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THE VERTICAL DISTRIBUTION OF RADAR FIELD STRENGTH OVER THE SEA UNDER VARIOUS CONDITIONS OF ATMOSPHERIC REFRACTION

By J. A. RAMSAY,
Ministry of Supply

§1. INTRODUCTION

THE experiments to be described in this paper formed part of a more extensive investigation which was undertaken to examine and compare the behaviour of centimetre-wave radar under conditions of normal and anomalous propagation. The wave-lengths studied were 10 cm., 3 cm. and 1·25 cm. No test gear was available to determine the power output and receiver sensitivity of the transmitting and receiving equipment: relative measurements of signal strength were obtained by calibration of the gain control by means of pulsed signal generators at intermediate frequency.

The experiments were carried out at a site in North Wales on the shore of the Conway Estuary, looking out over Beaumaris Bay and the Irish Sea. Various positions were chosen and tested, giving aerial heights of between 20 and 30 feet above mean sea level. The tide range was up to ± 12 feet at springs.

§2. METHOD

The principal method used consisted in raising a corner reflector to known heights above the sea by means of a balloon. This was carried out by a party of men at sea in a small dinghy in radio-telephonic contact with the radar station on shore. The reflector used was in fact four corner reflectors set together in the form of a cup anemometer which rotated slowly in the breeze and gave a regular pulsating signal whose maximum could be easily measured. The balloon was spherical, 10 feet in diameter, of rubberized fabric, and gave no measurable signal. At ranges of 8000 yards or more the dinghy gave no signal.

This improvised method suffered severely from the limitations that—

(*a*) it took some hours to organize the boat party and balloon;

(*b*) it could only be used in very calm weather. In winds of 3 miles per hour or more the balloon was carried down wind to such an extent that the cord made an angle of more than 15° with the vertical. It was necessary to use an outboard motor in the dinghy and to proceed down wind, which of course meant that the range was changing during a series of readings;

(*c*) at 1·25 cm. the signal was seldom of workable strength beyond 17 000 yards.

Observations were also taken on an autogyro at longer ranges, but as it was not possible to detect the autogyro on 1·25 cm. the results of these experiments are not reported.

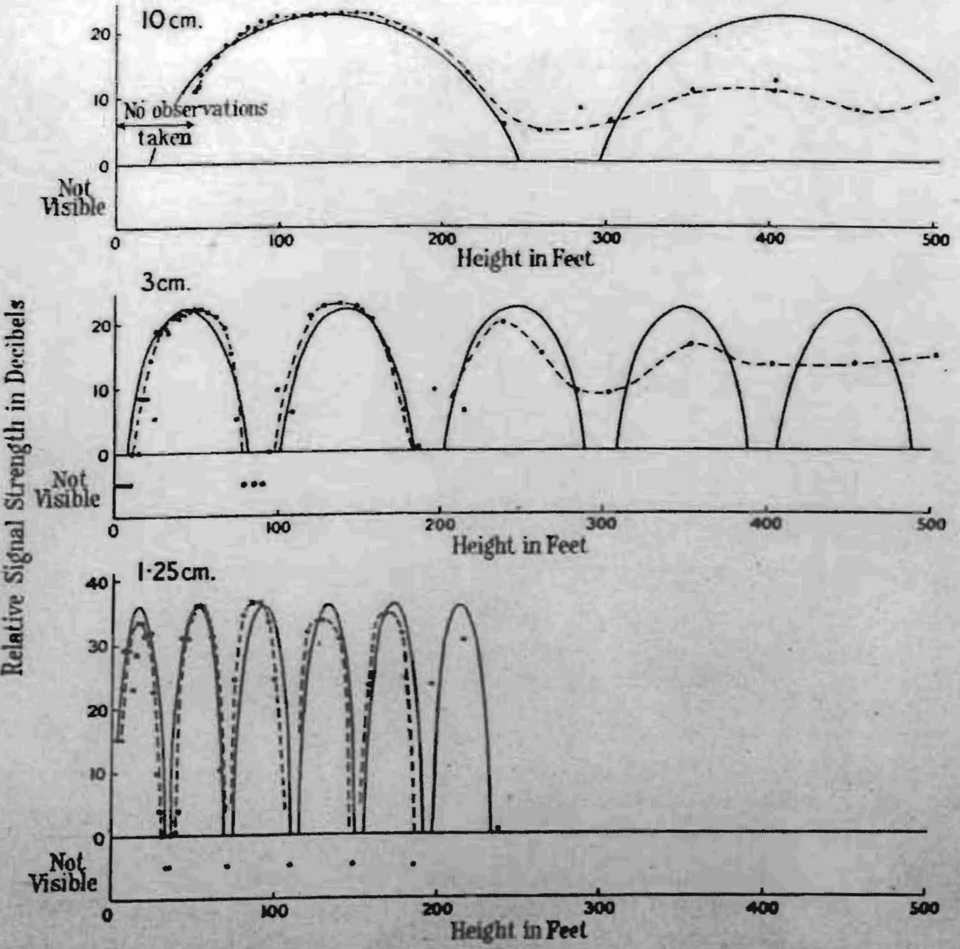

Figure 1. Corner reflector. 3.8.44. 22.00–23.00 h. A.P. very weak.

Range : 6500–7000 yards, adjusted to 6750 yards.
Height of radar : 13·2 ft. to 10·4 ft. for 1·25 cm.
13·5 to 10·7 ft. for 3 cm.
15·0 ft. to 12·2 ft. for 10 cm.

§ 3. RESULTS

The records gave the signal strength in decibels measured on each set entered against the height of the target. The range to the target and the time of day were noted, these being necessary in order that allowance should be made for the change in range and change in height of tide during a series of readings. The allowance thus made were very approximate: a range lying midway between the two extremes was chosen as standard and all the readings were related to this range on the assumption that signal strength varied inversely as the fourth power of the range and by scaling the target heights up or down proportionately to the range. The observations were then plotted in the form of decibels against height and were compared with a theoretical signal-strength distribution calculated on the basis of flat-earth theory; flat-earth theory is of course inappropriate to the

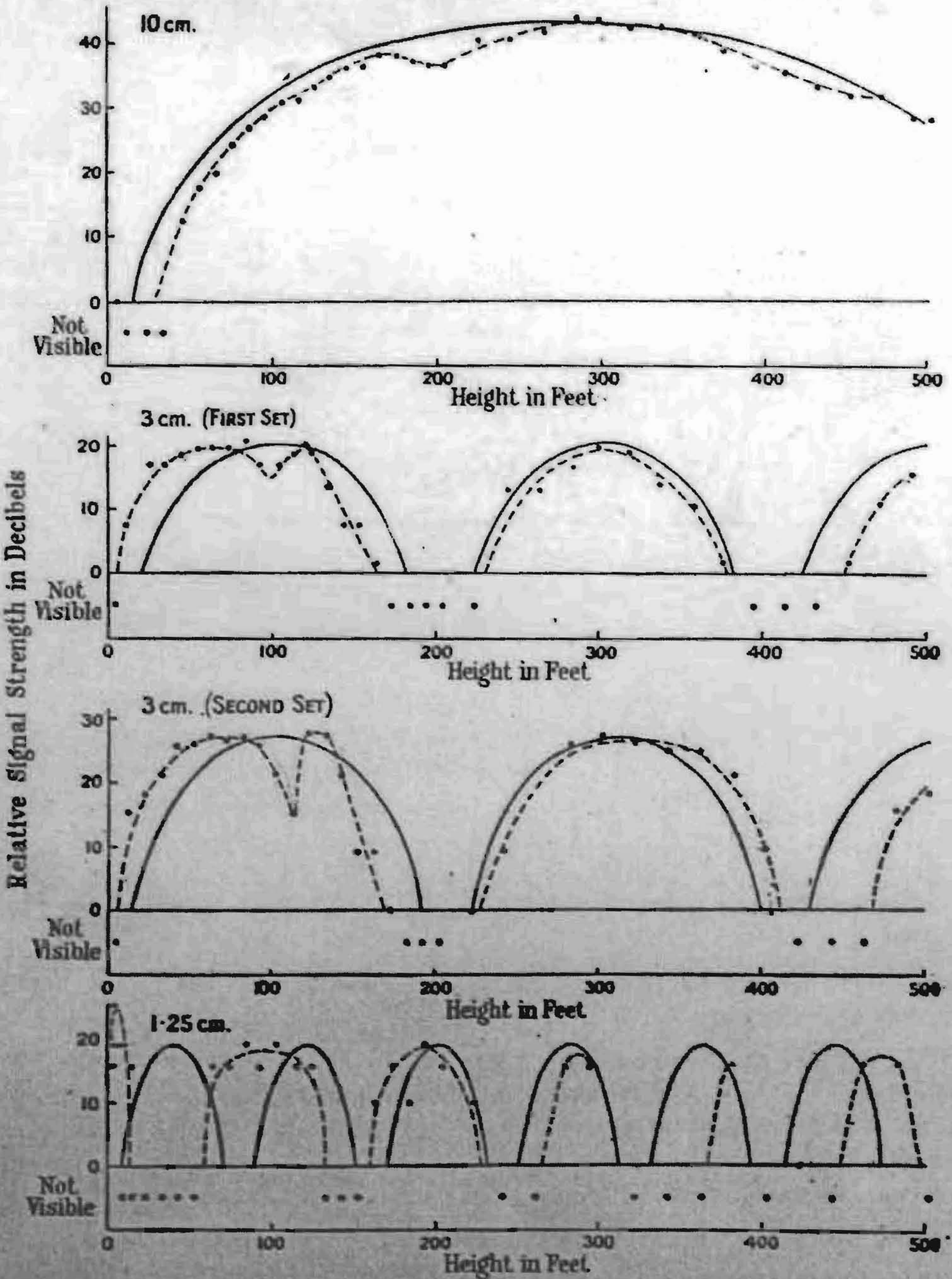

Figure 2. Corner reflector. 14.8.44. 21.45–22.15 h. A.P. weak.
Range : 17 000–17 500 yards, adjusted to 17 000 yards.
Height of radar : 12·7 ft. to 13·0 ft. for 1·25 cm.
13·2 ft. to 13·5 ft. for 3 cm. (first set).
12·7 ft. to 13·0 ft. for 3 cm. (second set).
14·4 ft. to 14·7 ft. for 10 cm.

situation under examination, but the accuracy of the observations does not warrant the greater labour which curved-earth theory involves, and the theoretical flat-earth signal-strength distribution may be used as a convenient but arbitrary yard-stick against which the effects of refraction can be assessed.

Meteorological measurements formed a part of the data which were collected for purposes of the wider aspect of these investigations but no data are available which can be regarded as relevant to the experiments now under discussion. As a matter of day-to-day routine it was customary to recognize "weak A.P." (i.e. Anomalous Propagation) or, "strong A.P." Under weak A.P. long-distance land echoes, e.g. from the Isle of Man, were visible on 10 cm. and 3 cm. but no echoes from shipping at corresponding ranges; under strong A.P. echoes from shipping were observable at ranges of 100 000 yards or more on 10 cm. and 3 cm.

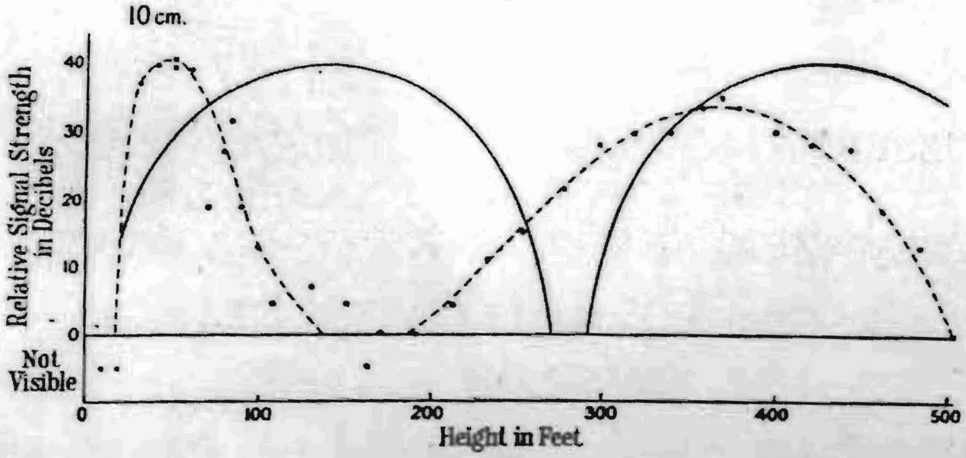

Figure 3. Corner reflector. 29.5.44. 16.16–16.45 h. A.P. strong.
Range: 15 500–15 000 yards, adjusted to 15 350 yards.
Height of radar: 27·6 ft. to 26·7 ft. for 10 cm.

Figure 1 shows a set of observations made under conditions which were adjudged as normal, and it will be seen that for all three bands the observed distribution of signal strength is approximately in agreement with expectation of flat-earth theory.

The observations of figure 2 were taken under conditions of weak A.P. They show a very marked downward bending of the first lobe on 1·25 cm., an apparent splitting of the first lobe on 3 cm. (confirmed by two different sets) and a very small irregularity of the first lobe on 10 cm.

Unfortunately, on such occasions, as strong A.P. developed, circumstances combined to prevent a complete series of observations being taken. Figure 3 shows the signal-strength distribution on 10 cm. only under strong A.P., and it can be seen that this distribution has much in common with the distribution on 1·25 cm. under weak A.P.

The accuracy of the signal-strength measurements in these experiments was indifferent, and it is only claimed that the results do show the existence of reproducible maxima and minima whose position is affected by the state of the atmosphere. It appears that the shorter wave-lengths are more strongly affected. Confirmation of a qualitative kind was obtained from observation of certain small floating objects, such as buoys, which were often detectable on 1·25 cm. but not on 10 cm. in spite of the very much greater power of the latter.

These results provide some practical confirmation of the prediction, made on theoretical grounds by Scott and Pearcey (1943), that anomalous propagation would be experienced on 1·25 cm. to a much greater extent than on 10 cm.

