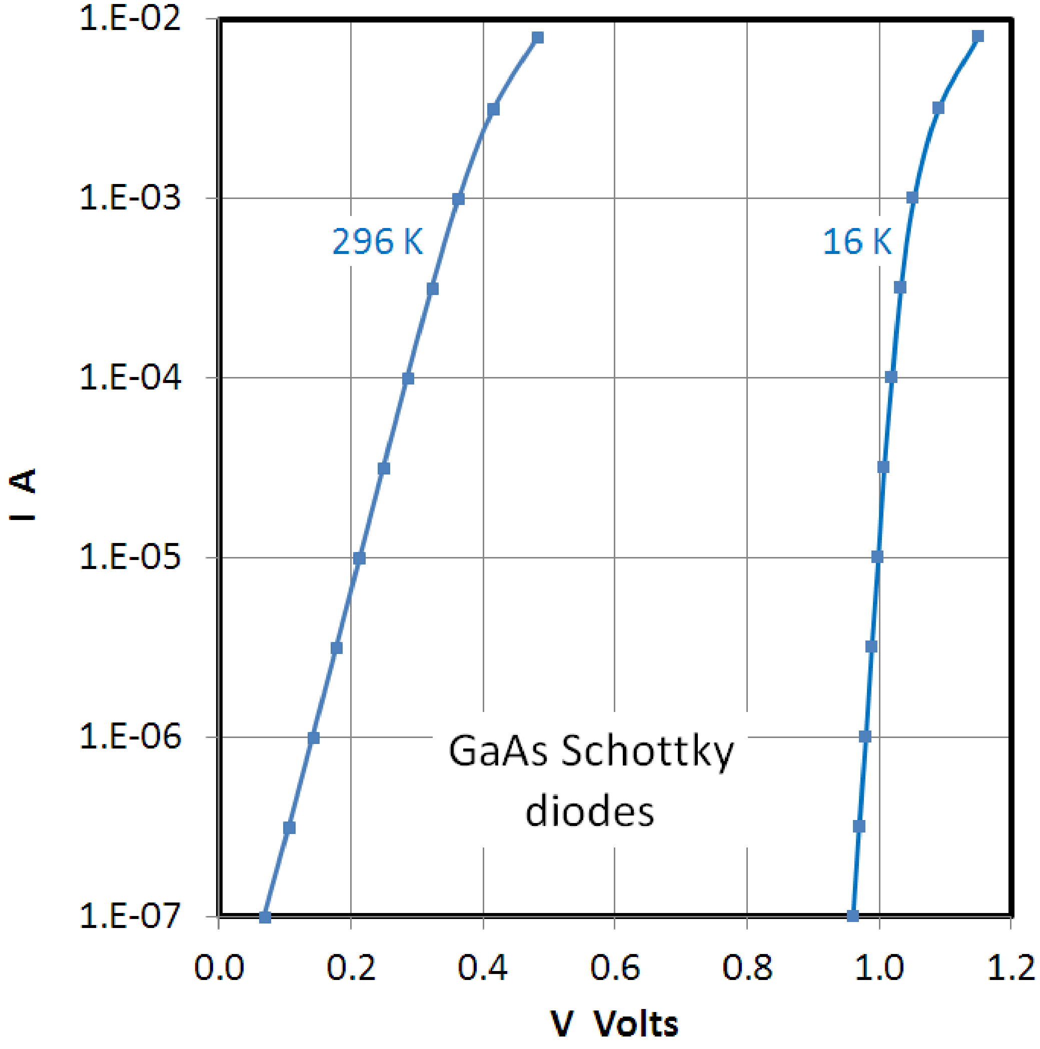
IMS 2015
Phoenix

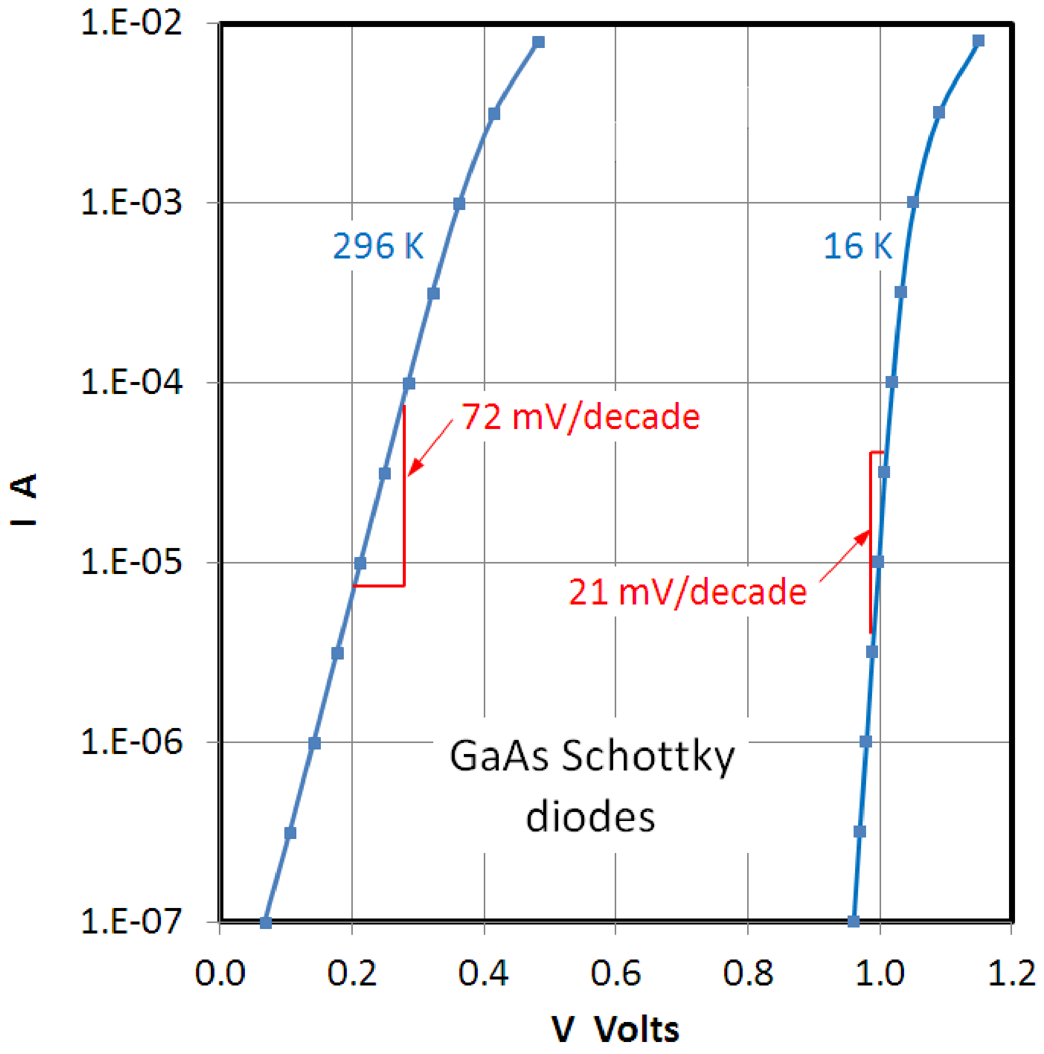
Schottky diode



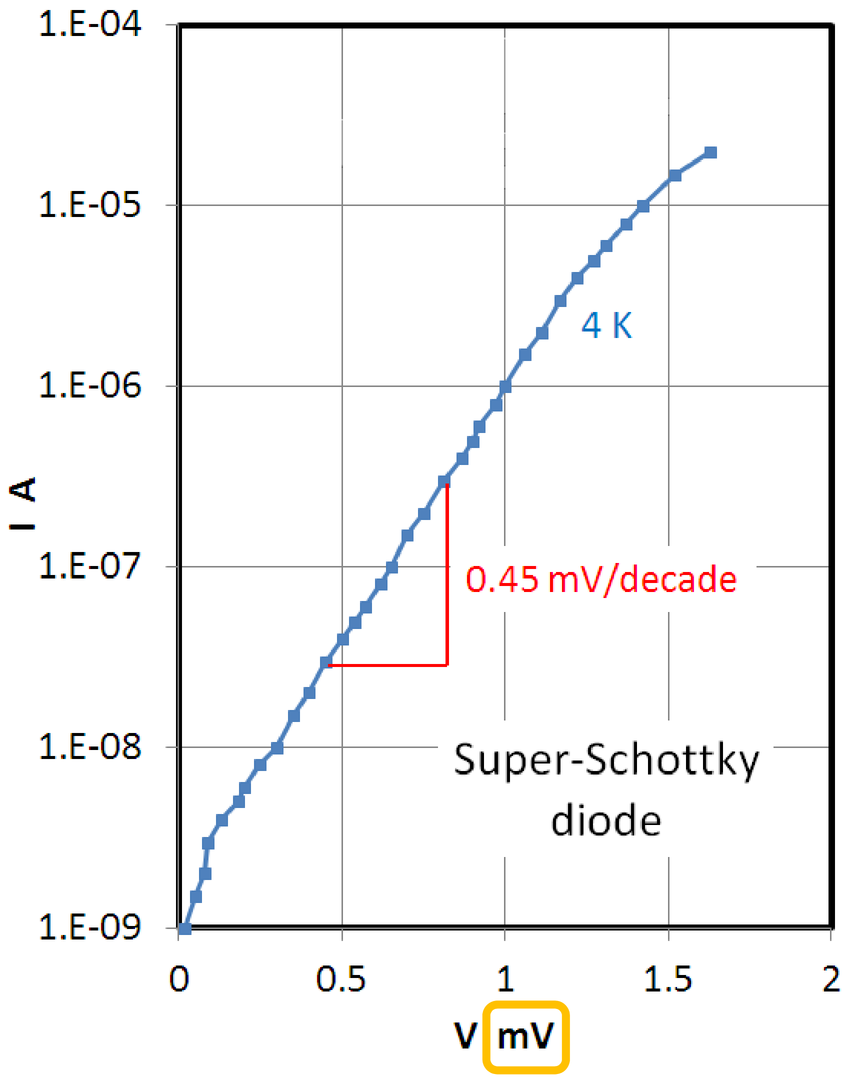
For an exponential diode the incremental conductance:

$$\frac{di}{dv} \approx \left[\frac{e}{\eta kT}\right] i$$

Schottky diode



Super-Schottky diode



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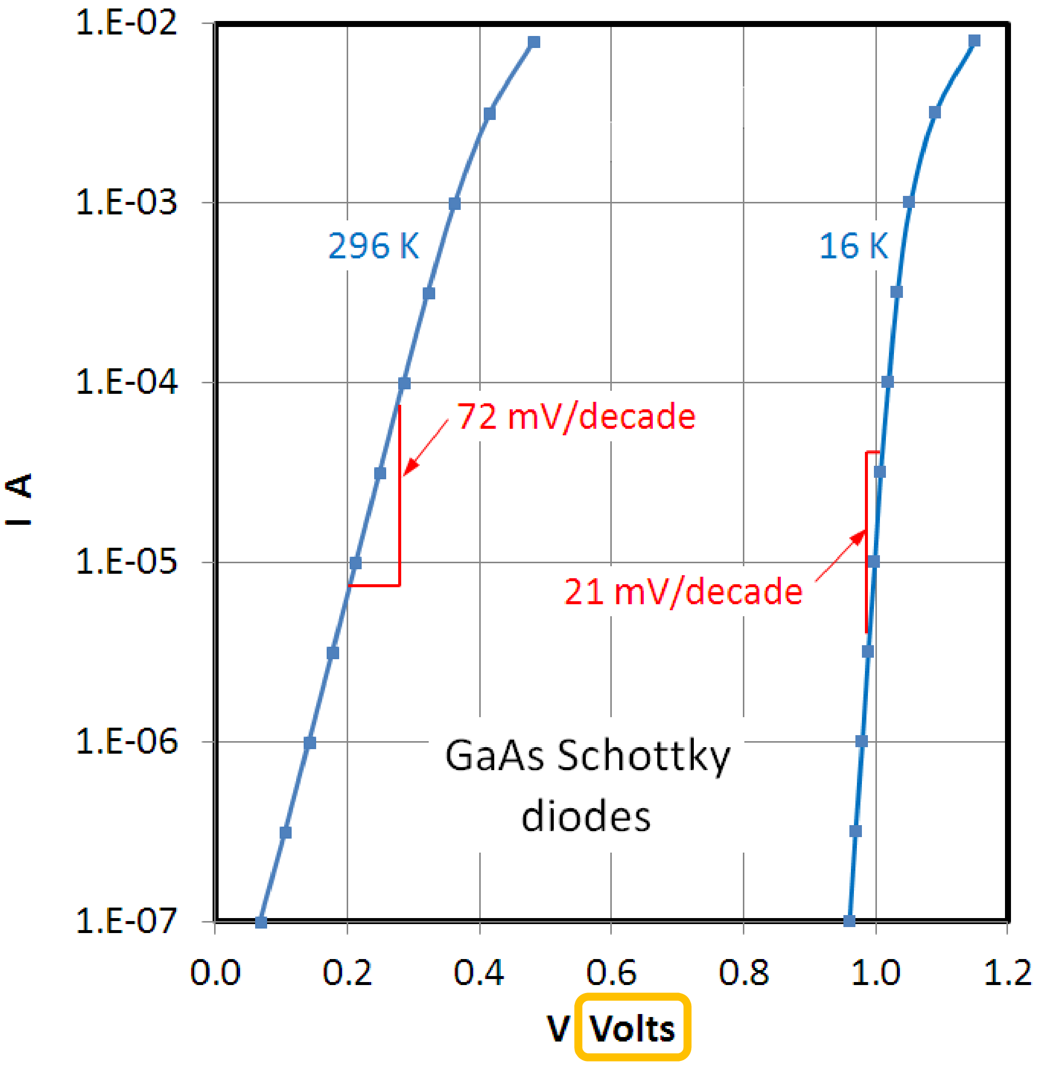
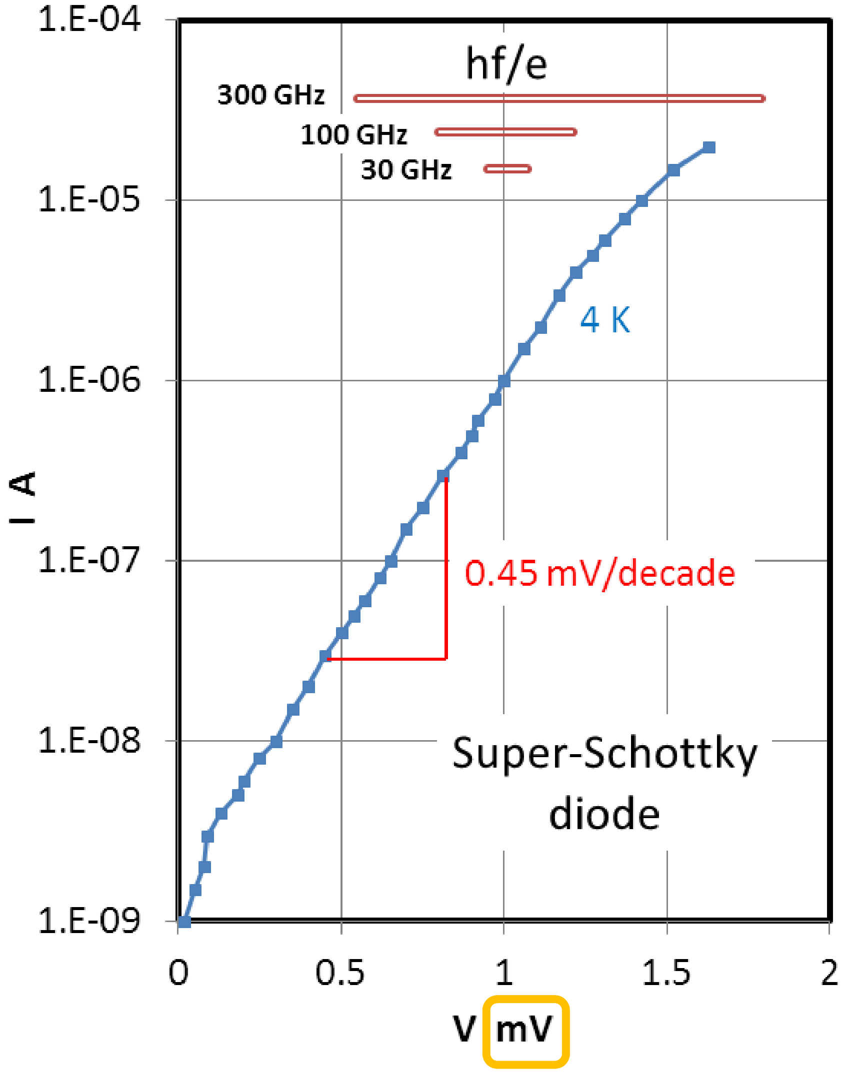


Quantum mixing effects become significant when the diode conductance changes significantly on the scale of the photon voltage hf/e.