ACKNOWLEDGEMENT

This work is published by kind permission of D.G.S.R.(D), Ministry of Supply.

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FURTHER MEASUREMENTS OF 3- AND 10-CM REFLECTION COEFFICIENTS OF SEA WATER AT SMALL GRAZING ANGLES

Abstract

Results of further measurements of 3- and 10-cm reflection coefficients of sea water for small grazing angles are reported. The values for vertical polarization are in good agreement with theory for a smooth sea, while the values for horizontal polarization are lower than those predicted by theory, falling as low as 0.6. The values obtained for horizontal polarization are higher, however, than those published previously in RL Report 478.

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P.J. Rubenstein

Approved by:

/s/ Donald E. Kerr
Chairman, Group 42

/s/ E. M. Purcell
Acting Division Head

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5 pages of figures

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INTRODUCTION

This report describes measurements of the effective plane-wave reflection coefficient of sea water at wavelengths of 3 and 10 cm. These measurements are an extension of the work begun last year.[1] The magnitude of the reflection coefficient was determined from the ratio of signal strengths at the maxima and minima in an interference pattern, measured with a one-way transmission path between a fixed ground station and an airplane flying at constant height toward the ground station. The phase shift on reflection was not determined.

Figures 1 and 2 show the results at 10 cm for calm sea water on vertical and horizontal polarizations, respectively; Figures 3 and 4 show the corresponding results at 3 cm. The dots represent the observed values of p, the magnitude of the reflection coefficient, plotted as a function of grazing angle (complement of the angle of incidence); the theoretical values for smooth sea water are given for comparison (solid lines).

It will be seen that the observed values for vertical polarization scatter about the theoretical curve at both wavelengths. With horizontal polarization, however, the observed values fall consistently below the theoretical line. The observed points indicate values of p between 1.0 and 0.6, the scatter being large even within a single set of observations. The results at 10 cm indicate higher values of p than those previously reported.[1]

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[1] For numbered references consult the bibliography at the end of this report.

EQUIPMENT

The transmitter used for the 10-cm measurements employed a Klystron operating at 10.0 cm and delivering a CW power output of about 3 watts. The output was monitored by a temperature-compensated thermistor bridge circuit. The 3-cm transmitter used a British CV-129 reflex-type velocity-modulated tube operating at 3.2 cm and delivering a CW output of about 75 milliwatts. A crystal monitor was used.

The CW receivers used were developed for other purposes and will be described in detail in another report.[2] They were equipped with automatic frequency control and with automatic gain control on one, two, or three of their four intermediate-frequency stages, the automatic gain control voltage being amplified and made to drive a 0-1 milliampere Esterline-Angus recording milliammeter. The minimum detectable power was about 130 db below 1 watt for the 10-cm receivers and 116 db below 1 watt for the 3-cm receiver. With the automatic gain control settings usually used, a full-scale range of 50 db was available at 3 cm and of 65 to 75 db at 10 cm.

On the 10-cm path 18-inch paraboloidal antennas were used on the ground stations while the plane carried a 12-inch paraboloid; both used dummy dipole feed. The 3-cm equipment originally used 8-inch wave-guide-feed paraboloids. Later the antenna in the plane was replaced by one of a set of 8-inch paraboloids cut to have an aperture of 8 x 5 inches. These special antennas (one for each polarization) increased the beam width in azimuth to give additional tolerance for errors in the plane's heading. The measured full beam widths to half power points in the horizontal plane for these antennas are 13 and 14.5 degrees for horizontal and vertical polarizations, respectively.*

The source of power in the plane was a 24-volt d.c. to 115-volt 400-cycle rotary converter with a voltage regulator placed between the converter and the equipment. In most of the work the ground station was a mobile truck system which carried its own well-regulated 115-volt 60-cycle motor-generator; a voltage regulator was always used with the 3-cm equipment. Radio communication was maintained between the truck and plane.

MEASUREMENT TECHNIQUE

The course used during the experiment is at a bearing of 43 degrees True from Deer Island, in Boston Harbor, passing through Eastern Point in Gloucester. It is shown in Figure 5. The plane flew out as far as desired on this course, turned, and headed toward Deer Island, flying with as nearly constant altitude, bearing, and speed as possible. At the beginning of the run the antenna in the plane was leveled and corrected in azimuth for crab. When informed by radio that the plane was on course, the operators of the ground station tuned in the signal on the receivers and rotated the antennas in azimuth for maximum signal. The automatic frequency control on the receivers usually held the signal in tune throughout the flight. An observer riding in the co-pilot's seat of the plane noted all errors in altitude and bearing, and time of passage over or near known landmarks (Thacher Island, Eastern Point, Halfway

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*We are indebted to Group 54, the Radiation Laboratory, for the construction and pattern measurements on these special antennas.

Rock, and the receiver station). This information was placed directly on the record with a side pen recorder, making possible a correlation with navigation errors and an approximate distance calibration.* The receiver and recorder combinations were calibrated each day with 10- and 3-cm signal generators.

Three general procedures were used during the experiment: originally the 10-cm receiver was carried in the plane and the transmitter was placed on Deer Island, 25 feet above mean sea level; later, the transmitter was carried in the plane and receivers were placed on the Island, 25 and 70 feet above mean sea level; finally, both 3- and 10-cm transmitters were carried in the plane with their corresponding receivers placed on the Island, 18 and 25 feet above sea level, respectively.

Because the lobe structure becomes increasingly fine as transmitter and receiver heights are raised, the plane was always flown at altitudes of 1000 feet or less when the 3-cm equipment was being operated. Flights at higher altitudes would have put impossibly severe requirements on the navigation accuracy of the plane in order to obtain a satisfactory cross-section of the lobe structure. When only 10-cm equipment was being operated, it was possible to fly as high as 5000 feet; indeed, it was often preferable since the greater stability of the air made navigation easier at that height.

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*When the receivers were at the ground station, the information was transmitted by coding on the 6-mc communications band. A relay operating from the communications receiver activated the side pen recorder.

The greatest difficulty with this technique of measuring reflection coefficients is keeping the plane accurately on course. In order to make a flight at all, it was necessary that visibility be exceptionally good and that wind speeds in the first 1000 feet be quite low. Consequently all results published here are for calm sea water; the extent to which the reflection coefficient is a function of water roughness cannot be measured by this method.

Figures 6 and 7 are records of a flight at 500 feet made on February 16, 1944, with vertical polarization; Figures 8 and 9 show a similar flight made with horizontal polarization the same day. The distance of the plane from Deer Island and the receiver calibration in db below 1 watt are marked directly on the record. The navigation difficulties are very clearly shown in these flights: since the beam widths of the 3-cm antennas are less than those at 10 cm and since the finer lobe structure at 3 cm makes altitude errors more important, it would be expected that the 3-cm records should be more erratic than those at 10 cm. This is clearly the case in the records shown.

REDUCTION OF THE RECORDS

When the antenna pattern is broad, it is easily shown that for a smooth spherical surface ρ, the magnitude of the reflection coefficient, is given by

$$\rho = \frac{1}{D}\,\frac{E_{max}/E_{min} - 1}{E_{max}/E_{min} + 1}$$

where E_{max} and E_{min} are the electric field strengths at adjacent maxima and minima in the interference pattern and D is the divergence factor, a geometrical quantity which expresses the divergence of a wave reflected from a spherical surface.[*] The value of D and the grazing angle can be determined at the adjacent maxima and minima and used in conjunction with the difference in signal level at the maximum and minimum to determine ρ for the given value of the grazing angle. The method cannot be used when either the divergence factor or the grazing angle varies appreciably between the maximum and minimum being considered.

It will be noticed that as ρ approaches 1.0 the db difference between maxima and minima becomes greater and approaches infinity. If the db difference becomes great enough, the signal level at a minimum will fall below minimum detectable power and it will be impossible to measure the db difference between the maximum and minimum. It has been found that values of $D\rho$ higher than 0.96 cannot be measured accurately with this equipment. In general, however, the signal level at a minimum did not fall below minimum detectable power during these experiments.

Before any record was accepted as satisfactory it was required that the maximum signal values obey the range attenuation law

$$\text{Power Received} = \text{Constant}/\text{Range}^{2}$$

and that the positions of maxima and minima check the positions predicted by spherical earth theory. In view of the low heights of the receivers,

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[*] In these experiments it has been decided to assume the geometrical expression for the divergence factor correct and to lump all departures from theory in the quantity ρ.

this entailed fairly accurate knowledge of height of tide during the experiment. These two checks served to eliminate those records or parts of records in which plane navigation was poor and gave a partial check on the existence of anomalous propagation, to be discussed below.

DISCUSSION

Observations made this year on 3 cm and both this year and last year on 10 cm indicate that the effective reflection coefficient of calm sea water for vertical polarization is in quite good agreement with the theory for a smooth sea. The use of the theoretical values in determining radar coverage should prove satisfactory.*

Observed values for horizontal polarization fall below the theoretical curve on both wavelengths and show a much greater scatter than on vertical polarization. Much of this scatter is believed to be real and tends to bear out similar conclusions from the Admiralty Signal Establishment in England.[3] A suggested cause for this scatter and for the low values observed is the roughness of the reflecting surface. At one time it was even thought that a very striking correlation had been found between the value of ρ observed and the direction of wave travel with respect to the path. The correlation indicated that high values of

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* It should be pointed out that the theoretical curve shown in Figure 3 is drawn for a dielectric constant of 35 and a conductivity of 17 mho/meter. These values have been tentatively suggested by Professor A. R. von Hippel of the M.I.T. staff as an improvement over the values $\epsilon = 55$ and $\sigma = 11$ mho/meter frequently quoted for this wavelength. These latter values give a theoretical reflection coefficient curve that falls below the one shown. The observed points are in better agreement with the former set of constants.

ρ accompany wave travel along the path and low values accompany wave travel across the path. More exact information on wind (and presumably wave) direction and failure to reproduce the correlation has tended to discount its value. It is now believed that the reflection area on the path used would be so close to Deer Island when the low receiver locations were used that land reflection and disturbances created by underwater obstacles would prevent any unidirectional wave travel. It does seem probable, however, that the magnitude of the reflection coefficient should be dependent upon the condition of the surface: the effects of individual wavelets may cause a short period scatter superposed on a general level depending upon the wave amplitude and possibly direction of travel. If such a hypothesis is true, the results shown here can be construed to show the range of values to be expected with calm sea water.

If the magnitude of the reflection coefficient is a function of surface roughness, it might be expected that the extent to which roughness affects its magnitude would depend upon the surface area illuminated and consequently upon the transmitter and receiver heights. The various combinations of receiver and transmitter heights listed previously were chosen in an effort to study this variation. It was found that there was no appreciable variation of the magnitude of the reflection coefficient at 10 cm for a ground station height variation from 25 to 125 feet and for a plane altitude variation from 500 to 5000 feet. The heights were chosen to simulate actual tactical conditions, and the results should be directly applicable to operational systems.

Reflection coefficients have frequently been measured at constant and extremely short ranges using variable heights,[4,5] both heights being small. Since a very small reflection area is used with this method, roughness cannot become predominant and values obtained in this manner are usually higher and in better agreement with theory for a smooth surface than those obtained with the method discussed in this report. It is believed that the use of a technique involving long ranges and greater heights gives more information of tactical use.

The difference between these two techniques is even more pronounced in the case of land reflection. High reflection coefficients have been measured using the short-range, low-height method;[5] the use of the technique discussed here has indicated that no reflection can be depended upon in the microwave region.[1] The implication in these results is that the ground is capable of reflecting radio waves in the microwave region, but that surface roughness is great enough to prevent any specular reflection of radiation from most radar stations. If a station site is such as to approximate the geometry of the short-range, low-height method, specular reflection may occur. There are well-authenticated cases of land reflection in the microwave region, but in every one reported to this group the transmitter height has been less than 25 feet and the site has been near an airport runway or extremely flat land containing little or no vegetation. This is probably the only case in which specular reflection of microwaves can take place over land.

The results obtained this year for sea water reflection with horizontal polarization at 10 cm are consistently higher than those measured last year.[1] While surface roughness may be responsible for part of this difference, wind data indicate no great difference between the sets of observations. The most obvious explanation for this disparity lies in the time constant of the receiver-recorder combination. With the equipment used last year a time of 3 seconds was required for a 90 per cent response to full-scale deflection. That response time has now been reduced to 0.3 seconds, which corresponds to approximately 0.01 miles for the plane's usual ground speed. Observations made this year on a high speed meter and the receiver output meter, both in series with the recorder, show that the recorder may fail to register the full depth of minima even when the spacing between adjacent maxima and minima is considerably greater than 0.01 miles. Indeed, it has been frequently noticed in the records obtained in the last two years that minima in the interference pattern tend to be even sharper than would be predicted by theory. This point is well illustrated in Figures 6 and 8. In order to avoid errors introduced by the time constant, values of ρ at grazing angles greater than 5 degrees were not determined.* It is believed that the recorder will give full response up to this angle; it is quite possible, however, that with last year's slower time constant the recorder failed to register the full depth of minima.

In any experiments involving transmission over considerable distances the effects of the atmosphere cannot be neglected. The calculations of the divergence factor and grazing angle, as well as the positions of maxima and minima, were made for an idealized standard atmosphere having a gradient of modified index of refraction with height of

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*Navigation difficulties frequently confused the interference pattern enough at 3 cm that it was deemed wise to limit the grazing angle to even small values.

3.6×10^{-8} per foot (corresponding to an effective earth radius 4/3 times the true radius).[6,7] For other atmospheric conditions the positions of maxima and minima may be shifted, the pattern may be more or less "washed out," and the inverse square law variation with distance may be violated. The requirement that the received signal check the inverse square law and the position of maxima and minima was imposed in an attempt to detect the presence of non-standard conditions. Failure of the record to meet either of these requirements was construed as evidence of anomalous propagation, and the record was not worked up. As a further check airplane soundings of the lower atmosphere were made prior to the day's flights whenever possible.[8,9]

As a further atmospheric complication there is a growing amount of evidence that a low-lying duct may always be present over water due to the very sharp decrease in vapor pressure with height. The height of this duct is not known, but it is probable that it can "trap" neither 10 nor 3 cm waves.[6] While this duct apparently has no radical effect on the received signal, it may possibly affect the depth of the minima observed. If such a modification corresponds to a virtual decrease in reflection coefficient, the results printed here can be considered reliable and used in coverage calculations. If, however, the minima are filled up while the maxima are essentially unaffected, the results shown here indicate too low a value of ρ, and the maximum range obtained by a radar set (in the maxima of the lobe structure) will be greater than would be predicted using these values. Since so little is known about this phenomenon, about the only conclusion that can be drawn is that values of

ρ shown here can be considered as minimum values. It should be emphasized that the reflection coefficient values given here can be used in coverage calculations only for standard meteorological conditions; reliable results may possibly be obtained for the case of a very low duct such as may exist over water all the time, but once the anomalous effects become at all pronounced the interference pattern becomes so distorted that the reflection coefficient no longer has significant meaning at low grazing angles, especially below one degree.

CONCLUSIONS

The values of reflection coefficient reported here do not give a final answer to the problem of sea water reflection. At present no adequate explanation is available for the fact that the variation of ρ with grazing angle for vertical polarization is in good agreement with theory for a smooth sea, but that the variation for horizontal polarization does not agree with theoretical values and shows an extreme scatter in a very short time interval. In addition, while surface roughness is suspected to be an important factor in determining the magnitude of ρ, no quantitative measurements of the effect it produces have been obtained, nor is it likely that they ever will be obtained using the technique described here. The results given in this report are probably reliable for the limited case of reflection from calm sea water. They indicate that the coverage resulting from sea reflection will not be reliably increased by as much as a factor of two in the case of horizontal polarization and in the case of vertical polarization it will be that predicted using theoretical values for a smooth sea.

W. T. Fishback
P. J. Rubenstein
May 4, 1944

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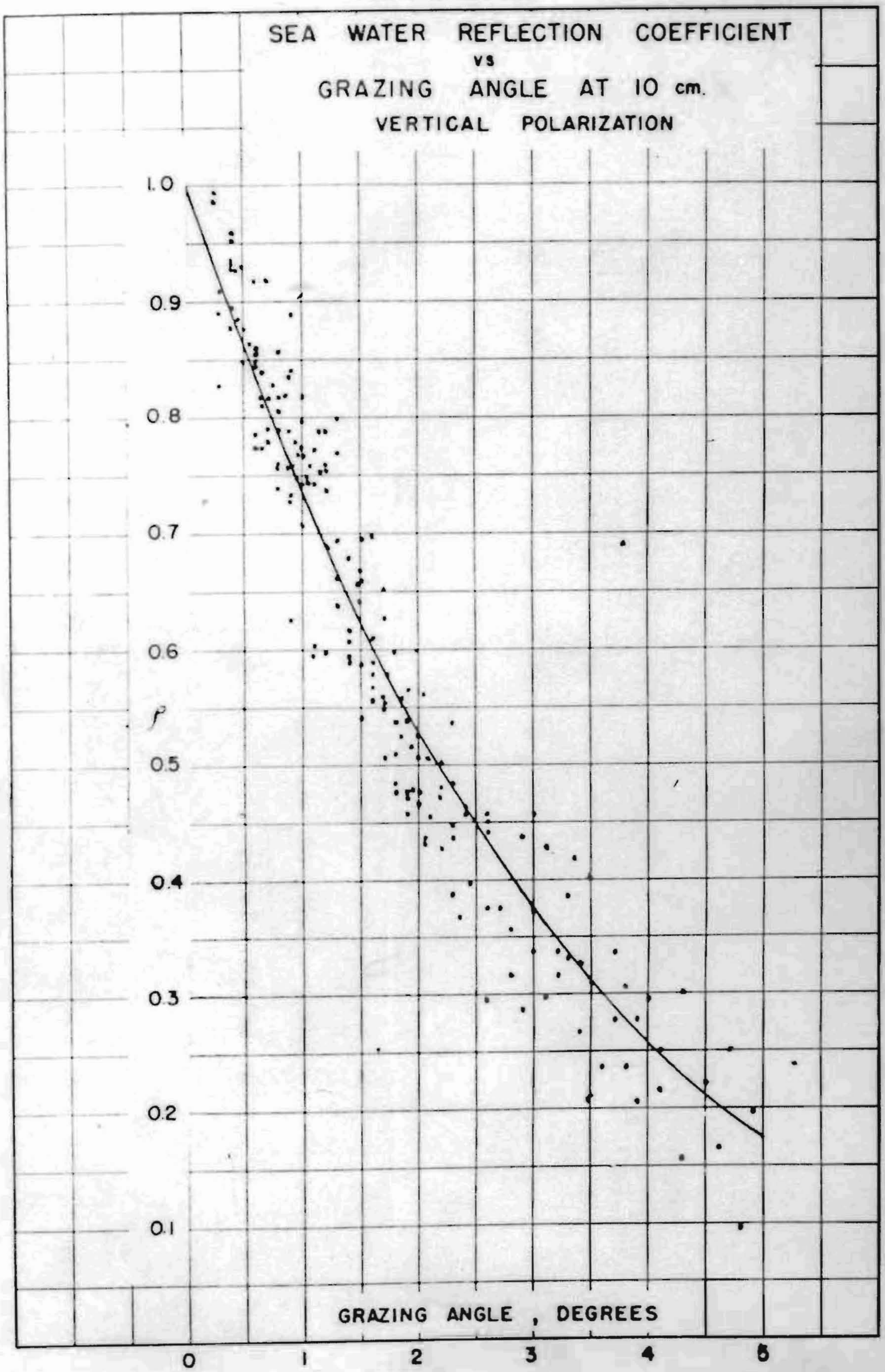

SEA WATER REFLECTION COEFFICIENT
VS
GRAZING ANGLE AT 10 cm.
VERTICAL POLARIZATION
1.0
0.9
0.8
0.7
0.6
ρ
0.5
0.4
0.3
0.2
0.1
GRAZING ANGLE , DEGREES
0
1
2
3
4
5

FIGURE I

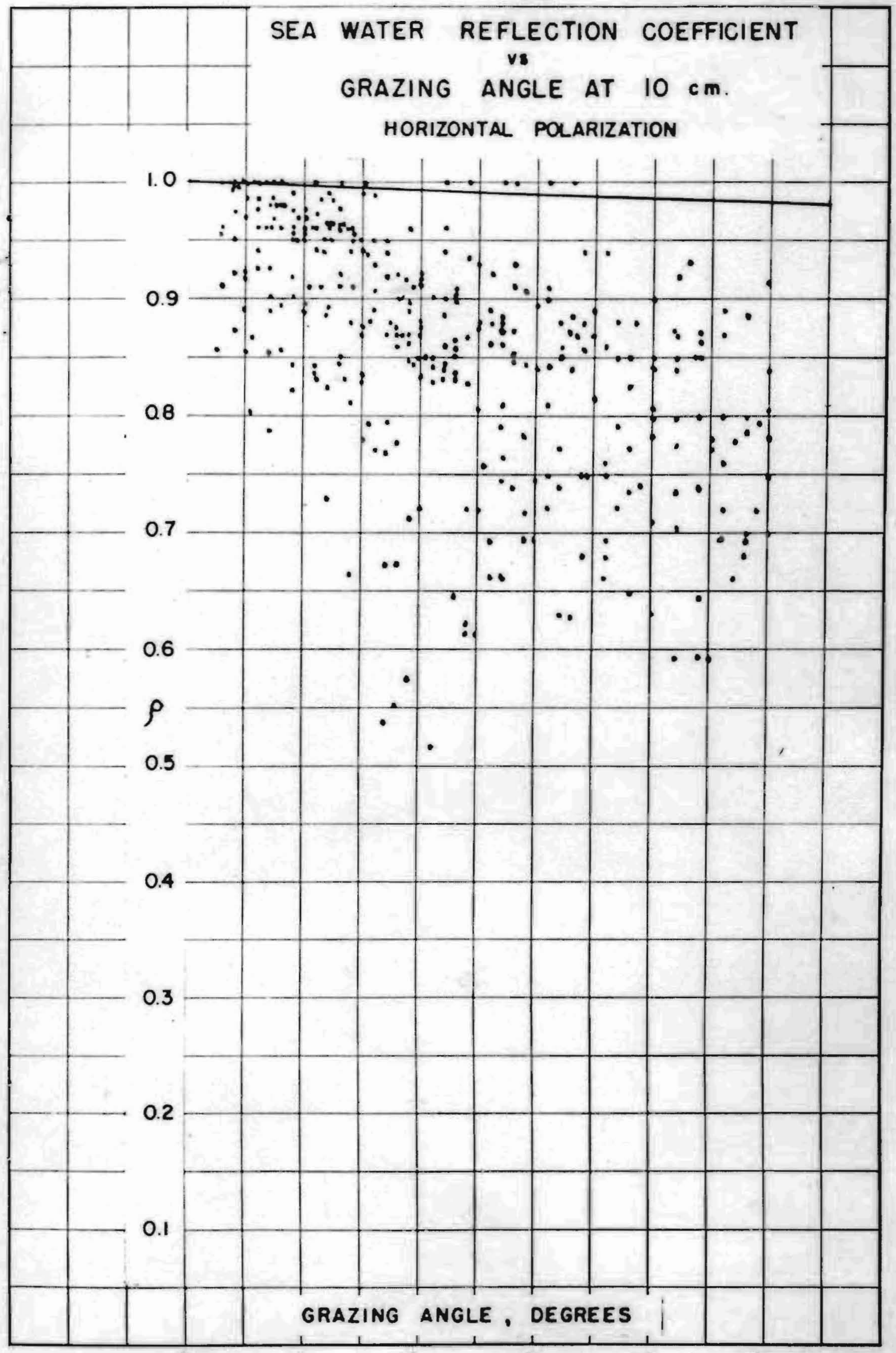

SEA WATER REFLECTION COEFFICIENT
VS
GRAZING ANGLE AT 10 cm.
HORIZONTAL POLARIZATION
1.0
0.9
0.8
0.7
0.6
ρ
0.5
0.4
0.3
0.2
0.1
GRAZING ANGLE, DEGREES
0
1
2
3
4
5