Super-Schottky diode



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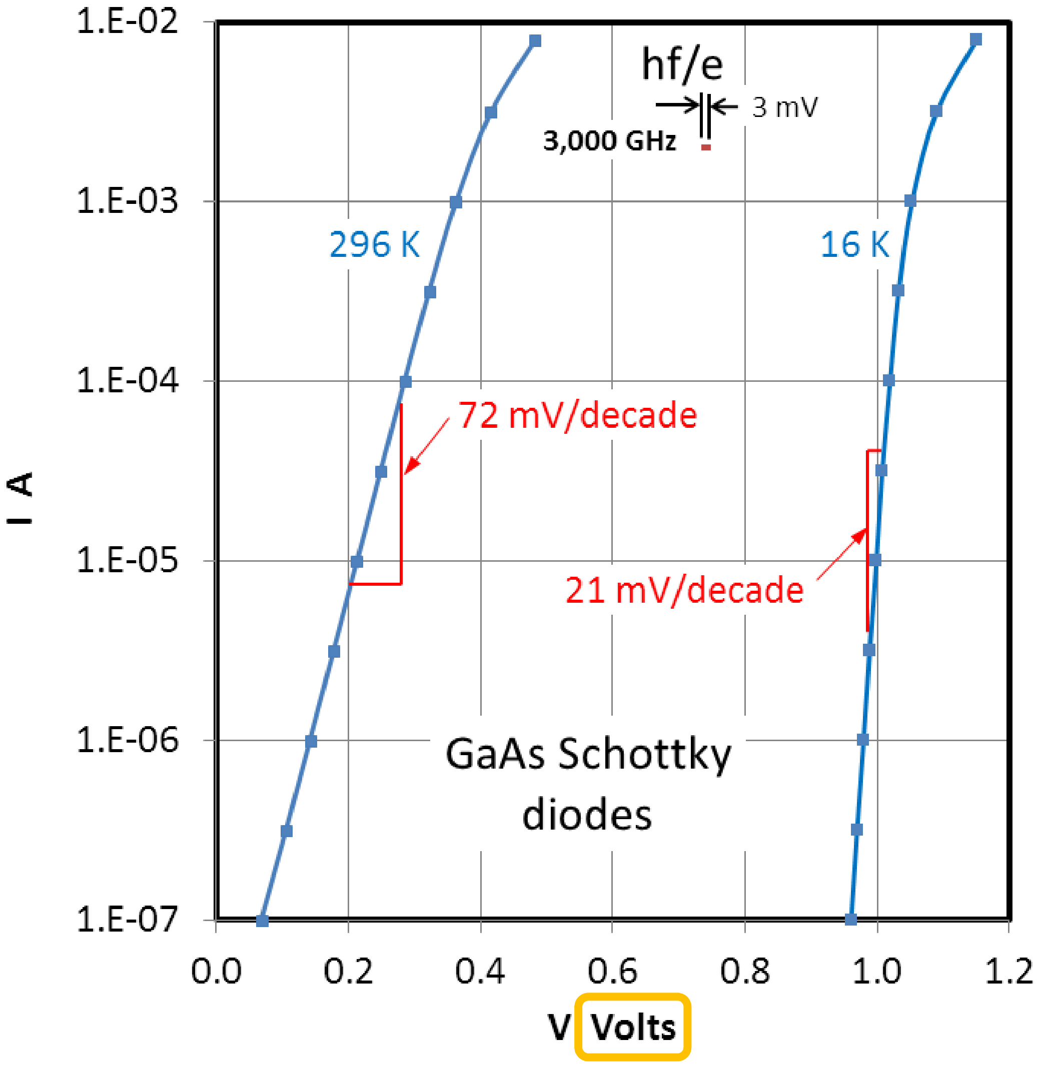
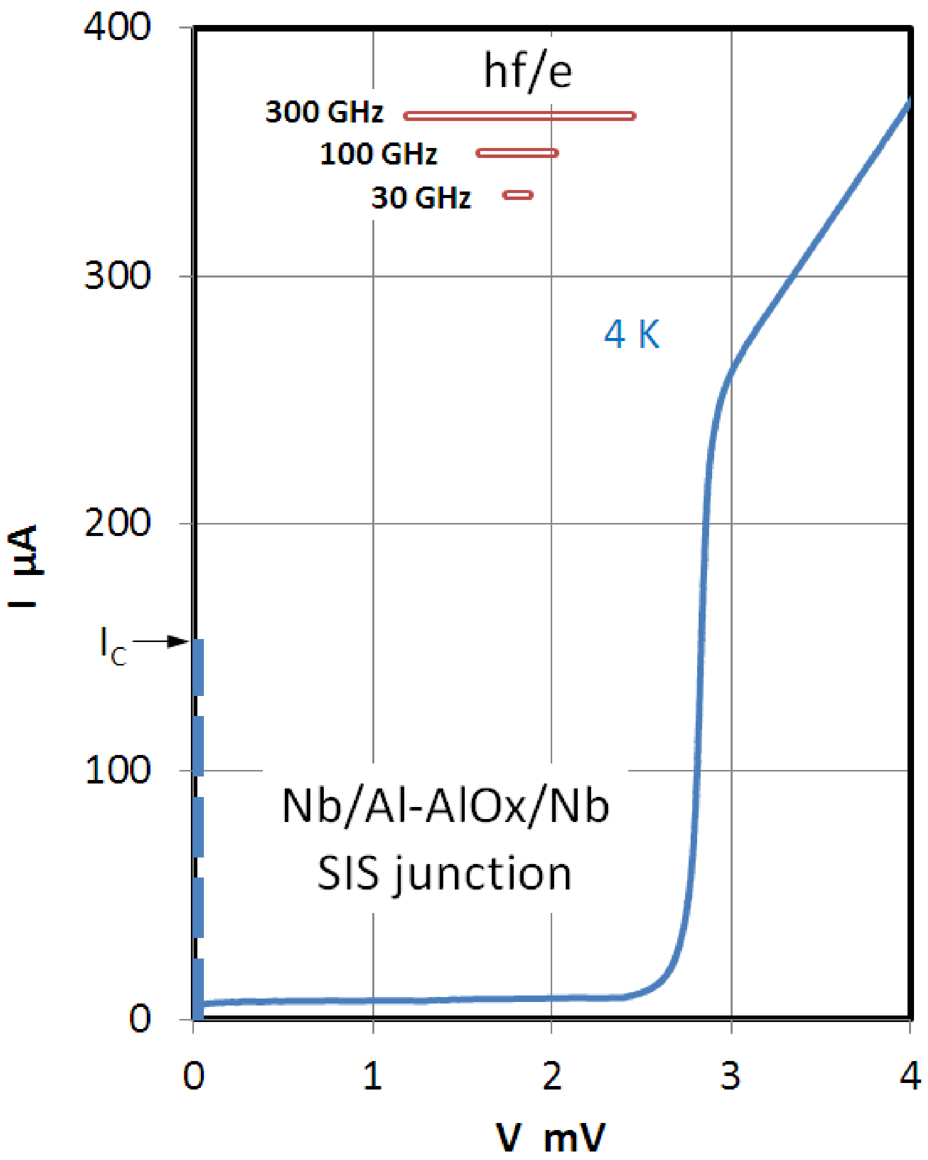


Superconductor-Insulator-Superconductor (SIS) junction



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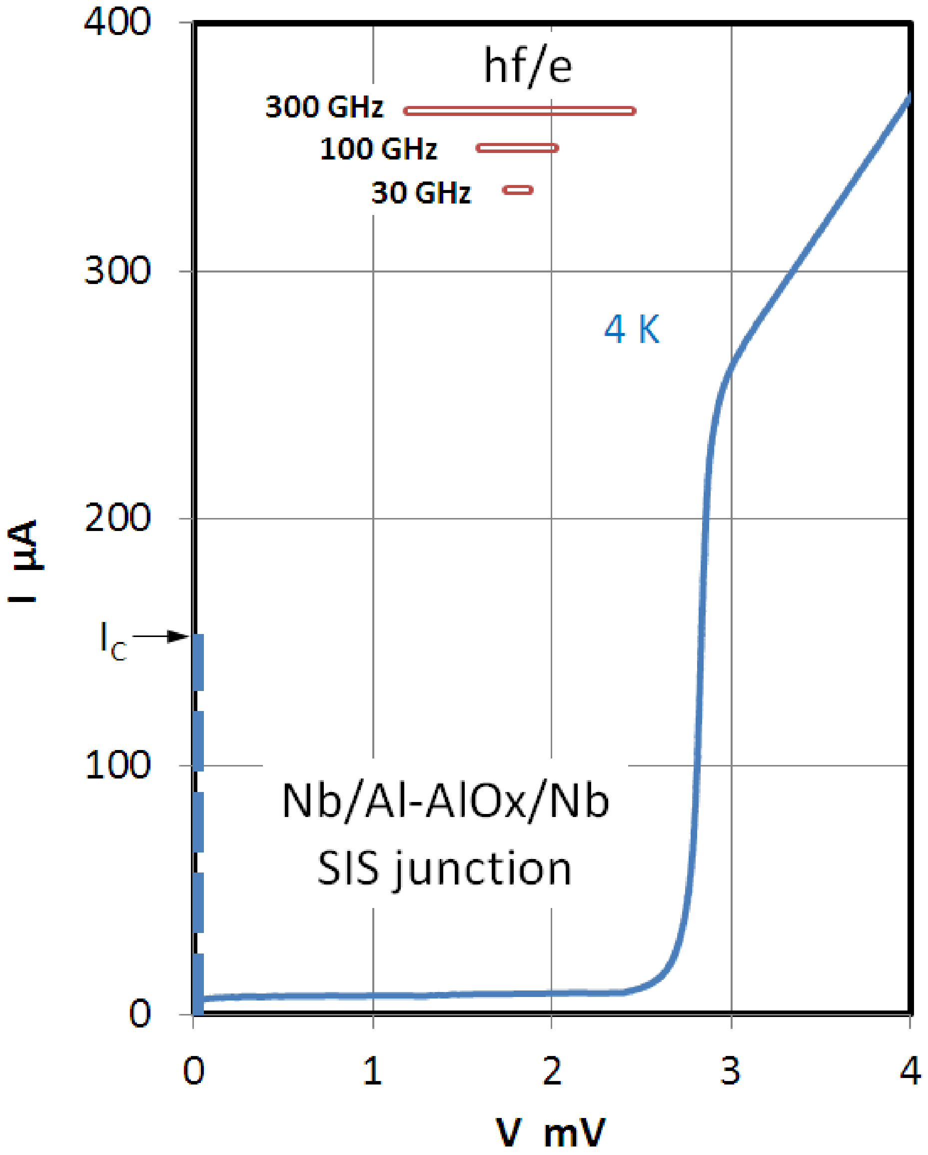
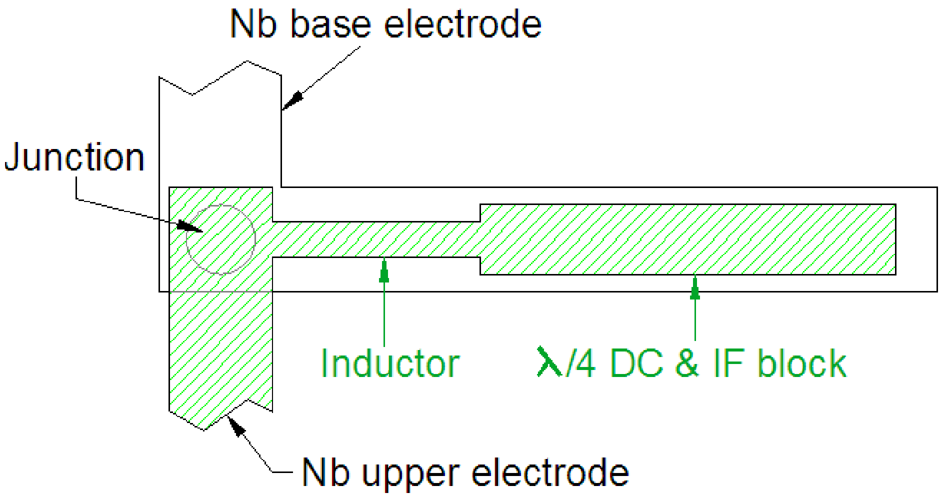
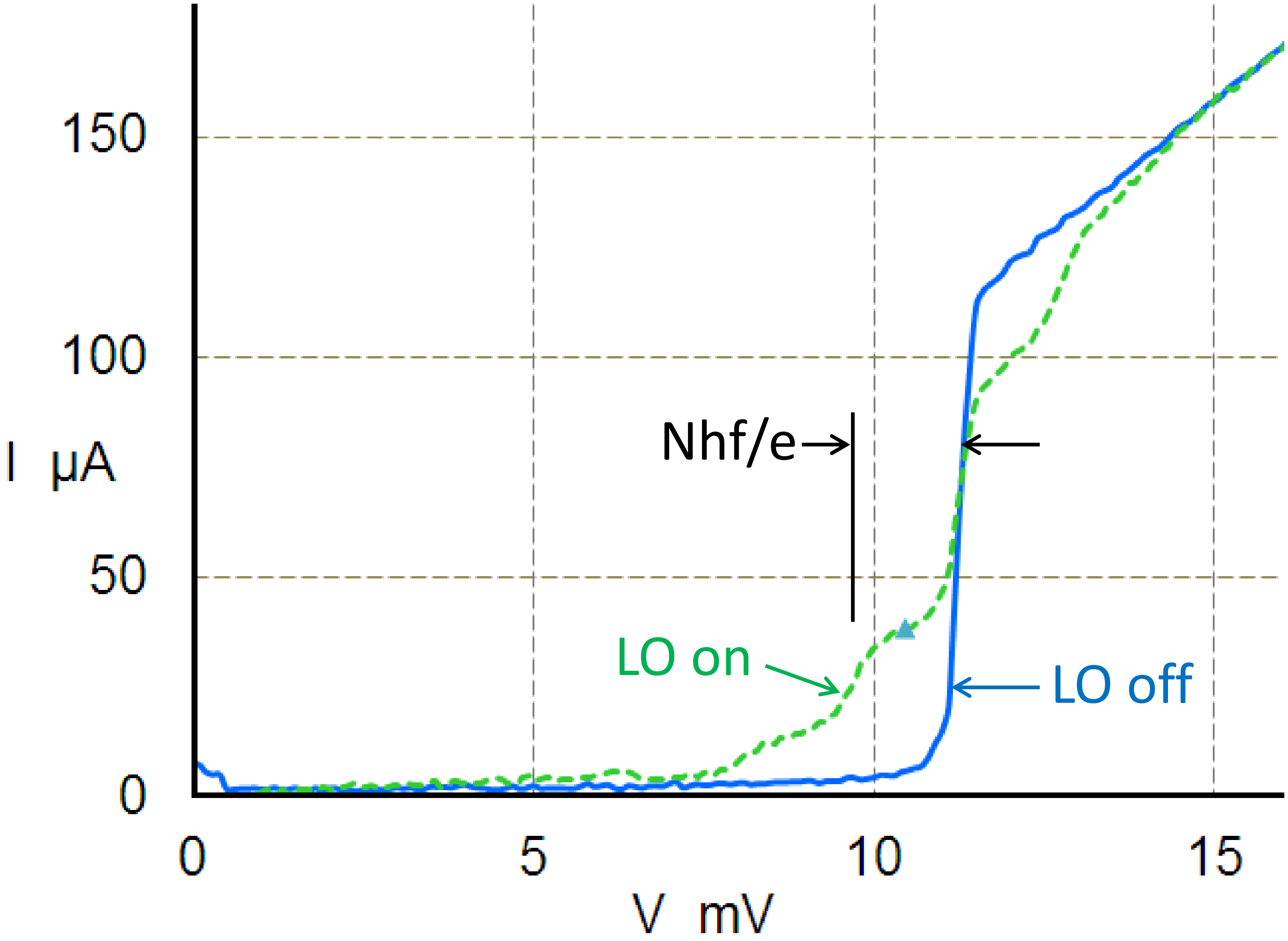