FIGURE 2

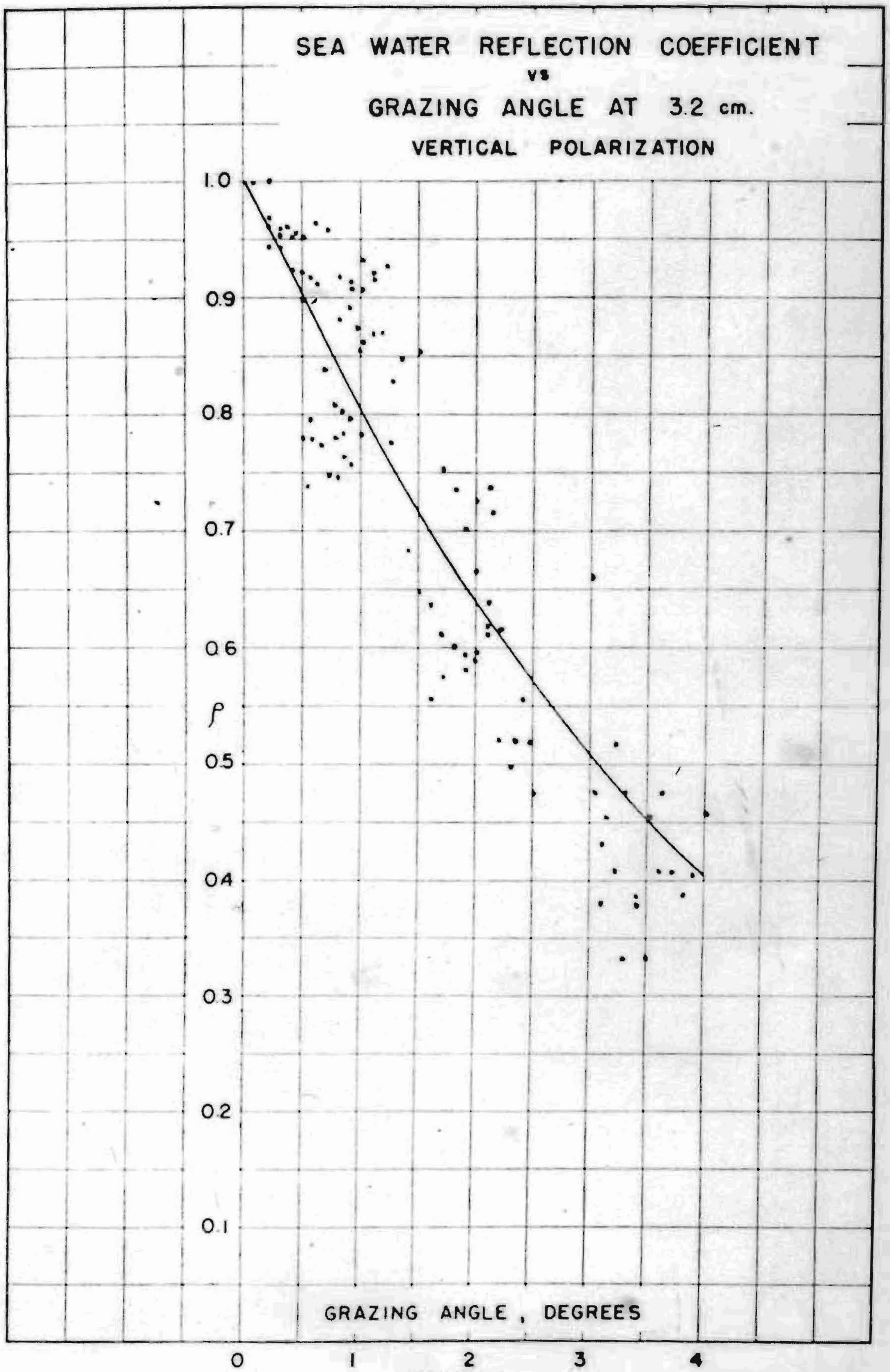
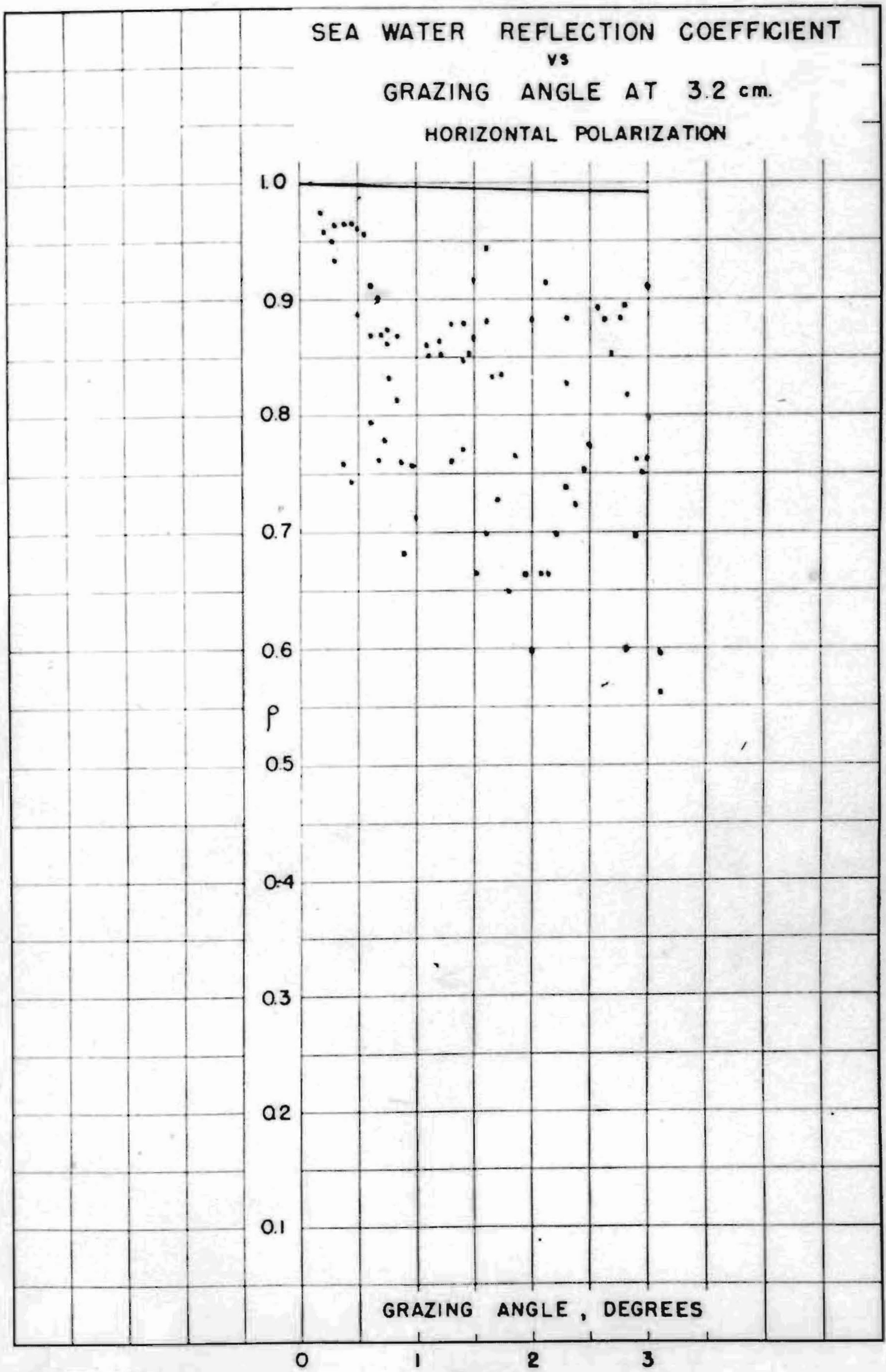

SEA WATER REFLECTION COEFFICIENT
vs
GRAZING ANGLE AT 3.2 cm.
VERTICAL POLARIZATION
1.0
0.9
0.8
0.7
0.6
ρ
0.5
0.4
0.3
0.2
0.1
GRAZING ANGLE, DEGREES
0
1
2
3
4

FIGURE 3



SEA WATER REFLECTION COEFFICIENT
VS
GRAZING ANGLE AT 3.2 cm.
HORIZONTAL POLARIZATION

FIGURE 4

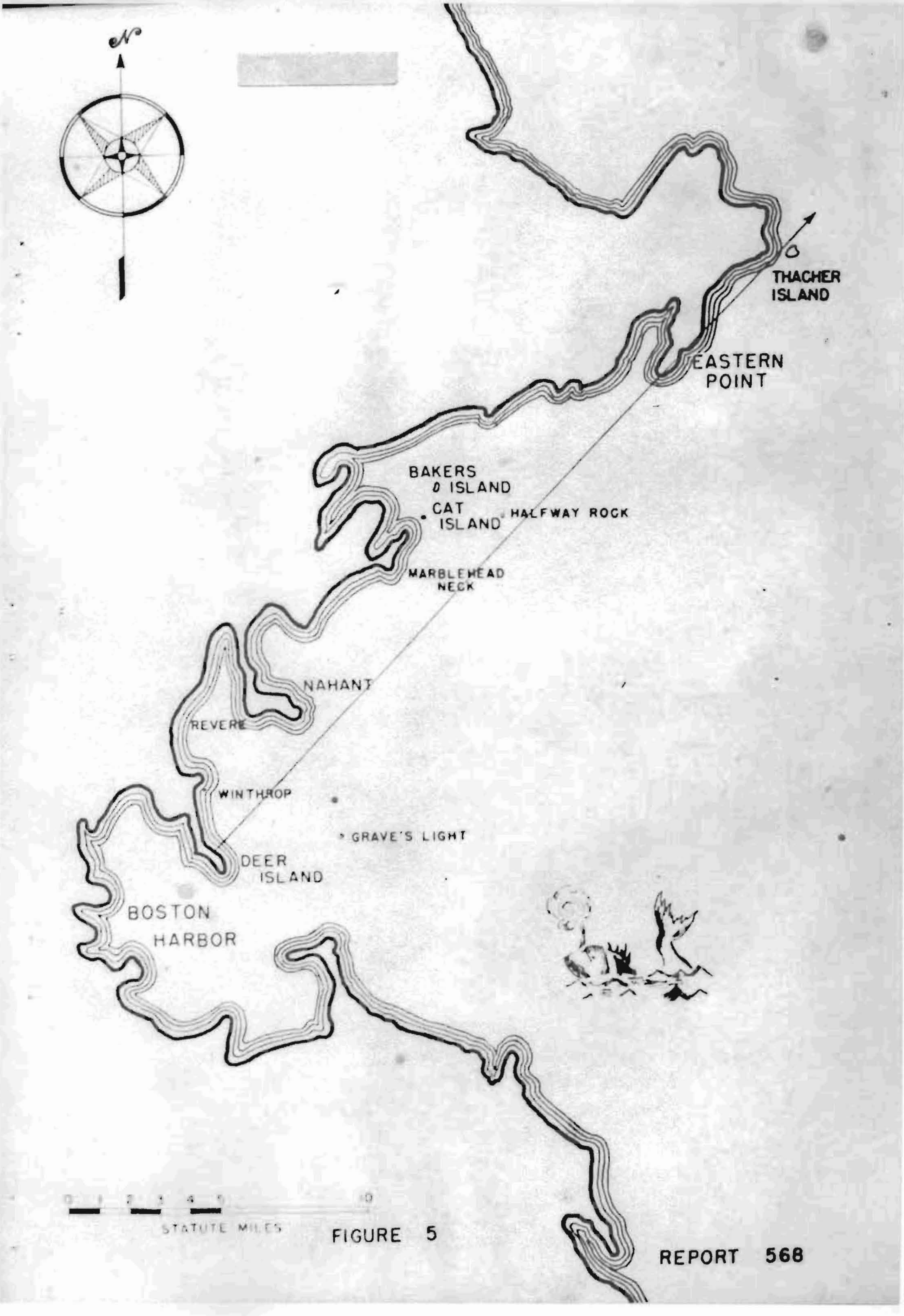


FIGURE 5

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[redacted]

SIGNAL STRENGTH VS. DISTANCE OVER SEA WATER AT 10.0 CM.

TRANSMITTER AT 500 FT. VERTICAL POLARIZATION
RECEIVER AT 25 FT. FEBRUARY 16, 1944

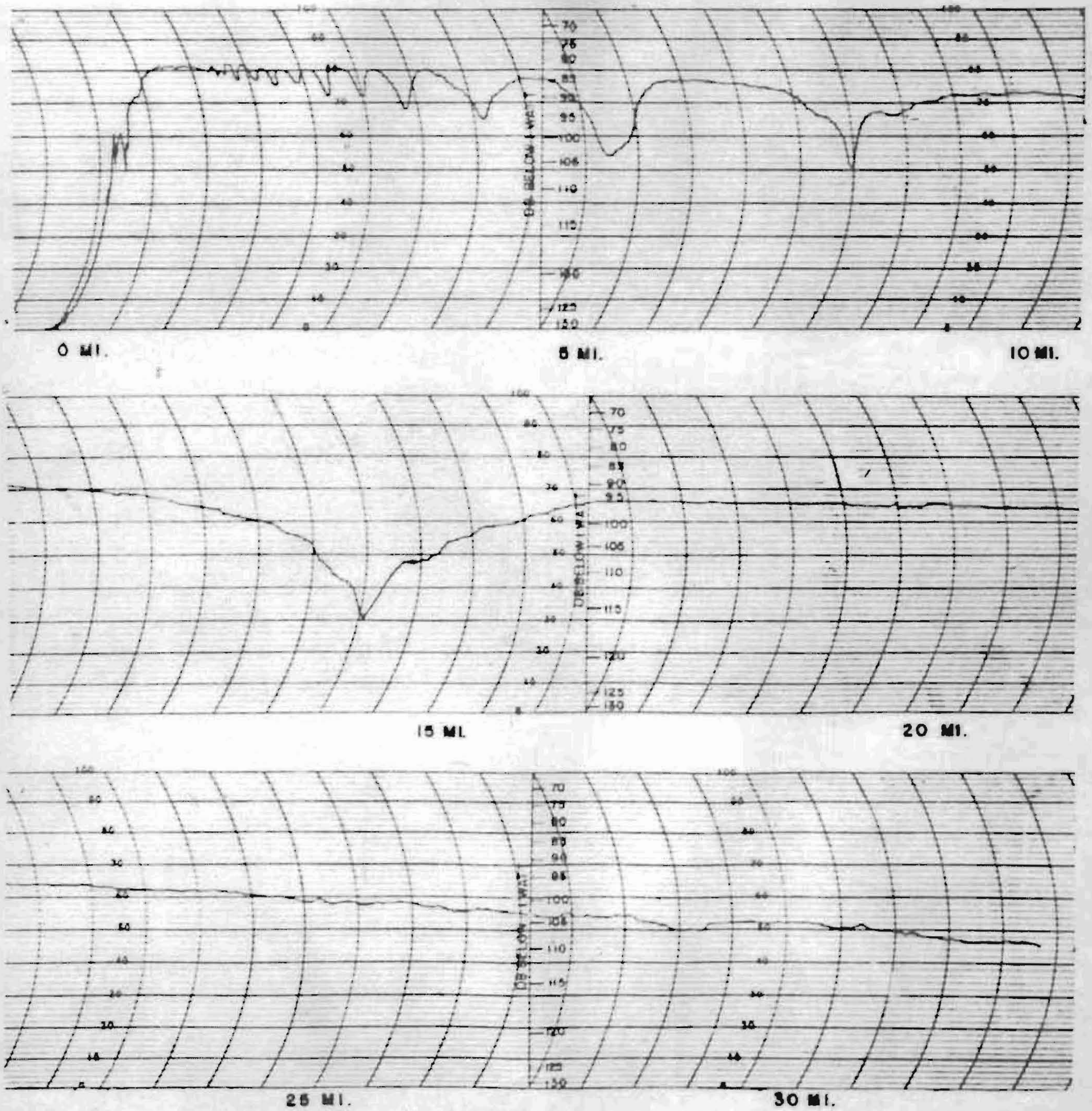

FIGURE 6

REPORT 568

[redacted]

SIGNAL STRENGTH VS. DISTANCE OVER SEA WATER AT 3.2 CM.