Ref. JIRMM 1990.

Photon assisted tunneling in an SIS mixer at 100 GHz.

(N = 4 junctions in series)



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Classical mixer analysis

Sideband frequencies:

$$\boxed{f_m = f_{IF} + mf_{LO},} \quad \text{where } f_{IF} = f_1 - f_{LO}$$

The small-signal currents and voltages at the sideband frequencies are related by the small-signal *conversion admittance matrix,* [*Y*]*:*

$$[I] = [Y][V]$$

For an exponential diode mixer, the elements of $[Y]$ are given by:

$$Y_{mm'} = G_{m-m'}$$

which are the Fourier coefficients of the diode conductance waveform produced by the LO.

Classical mixer analysis

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Tucker's quantum mixer analysis

In the quantum-limited case $Y_{mm'}$ becomes complex:

$$\boxed{Y_{mm'} = G_{mm'} + jB_{mm'}}$$

where

$$G_{mm'} = \frac{e}{2hf_{m'}} \sum_{n,n'=-\infty}^{\infty} J_n(\alpha) J_{n'}(\alpha) \delta_{m-m',n'-n} \times \{[I_{dc}(V_0 + n'hf_{LO}/e + hf_{m'}/e) - I_{dc}(V_0 + n'hf_{LO}/e)] + [I_{dc}(V_0 + nhf_{LO}/e) - I_{dc}(V_0 + nf_{LO}h/e - hf_{m'}/e)]\}$$

Pumping parameter:
$\alpha = eV_{LO}/hf_{LO}$

$$B_{mm'} = \frac{e}{2hf_{m'}} \sum_{n,n'=-\infty}^{\infty} J_n(\alpha) J_{n'}(\alpha) \delta_{m-m',n'-n} \times \{[I_{KK}(V_0 + n'hf_{LO}/e + hf_{m'}/e) - I_{KK}(V_0 + n'hf_{LO}/e)] - [I_{KK}(V_0 + nhf_{LO}/e) - I_{KK}(V_0 + nhf_{LO}/e - hf_{m'}/e)]\}$$

$I_{KK}(V)$ is the Kramers-Kronig transform of $I_{DC}(V)$

In the low-frequency limit, the equation for $G_{mm'}$ reduces to the classical expression on the previous slide, and $B_{mm'}$ → 0.

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Shot noise in a mixer

The shot noise currents and voltages at the sideband frequencies are related by the *noise current correlation matrix,* $[H]$.

In the classical case:

$$\boxed{H_{mm'} = \langle \delta I_{S,m} \delta I_{S,m'}{}^{*} \rangle / \delta f = 2eI_{m-m'}}$$

where the I_i are the Fourier coefficients of the diode current waveform produced by the LO.

Tucker's theory for the quantum-limited case:

Pumping parameter:
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In the low-frequency limit this equation reduces to the classical expression above.

Non-classical behavior in an SIS mixer at 100 GHz.

ALMA Nb/Al-AlOx/Al SIS mixer at 100 GHz.