TRANSMITTER AT 500 FT. VERTICAL POLARIZATION
RECEIVER AT 18 FT. FEBRUARY 16, 1944

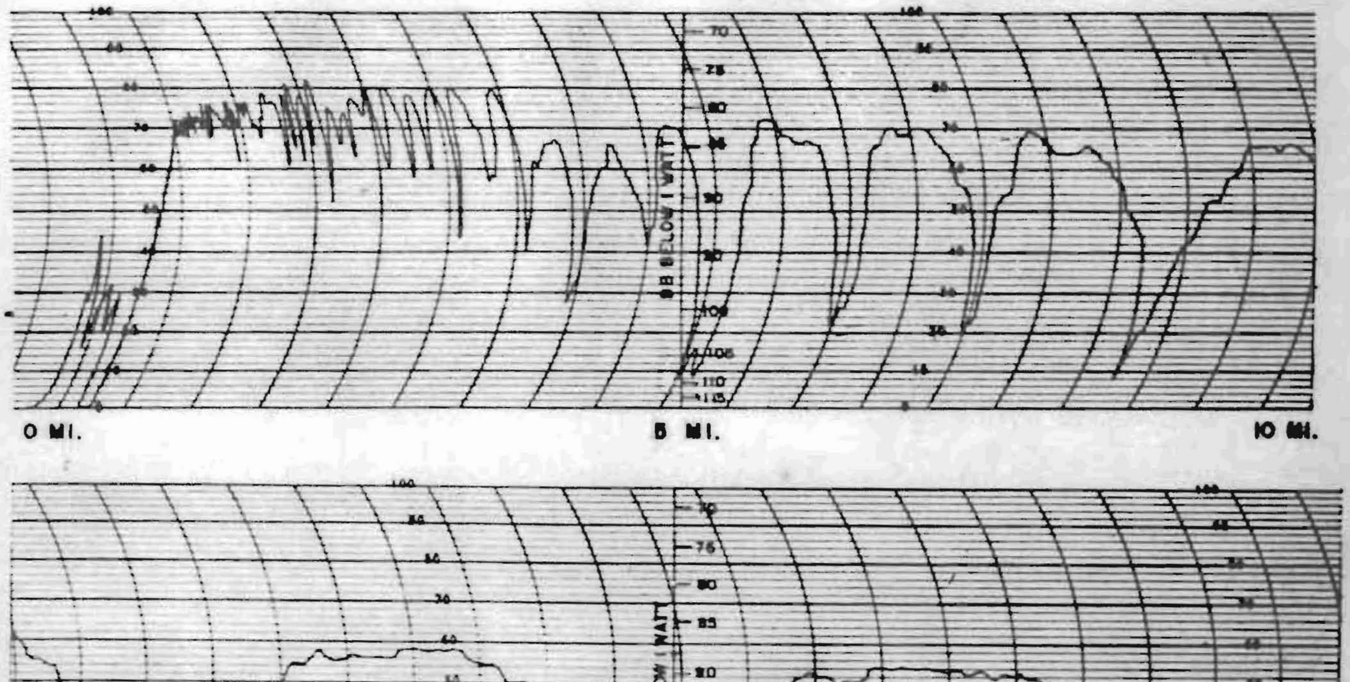

FIGURE 7

SIGNAL STRENGTH VS. DISTANCE OVER SEA WATER AT 10.0 CM.

TRANSMITTER AT 500 FT.
RECEIVER AT 25 FT.

HORIZONTAL POLARIZATION
FEBRUARY 16, 1944

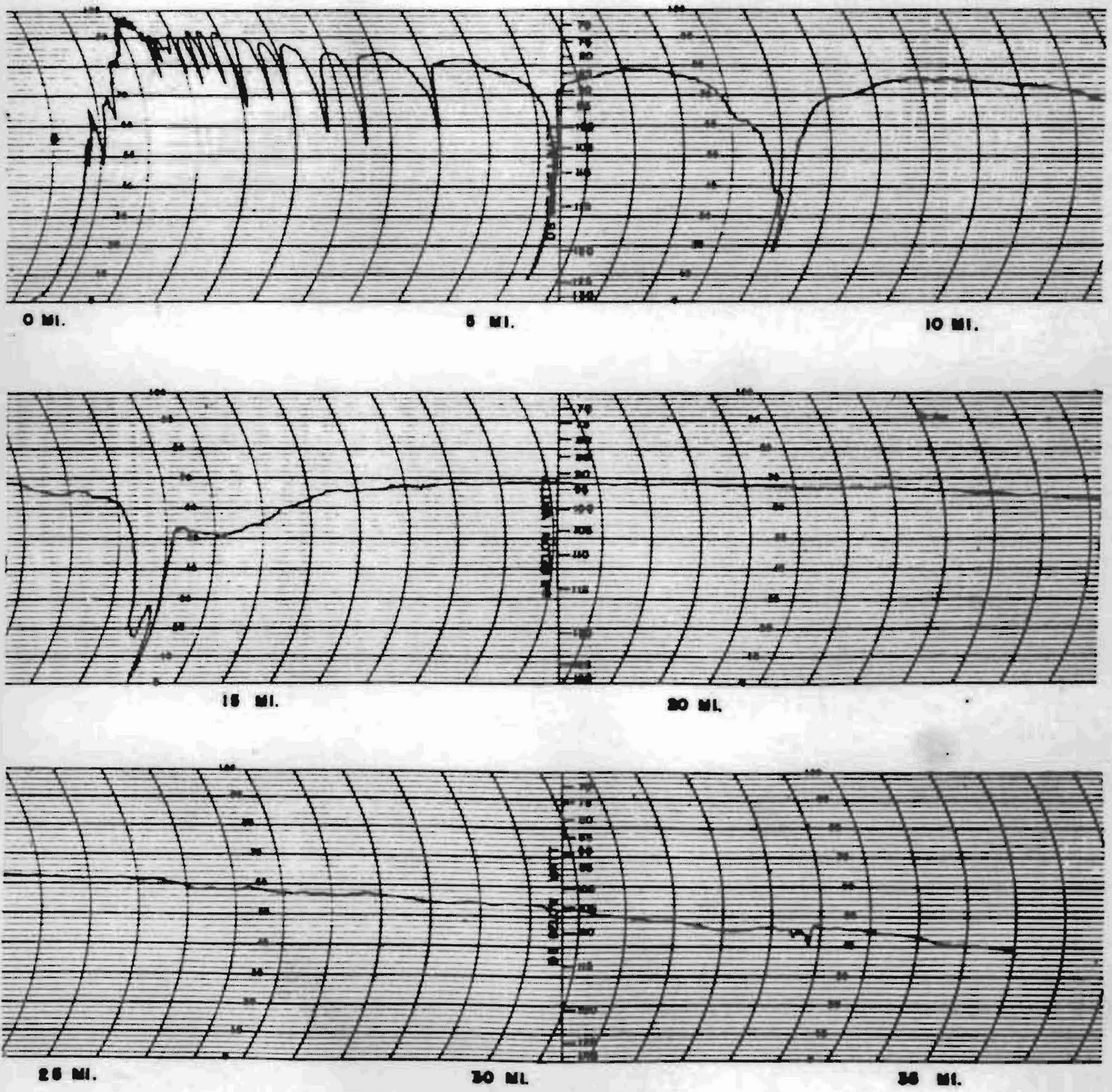

FIGURE 8

SIGNAL STRENGTH VS. DISTANCE OVER SEA WATER AT 3.2 CM.

TRANSMITTER AT 500 FT.
RECEIVER AT 18 FT.

HORIZONTAL POLARIZATION
FEBRUARY 16, 1944

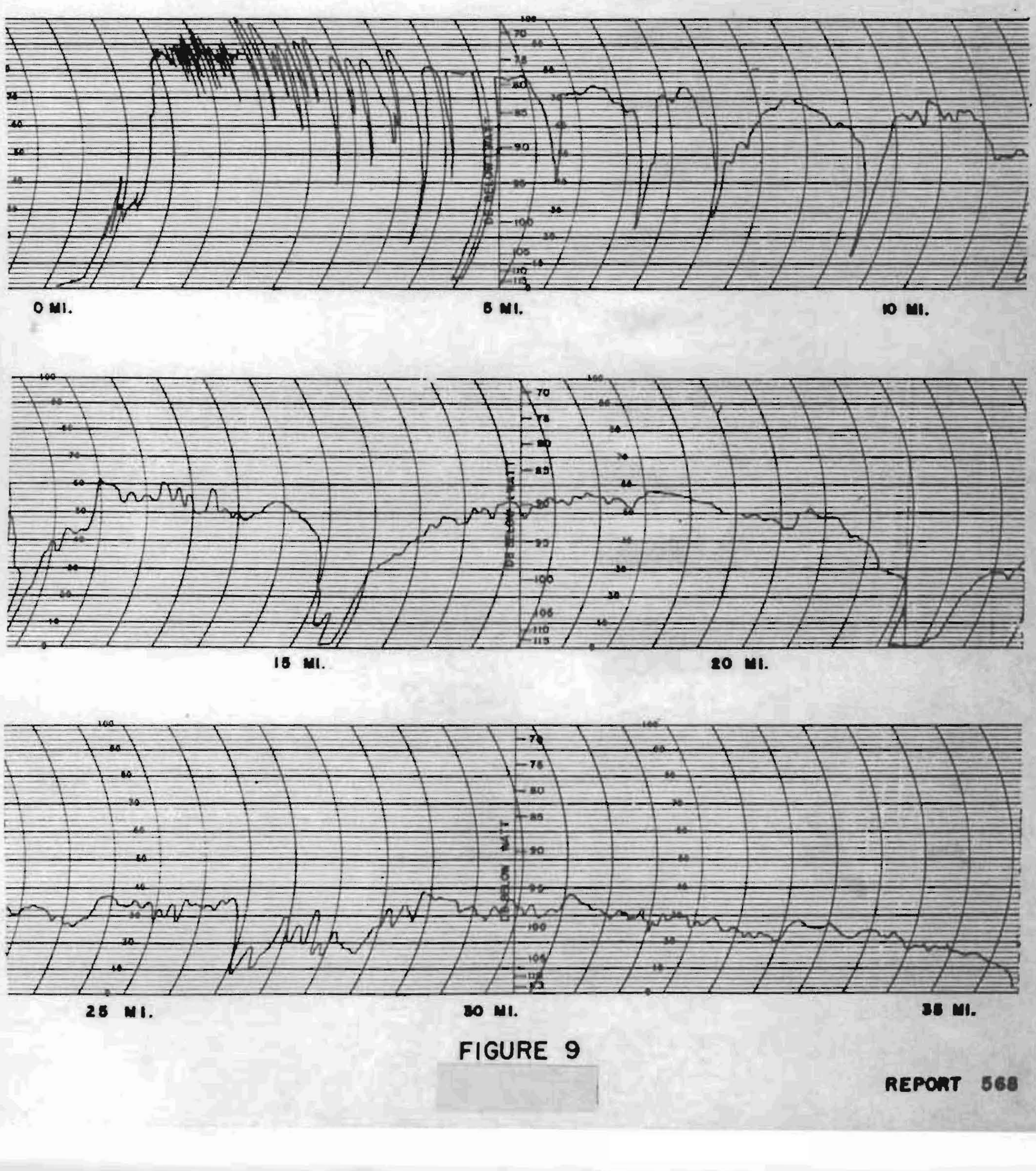

FIGURE 9

CRPL-4-M-12 JULY 11, 1949

NATIONAL BUREAU OF STANDARDS
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Washington, D. C.

SCATTERING OF RADIO GROUND WAVES IN PROPAGATION OVER IRREGULAR TERRAIN

By

Kenneth A. Norton

Paper presented by Kenneth A. Norton on July 28, 1949, as a part of a Symposium on Radio Wave Propagation at the Naval Electronics Laboratory in San Diego, California.

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By

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The application of statistical methods in any field of engineering or science usually becomes necessary at some stage in its development in order that further progress can be made. Today, I will first describe some of the characteristics of two important mathematical tools useful in statistical predictions of ground-wave propagation and will then demonstrate their applicability to the description of radio propagation over irregular terrain by means of an example. It should be stressed at the outset that this is an interim report on methods which have been found useful in a field in which new and better methods of analysis are being developed almost daily. The methods I will describe have been useful to us in stimulating our thinking on this subject and I pass them along to you at this time merely as a progress report and not as completed research.

SLIDE NO. 1

The first mathematical tool I will describe is the Rayleigh distribution or Random Walk. Lord Rayleigh solved the problem of determining the probability distribution of the intensity and phase of the resultant vector, E_s, obtained by adding together, with random relative phase, a large number of vectors, E_1, E_2, etc. up to E_n. When the four conditions listed are satisfied, it is found that the amplitude of the resultant, E_s, may be calculated by means of the simple formula: the probability, p, that a value of E_s greater than X will occur is e to the minus the quantity X^2 over E_r^2. The four requirements for this distribution are: (1) the sum of the energies of the individual components is a constant, (2) each individual vector must be small compared to the root-sum-square value of all of them, (3) the phase of each component vector must be random, that is, all values of the phase between 0 and 2 π must be equally likely, and (4) the number of component vectors must be sufficiently large.

SLIDE NO. 2

I will now show what happens as a result of deviations from each of the above four requirements. We see here the effect of reducing the number, n, of component vectors. This particular graph paper has been constructed in such a way that a Rayleigh distributed vector of amplitude, E_s, will lie on a straight line with a slope of minus one. The distributions shown here are for n unit vectors added with random relative phase. Thus, the root-sum-square value in each case is equal to the square root of n. The ordinate gives E_s divided by this root-sum-square value. It can easily be shown that the root-mean-square value of the resultant amplitudes, E_s, is simply equal to the root-sum-square value of the amplitudes of the individual components. Thus, the $\sqrt{n}$ is also equal to the rms value of E_s. The maximum amplitude of the resultant of n unit vectors is, of course, simply equal to n. The rapid approach to a Rayleigh distribution is clearly shown by the curves for n = 2, 3 and 4. Throughout the range of this particular graph, that is, from a value of 0.01% up to 99.99%, the distribution for n = 10 deviates from the true Rayleigh distribution by less than the width of the line. It is important to notice that a complete specification of a Rayleigh distributed wave can be made in terms of a single parameter, which is, in effect, a measure of the total energy in the wave. This single parameter might be the root-mean-square value which is exceeded by 36.8%

of all of the values which the amplitude of E_s may have. A Rayleigh distributed amplitude could equally well be defined in terms of a value exceeded for some specified percentage of the time, for example, the median, 50%, amplitude is equal to 0.8326 times the root-mean-square value of E_s.

SLIDE NO. 3

We will now consider the distribution of the instantaneous voltage, v, to be expected from a Rayleigh distributed vector with amplitude, E_s, and random phase $(\omega t + \phi)$. This voltage may be determined simply by obtaining the component of the vector along the real axis and we find that this voltage is distributed in a normal distribution with a mean value of zero and a standard deviation $E_r / \sqrt{2}$.

In most of our radio propagation studies the received fields are rectified before being recorded and thus our continuous recorders ordinarily provide records of the variations of the inherently positive amplitudes, E_s, of the envelope rather than of the instantaneous voltage, v, which may be either positive or negative. For this reason, in the remainder of my discussion of the Rayleigh distribution no further mention will be made of the instantaneous voltage v. This brief discussion of the normal distribution of instantaneous voltage was presented because it is sometimes confused with the Rayleigh distribution of the amplitude of E_s and today I will use only the latter distribution. Before leaving this slide I wish to point out an error in equation (1). The exponent should be X^2 over E_r^2 rather than X over E_r^2.

SLIDE NO. 4

I will now discuss the application of the above theory to a problem of ground-wave propagation over irregular terrain. It is well known that the resultant field, E, to be expected in propagation over a smooth earth at points within the line of sight may be considered to be the vector sum of a direct wave, E_o, plus a ground-reflected wave, E_g. Over a smooth spherical earth the ground-reflected wave will be weaker than the direct wave not only because some of its energy is lost by absorption but also because of a divergence of the energy on reflection at the curved surface of the earth.

Over a rough earth it is convenient to consider the ground-reflected wave to be the resultant of a large number of component vectors with random relative phases. Thus, each component vector may be considered to have its phase determined by the length of the path from the transmitting antenna to the corresponding bounce point and thence to the receiving antenna. The bounce points on the rough reflecting surface, corresponding to the several component vectors, are the locations on the surface for which the phase is stationary, that is, the path length is either a minimum or a maximum. When the earth is sufficiently rough, the frequency sufficiently high, and the angle of incidence sufficiently small, that is not too near to grazing incidence, it will be found that the relative phases of the individual component vectors will be comparable to or greater than 2π radians. Under these circumstances all values of the relative phase between the component vectors are equally likely. The above description of the ground-reflected wave over a rough earth is simply that of a Rayleigh distributed wave. We have already seen that such a wave is completely described by its root-mean-square amplitude, E_r. If there were no additional loss in the ground-reflected wave energy due to roughness, this rms amplitude, E_r, would simply be equal to the amplitude, E_g, of the wave reflected from the smooth earth. For the present, it will be sufficient to let $E_r^2 = k\ E_o^2$ where k is a constant, usually less than unity.

which denotes the energy in the rough-earth ground-reflected wave relative to that in the direct wave.

It might at first sight appear that the vector sum of the direct wave, E_0, plus the Rayleigh-distributed rough-earth ground-reflected wave would also be distributed in a Rayleigh distribution but this does not follow because of the fact that the component vector, E_0, is, in this case, not small compared to the root-sum-square value of all of the components.

SLIDE NO. 5

On this slide is shown the expected distribution of a resultant when the energy of one of the individual component vectors, represented by E_0^2, is not small in comparison to the total energy represented by $(E_0^2 + E_r^2)$. It will be noted that the distribution of the resultant, E, is Rayleigh distributed only for very large values of k, that is, only for very large values of multiple-component Rayleigh-distributed energy compared to the single-component direct wave energy. Such large values of k would be expected only in an unusual situation where the direct wave is suppressed, for example, by means of a transmitting array directed away from the actual receiving antenna towards the center of gravity of the images of the receiving antenna in the rough ground.

GR note!
This is case in question of sea reflection only

As k becomes smaller and smaller, that is, as the random energy becomes small in comparison to the direct wave energy, the slope of the distribution becomes smaller. This should be noted in connection with later experimental results.

The results shown here can also be used in the case where the individual component vectors do not have completely random relative phases. Consider, for example, the situation where the individual vectors have phases which vary at random only through a restricted total range of phase variation much less than 2π. In this case each individual component vector can be resolved into two other components, one of which can be considered to consist of coherent, specularly reflected energy and the other to consist of scattered energy. The coherent components can be added to give the vector designated as E_o on this slide while the root-sum-square value of the incoherent components is simply E_r. Thus, it becomes possible to determine the distribution of the amplitude of a wave reflected at a moderately rough surface simply by identifying a specularly reflected component, E_o, and a scattered component, E_r.

SLIDE NO. 6

Some of the concepts just described can best be illustrated by applying them to the explanation of the results of a propagation experiment. In this experiment two aircraft flew away from each other at an altitude of 9800 feet. Field intensity measurements were made by the Collins Radio Company in one of the aircraft of the simultaneous transmissions from the other on 123 Mc and 328 Mc. Two flights were made, one with the midpoint of the transmission path on land in the Midwest and the other with the midpoint on Lake Michigan.

SLIDE NO. 7

Two samples of the received field intensities are illustrated on this slide. In both cases the data shown are for 328 Mc over land. The upper sample corresponds to the case where the two aircraft were between 60 and 70 miles apart while the lower sample corresponds to the case where the aircraft were separated by a range varying from 128 to 148 miles. Calculations indicated that the regular fading is occurring at the rate which would be expected for interference between a direct and a ground-reflected wave. Under these conditions, the field intensity maxima should be equal in amplitude to the sum of the direct wave amplitude plus the ground-reflected wave amplitude and the observed minima equal to the difference in these two amplitudes. By measuring these maxima and minima, it is possible to calculate the value of the ground reflection coefficient. A separate value of this ground reflection coefficient can be obtained from each maximum and succeeding minimum.

SLIDE NO. 8

On this slide, I have shown the distribution of the ground-reflection coefficients determined in this way for these two distance ranges. The lines are the Rayleigh distributions determined from the root-mean-square value of the reflection coefficient. It will be noted that the measurements are in good agreement with the theory at the shorter of the two distance ranges corresponding to more nearly oblique incidence on the ground: the angle of incidence in this case being 87° so that the corresponding grazing angle at the earth's surface is 3°. The data at the larger range did not agree as well with the Rayleigh distribution, presumably because of the larger angle of incidence, that is, nearly 89° in this case. Thus, it appears that the ground is beginning to appear more nearly smooth to the radio waves at this grazing angle of only about 1°. By using the curves shown on an earlier slide, it is possible to estimate from the slope of this curve the relative amount of the energy scattered and specularly reflected; in this particular case, it has been determined that the energy received by specular reflection is about six times the scattered energy.

Before leaving this slide it should be noted that the root-mean-square reflection coefficient is somewhat larger at the larger range.

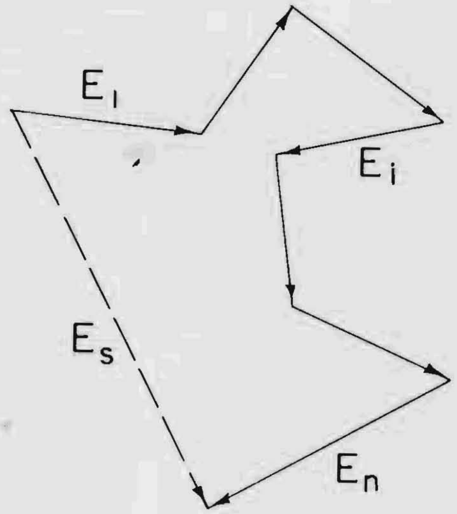
SLIDE NO. 9

I will turn now to a discussion of the magnitudes of these ground reflection coefficients. On this slide the points plotted represent root-mean-square values of the ground reflection coefficients. Each point is the rms value obtained from about 40 separate determinations. The circles joined by a dashed curve denote values measured over land on 328 Mc while the crosses joined by a dotted curve represent values measured over land on 123 Mc. The upper solid curve corresponds to a reflection coefficient calculated on the assumption that the earth is smooth and may be represented by a dielectric constant equal to 30; thus, these values correspond to the product of a plane wave reflection coefficient multiplied by a divergence factor to allow for the effects of earth's curvature in spreading the energy in the reflected wave. The low values of reflection coefficient at the shorter distances are caused by earth absorption near the pseudo-Brewster angle while the low values at large distances are due to the larger divergence expected near grazing incidence. The lower solid curve corresponds to Lambert's law of reflection from diffuse surfaces; thus, according to this law, the energy reflected is proportional to the cosine of the angle of incidence. Lambert's law does not specify the magnitude of the reflection and this has been assumed in these curves to be the plane earth reflection coefficient. Thus, it is assumed that the divergence factor, D, over a smooth sphere should be replaced by the factor, $\sqrt{\cos \phi}$, for a perfectly rough earth, the latter corresponding to a much greater divergence of the energy due to scattering. I will use the blackboard for a discussion of the two dashed curves which show the transition between the perfectly smooth and perfectly rough calculations.

SLIDE NO. 10

Finally on this slide are values similar to those on the previous slide but now for propagation over Lake Michigan. In this case it was assumed that the dielectric constant should be 80 and we see that the pseudo-Brewster effect occurs now at a large range. The large value of Δ h = 10 feet which seems to agree best with the experimental data is difficult to believe unless Lake Michigan was unusually rough. However, it is not known how far out over the lake the measurements were made and it may be that shore reflections played a big part in the measured results.

THE RAYLEIGH DISTRIBUTION OR RANDOM WALK



$$E_r^2 = E_1^2 + \cdots E_i^2 + \cdots + E_n^2$$

if (1) E_r^2 is a constant, i.e. the mean energy in the reflected wave is a constant

and (2) $E_i^2 << E_r^2$ for $i = 1$ to n

and (3) The phase of each component vector E_i is random

and (4) n is sufficiently large

then (1) The probability that the resultant E_s will be greater than X is given by the cumulative Rayleigh distribution:

$$p_{(E_s > X)} = e^{-(X^2/E_r^2)}$$

and (2) The phase of the resultant vector E_s will also be random

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Fig. 1

THE RESULTANT OF n UNIT VECTORS WITH RANDOM RELATIVE PHASE

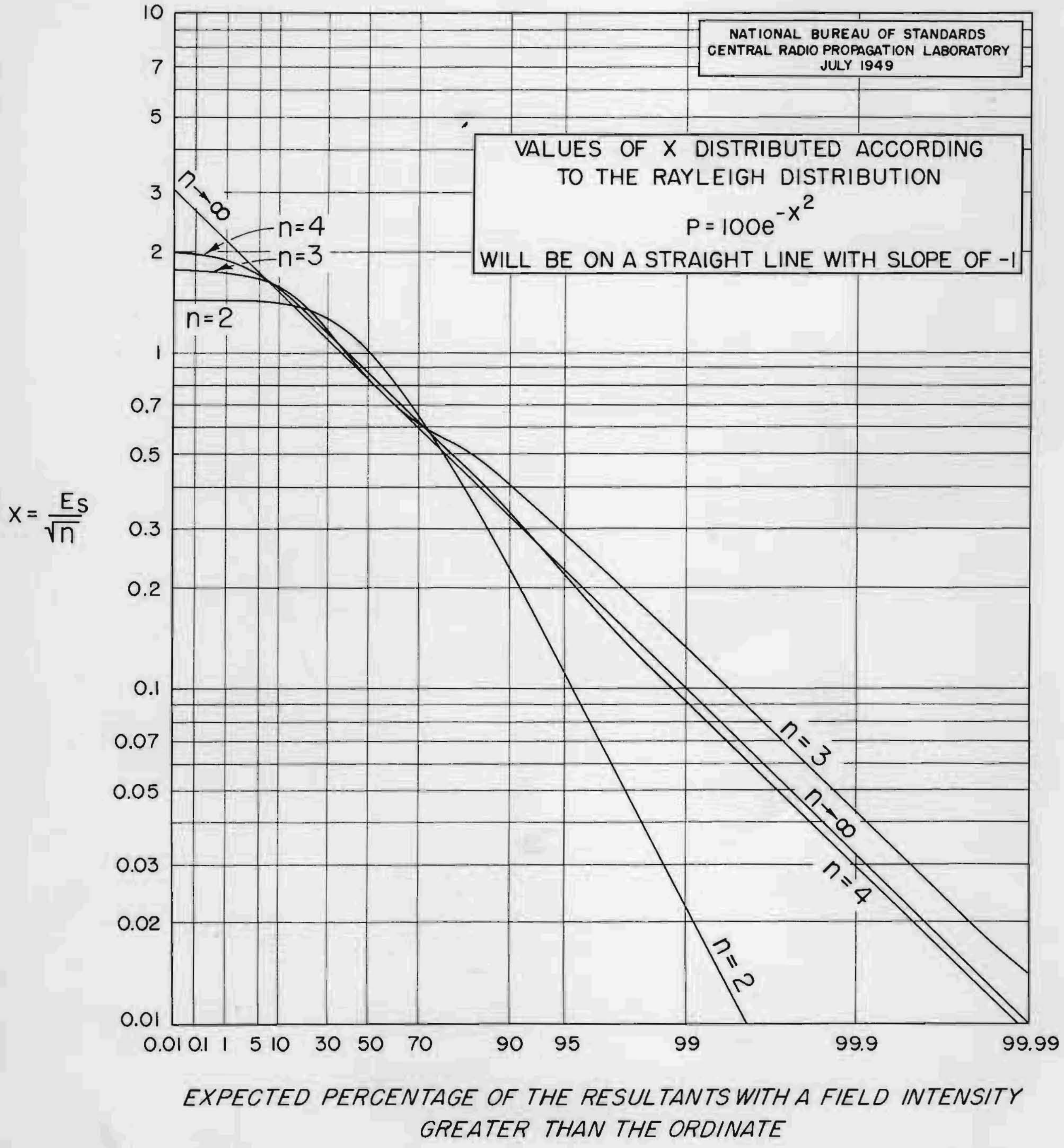

Fig. 2

THE NORMAL DISTRIBUTION OF VOLTAGE, v, DERIVED FROM A RAYLEIGH DISTRIBUTED VECTOR OF AMPLITUDE E_s

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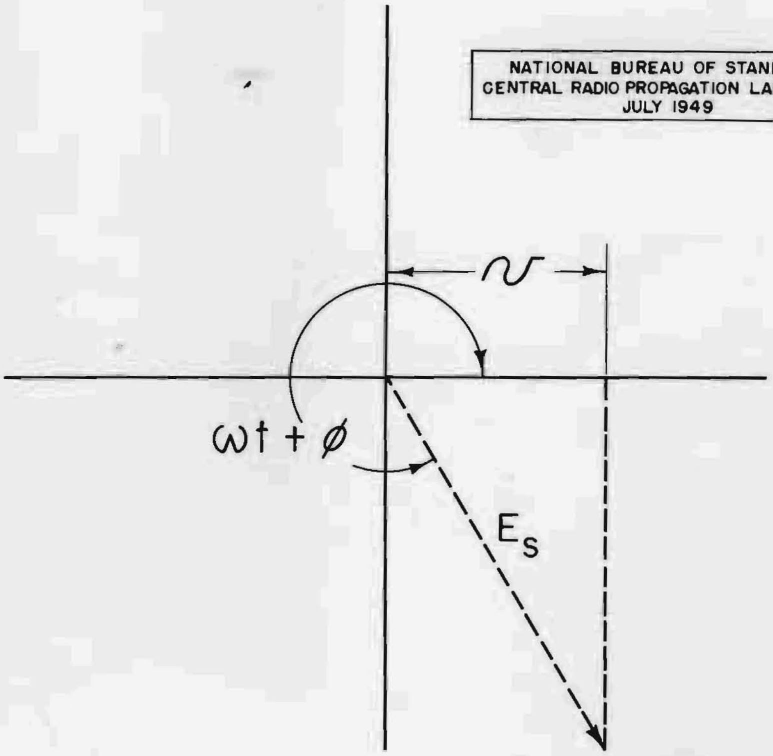


$$v = E_s \sin(\omega t + \phi)$$

if (1) $p(E_s > X) = e^{-(X/E_r^2)}$

and (2) all values of $(\omega t + \phi)$ are equally likely.