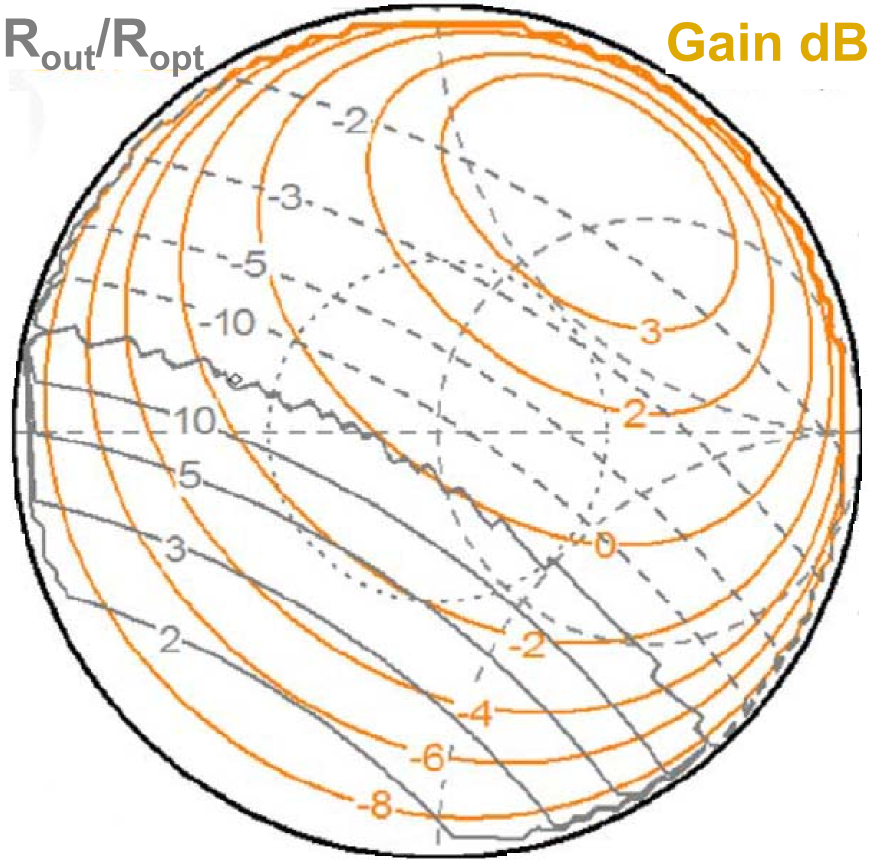
Contours:

——— mixer gain dB.

——— IF output impedance/R_{opt}

– – – – dashed where negative.

………. $|\rho| = 0.4$



Smith chart is **RF source impedance** normalized to $Z_0 = R_{opt}$.

Non-classical behavior in an SIS mixer at 100 GHz.

ALMA Nb/Al-AlOx/Al SIS mixer at 100 GHz.

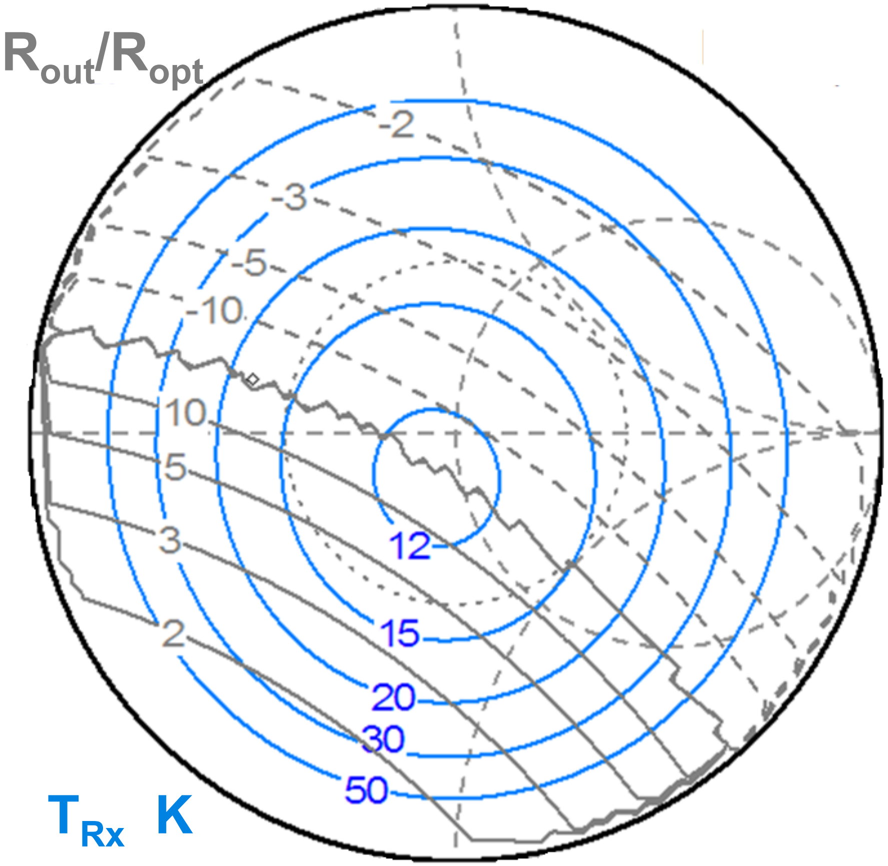
Contours:

——— T_{RX} K (with T_{IF} = 4 K)

——— IF output impedance/R_{opt}

– – – – dashed where negative.