then $p(v > X) = \frac{1}{\sqrt{2\pi}} \int_{(X\sqrt{2}/E_r)}^{\infty} e^{-y^2/2} dy$

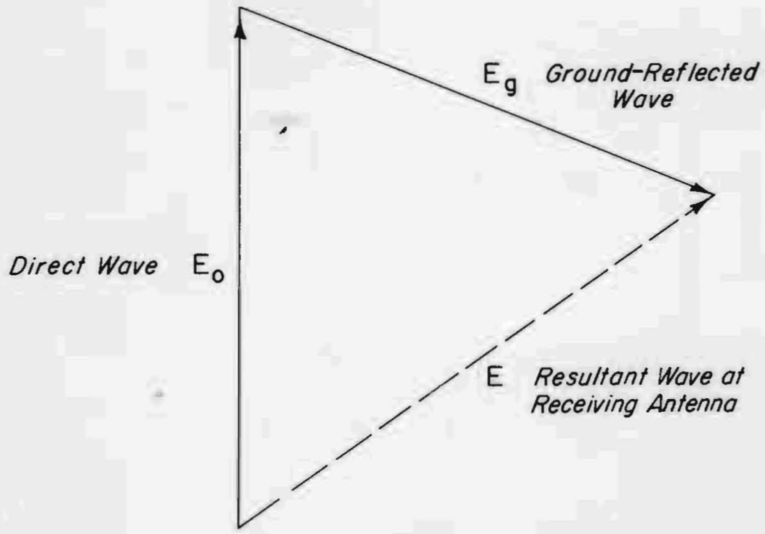
mean value of $v = 0$

standard deviation of v = root-mean-square voltage $= E_r/\sqrt{2}$

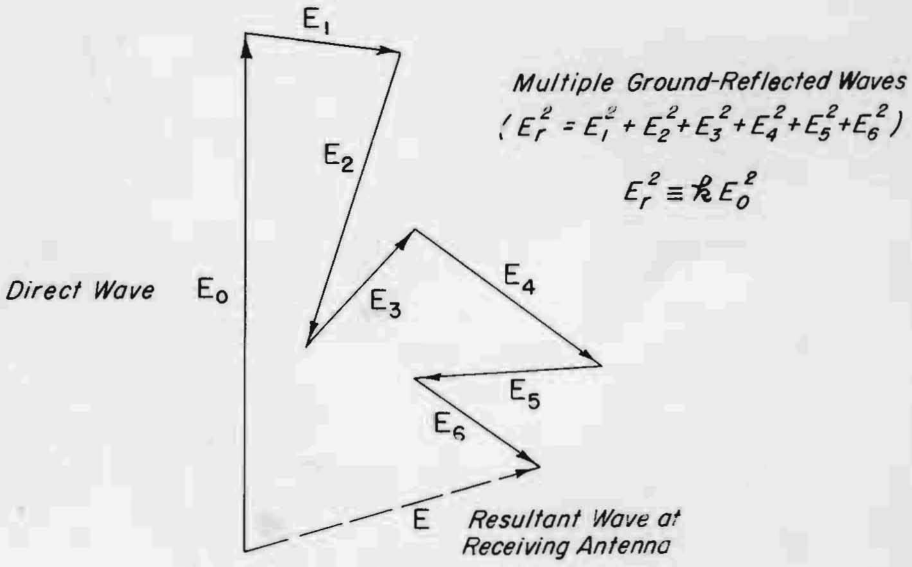
E_r = root-mean-square value of the amplitude E_s

Fig. 3

VECTOR REPRESENTATION OF DIRECT AND GROUND-REFLECTED WAVES FOR SMOOTH OR ROUGH EARTH



a. SMOOTH EARTH



b. ROUGH EARTH

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Fig. 4.

THE SUM OF A DIRECT WAVE PLUS A RAYLEIGH DISTRIBUTED GROUND-REFLECTED WAVE

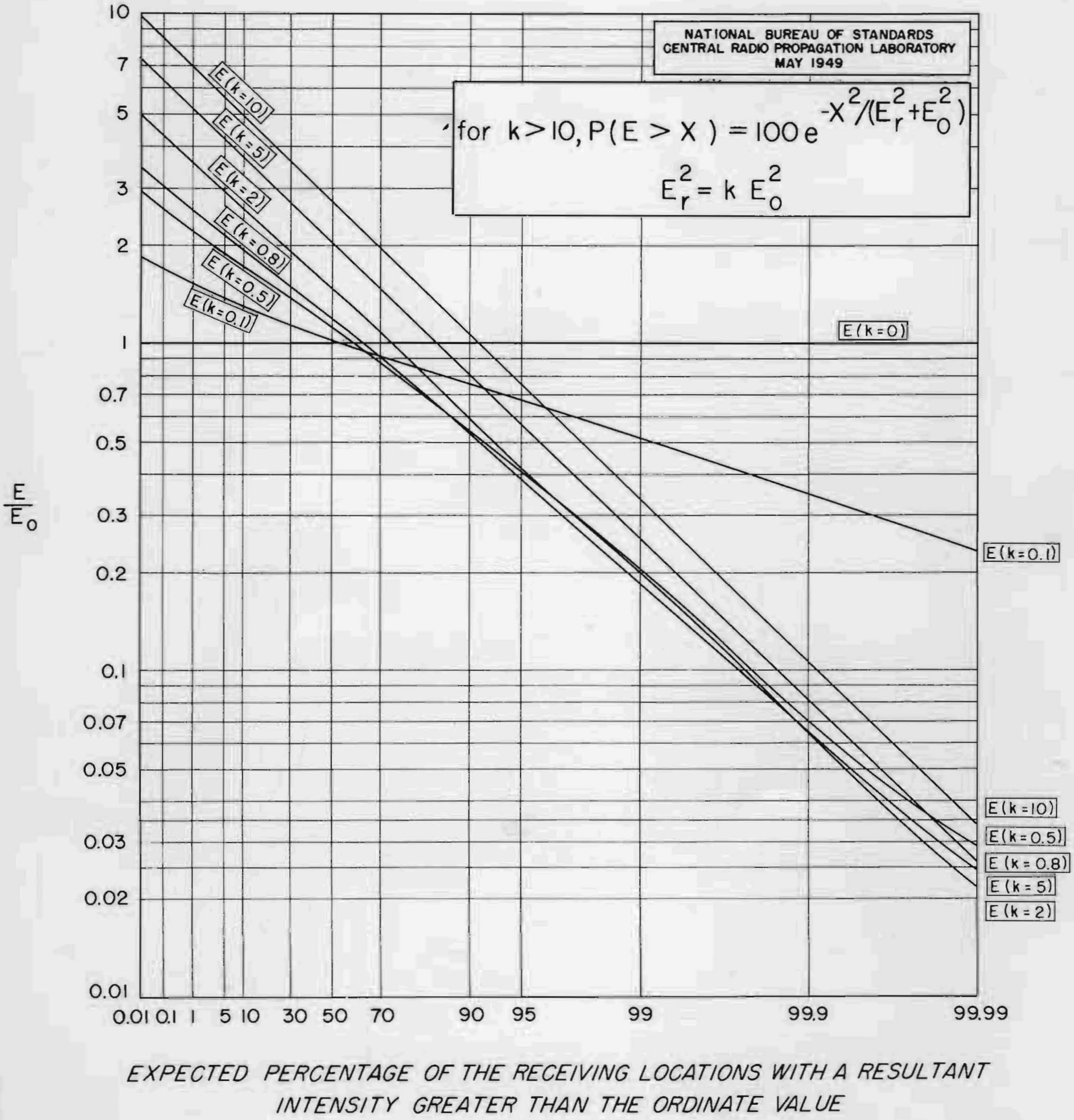


Fig 5

GEOMETRY OF AIR-TO-AIR PROPAGATION EXPERIMENT

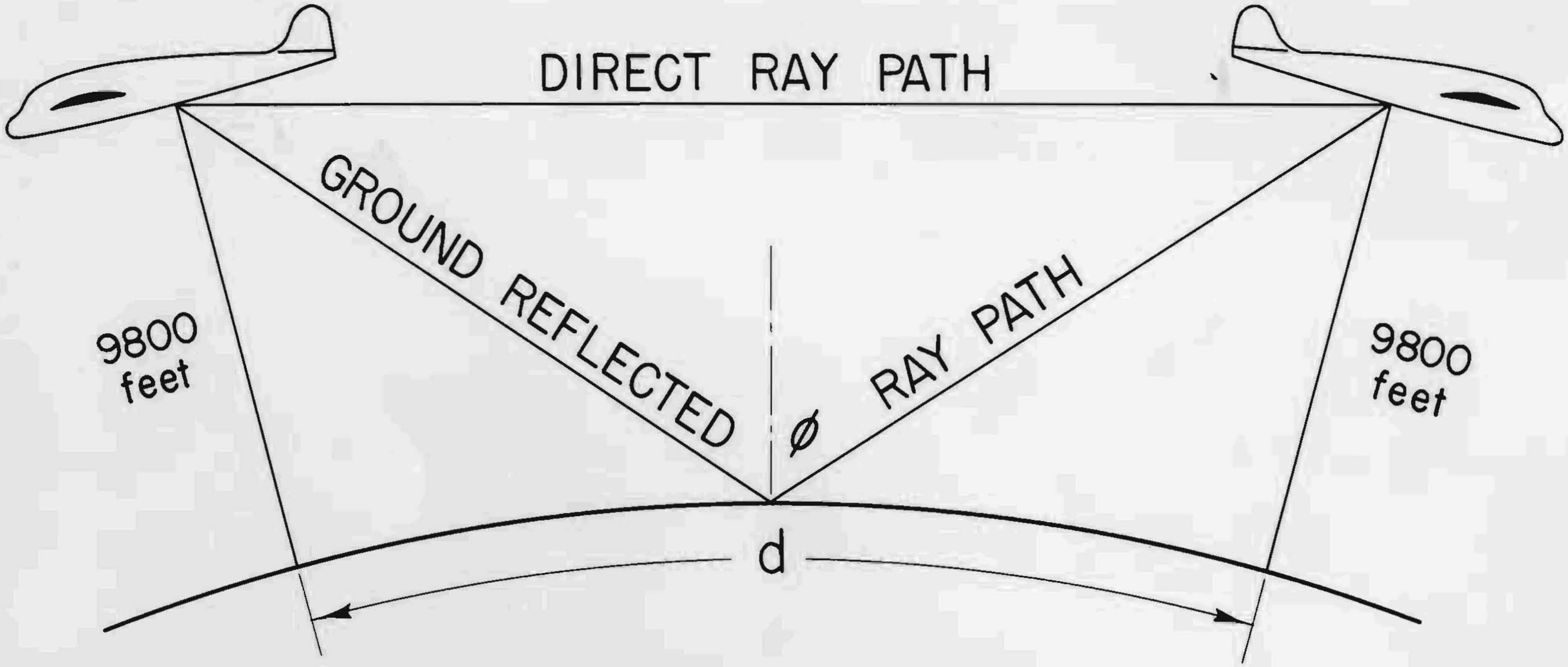


Fig 6.

FIELD INTENSITY VARIATIONS OBSERVED IN AIR-TO-AIR PROPAGATION OVER IRREGULAR TERRAIN

ALTITUDE OF BOTH AIRCRAFT 9800 FEET

FREQUENCY 328.2 Mc/s

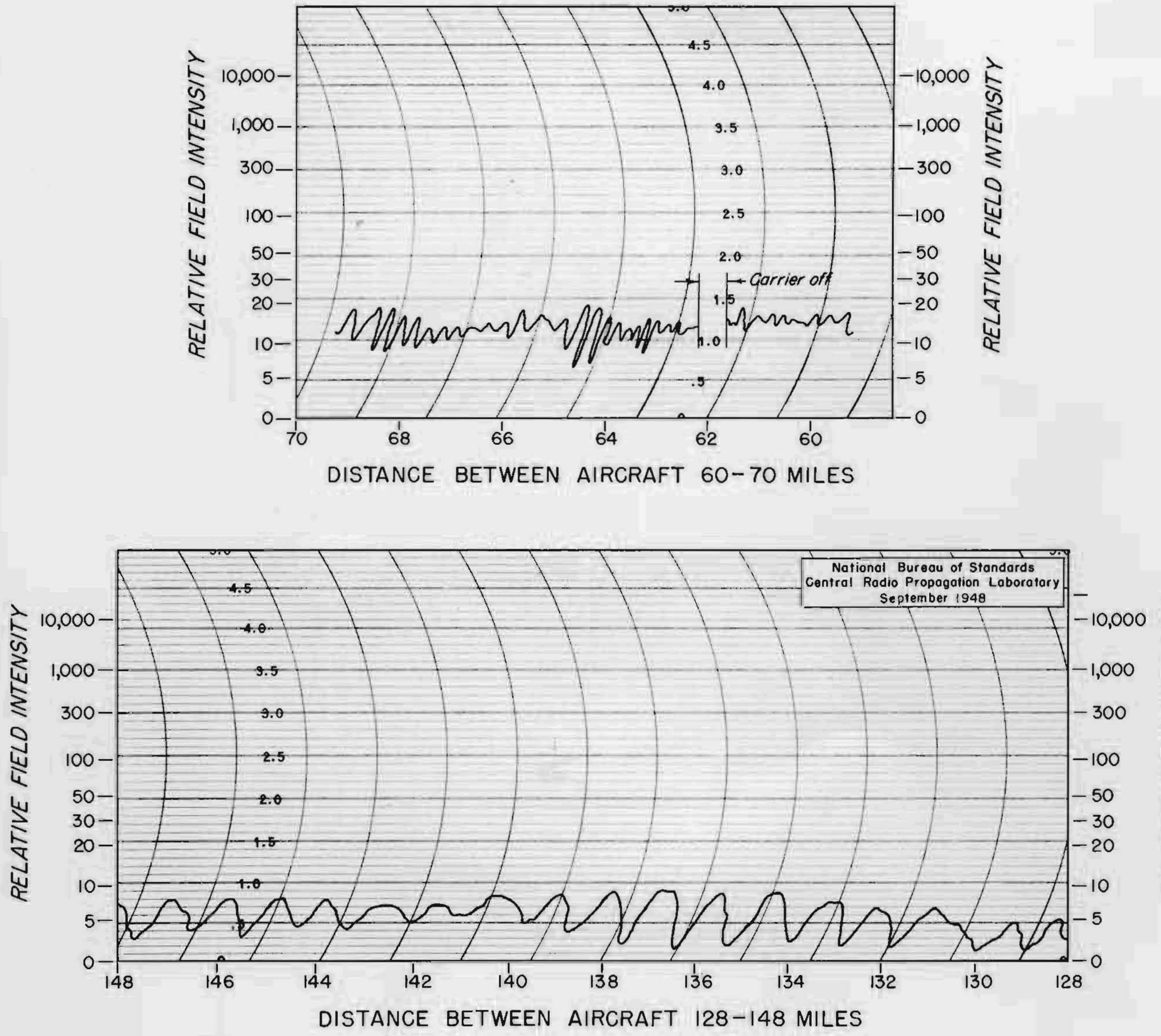
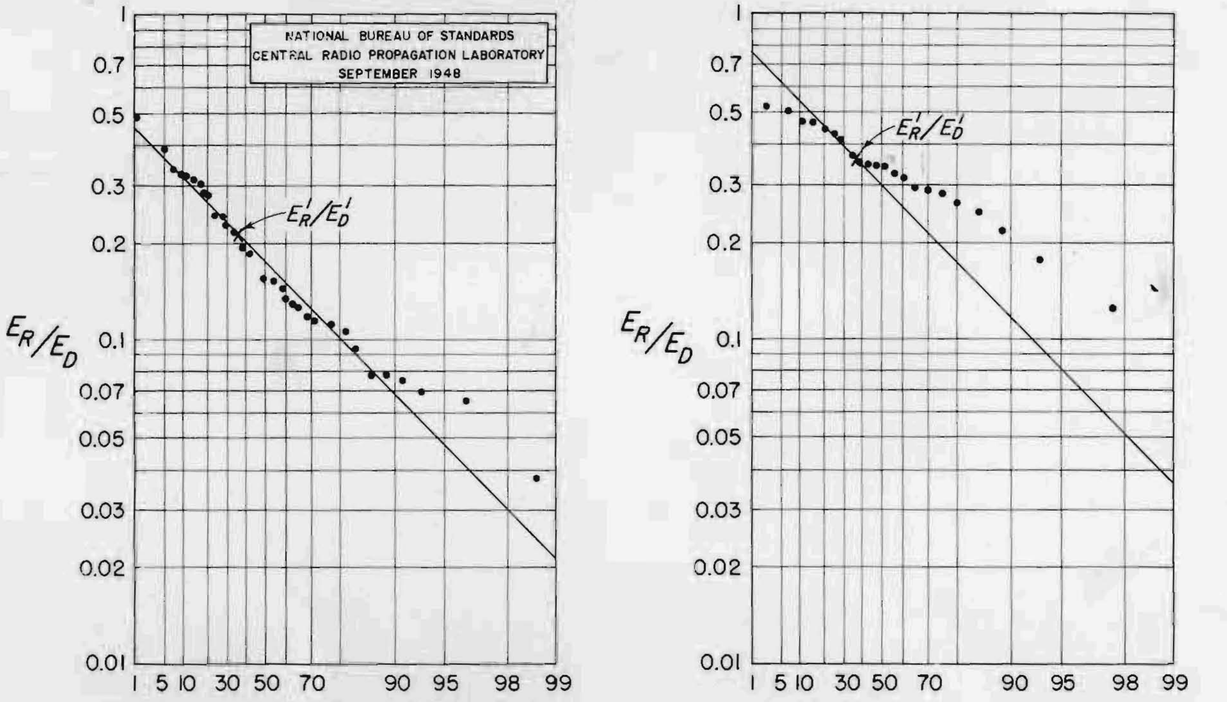


Fig. 7

Fig. 8

DISTRIBUTION OF OBSERVED EFFECTIVE INTENSITY OF GROUND REFLECTED WAVE, E_R, RELATIVE TO THE DIRECT WAVE, E_D, FOR TWO AIRCRAFT FLYING OVER IRREGULAR TERRAIN

FREQUENCY 328 MC/S; ALTITUDE 10,000 FEET



PERCENTAGE OF THE DISTANCE WITH RATIOS HIGHER THAN THE ORDINATE VALUE

60-70 MILES

PERCENTAGE OF THE DISTANCE WITH RATIOS HIGHER THAN THE ORDINATE VALUE

128-148 MILES

Values of E_R/E_D distributed in accordance with the Rayleigh distribution

$$P = 100\, e^{-(E_R/E_D)^2/(E_R'/E_D')^2}$$

would be on the straight line with slope -1

(From data taken by Collins Radio Company, Cedar Rapids, Iowa)

Fig 9

EFFECTIVE REFLECTION COEFFICIENT OF THE GROUND FOR AIR-TO-AIR RADIO PROPAGATION

BOTH AIRCRAFT AT AN ALTITUDE OF 9800 FEET OVER LAND
VERTICAL POLARIZATION

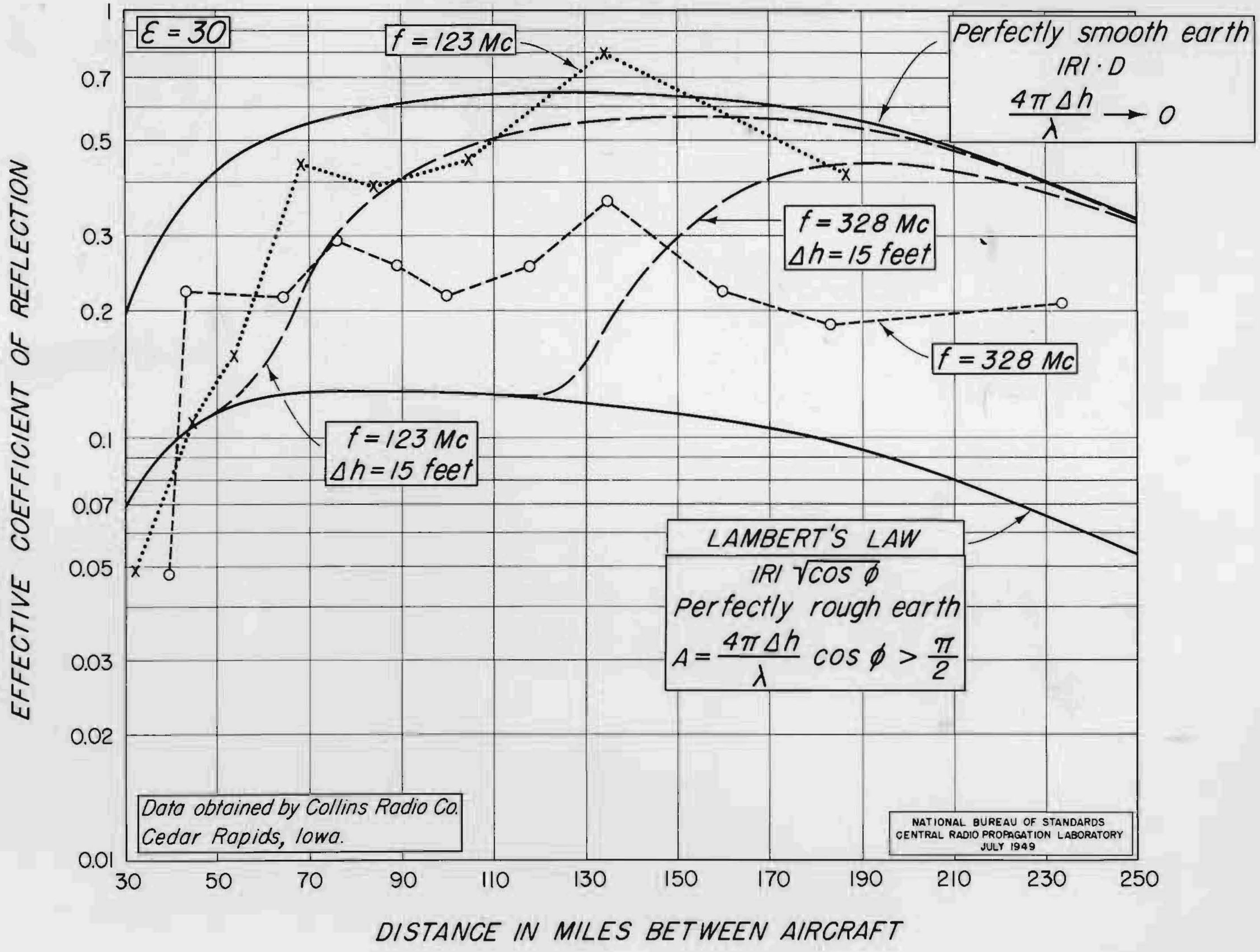
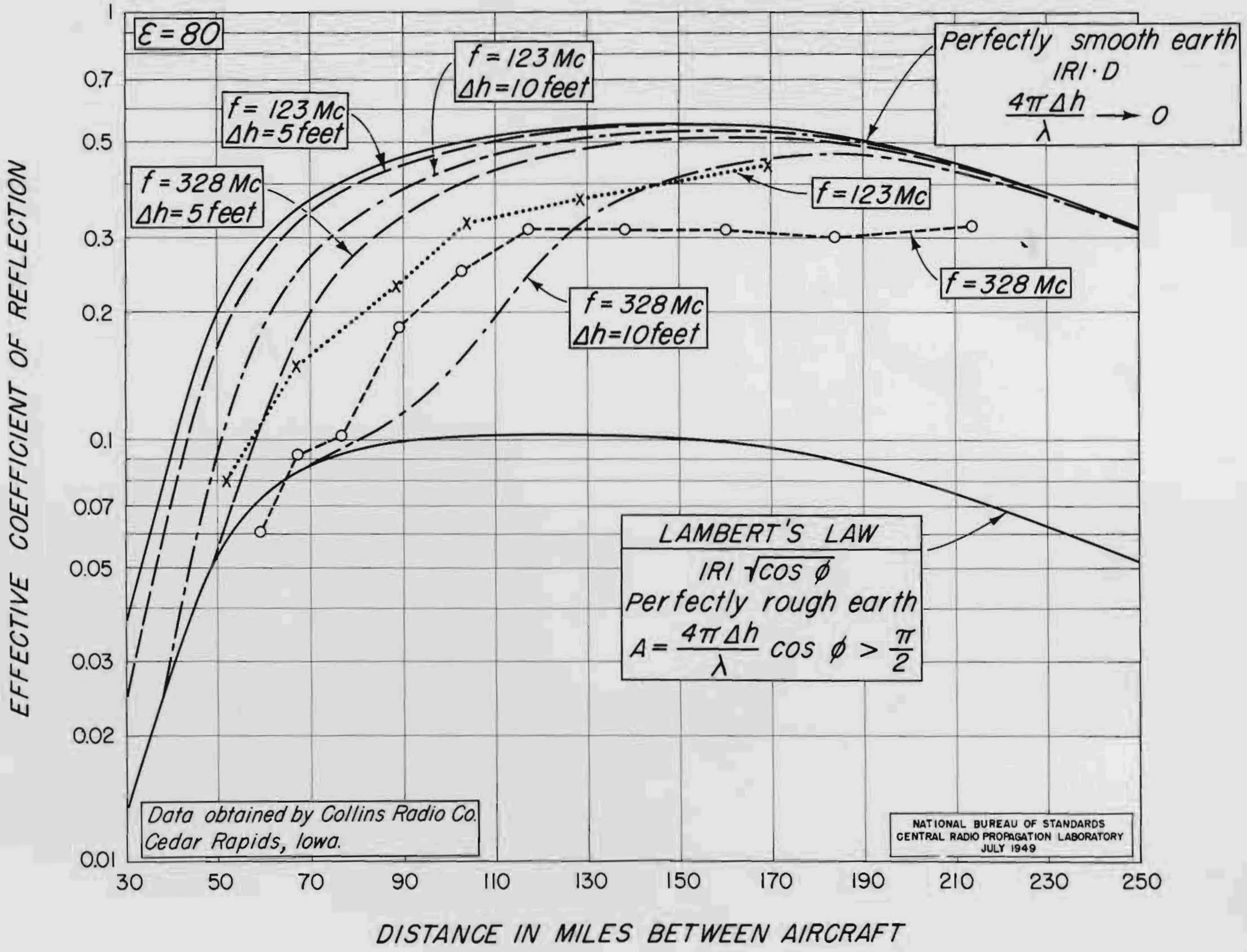


Fig 10

EFFECTIVE REFLECTION COEFFICIENT OF THE GROUND
FOR AIR-TO-AIR RADIO PROPAGATION

BOTH AIRCRAFT AT AN ALTITUDE OF 9800 FEET OVER LAKE MICHIGAN
VERTICAL POLARIZATION



Data obtained by Collins Radio Co.
Cedar Rapids, Iowa.

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