.... $|\rho|$ = 0.4



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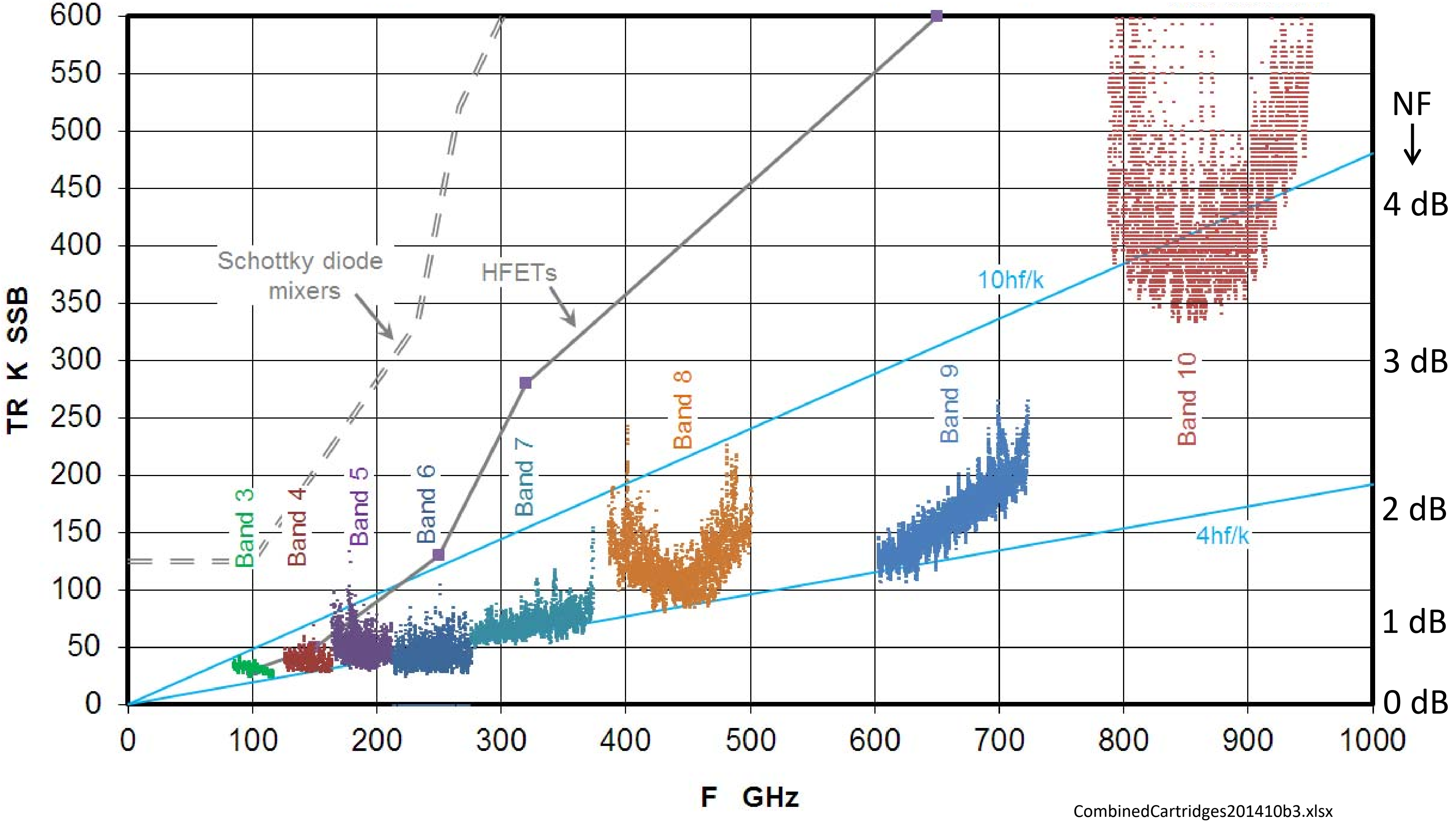
Typical ALMA Receiver SSB Noise Temperatures, 2014



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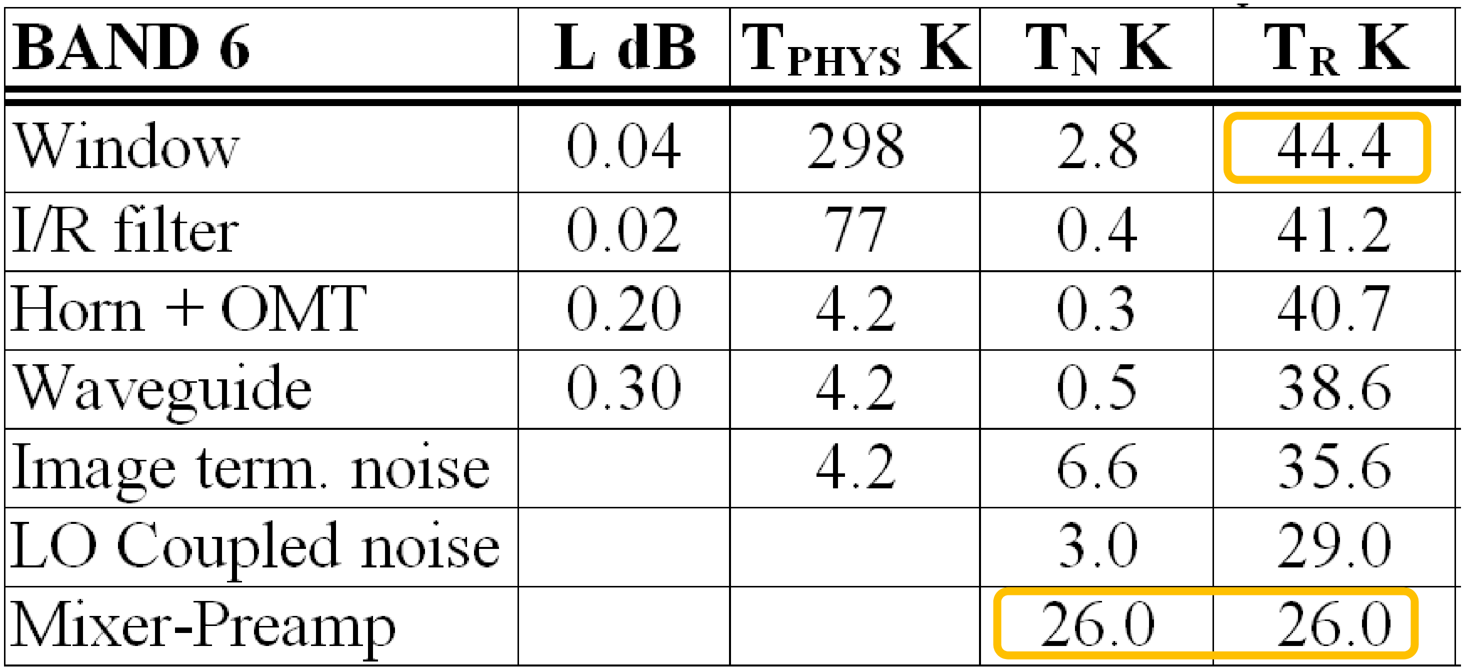
Typical noise breakdown for an ALMA Band-6 (211-275 GHz) SIS receiver



BAND 6	**L dB**	**T_{PHYS} K**	**T_N K**	**T_R K**
Window	0.04	298	2.8	44.4
I/R filter	0.02	77	0.4	41.2
Horn + OMT	0.20	4.2	0.3	40.7
Waveguide	0.30	4.2	0.5	38.6
Image term. noise		4.2	6.6	35.6
LO Coupled noise			3.0	29.0
Mixer-Preamp			26.0	26.0

All noise temperatures are SSB.