Notes on stellar mass input calculations

1 Assumptions

2 Deprojection of the surface-brightness distribution

The general relations are given by Binney & Merrifield, p. 180:

$$
\begin{aligned}
L(r) &= -\frac{1}{\pi}\int_r^\infty \frac{dI}{dR}\frac{dR}{(R^2-r^2)^{1/2}} \\
I(R) &= 2\int_R^\infty \frac{rL(r)}{(r^2-R^2)^{1/2}}dr
\end{aligned}
$$

where $L(r)$ is the luminosity density and $I(R)$ is the surface brightness.

2.1 Power-law deprojection (as in L&B 2002b)

For a surface-brightness distribution of the form

$$
\begin{aligned}
I(R) &= I_0 R^{-\delta} \\
\frac{dI}{dR} &= -\delta I_0 R^{-(1+\delta)} \\
L(r) &= \frac{\delta}{\pi} I_0 \int_r^\infty \frac{R^{-(1+\delta)}}{(R^2-r^2)^{1/2}}dR
\end{aligned}
$$

With the substitution $\cos\theta = r/R$, this reduces to

$$
\begin{aligned}
L(r) &= \frac{\delta I_0}{\pi} r^{-(1+\delta)} \int_0^{\pi/2} (\cos\theta)^\delta d\theta \\
&= L_0 r^{-(1+\delta)}
\end{aligned}
$$

The inverse problem starts with the luminosity density

$$
\begin{aligned}
L(r) &= L_0 r^{-(1+\delta)} \\
I(R) &= 2L_0 \int_R^\infty \frac{r^{-\delta}}{(r^2-R^2)^{1/2}}dr \\
&= 2L_0 R^{-\delta} \int_0^{\pi/2} (\cos\theta)^{\delta-1} d\theta
\end{aligned}
$$

This time the substitution is $\cos\theta = R/r$. The condition for the two relations to be consistent is

$$\frac{2\delta}{\pi}\int_0^{\pi/2}(\cos\theta)^{\delta}d\theta\int_0^{\pi/2}(\cos\theta)^{\delta-1}d\theta \quad = \quad 1$$

Gradshteyn & Ryzhik 3.621.1 gives the integrals as beta functions:

$$\int_0^{\pi/2}\cos^{\mu-1}x dx \quad = \quad 2^{\mu-2}\mathcal{B}\left(\frac{\mu}{2},\frac{\mu}{2}\right)$$

so

$$\begin{aligned}
& \frac{2\delta}{\pi}\int_0^{\pi/2}(\cos\theta)^{\delta}d\theta\int_0^{\pi/2}(\cos\theta)^{\delta-1}d\theta \\
= \; & \frac{2^{2\delta-2}\delta}{\pi}\mathcal{B}\left(\frac{\delta}{2},\frac{\delta}{2}\right)\mathcal{B}\left(\frac{\delta+1}{2},\frac{\delta+1}{2}\right) \\
= \; & \frac{2^{2\delta-2}\delta}{\pi}\frac{\Gamma(\delta/2)^2\Gamma(\delta/2+1/2)^2}{\Gamma(\delta)\Gamma(\delta+1)} \\
= \; & \frac{2^{2\delta-2}}{\pi}\left[\frac{\Gamma(\delta/2)\Gamma(\delta/2+1/2)}{\Gamma(\delta)}\right]^2 \\
= \; & 1
\end{aligned}$$

Where we have used GR 8.384.1 to express the beta functions in terms of gamma functions and the doubling formula for Gamma functions (GR 8.335.1):

$$\Gamma(\delta) \quad = \quad \frac{2^{\delta-1}}{\pi^{1/2}}\Gamma(\delta/2)\Gamma(\delta/2+1/2)$$

Program `plint` does a numerical check of this result and works out the value of L_0/I_0 given δ.

2.2 Sérsic/de Vaucouleurs deprojection

2.2.1 The Sérsic profile

The Sérsic brightness profile (Ciotti 1991; Graham & Driver 2005) is:

$$\begin{aligned}
I(R) &= I_e\exp[-b_n[(R/R_e)^{1/n}-1]] \\
&= I_0\exp[-b_n(R/R_e)^{1/n}] \\
I_0 &= I_e\exp b_n
\end{aligned}$$

where I_e is the surface brightness at effective radius R_e that encloses half of the light. The De Vaucouleurs profile is the special case with $n = 4$.

The coefficient b_n is defined by the condition

$$\Gamma(2n) = 2\gamma(2n, b_n)$$

Numerical Recipes provides a function `gammp(a,x)` for

$$P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)}$$

(this uses `gcf` and `gser`. We need to solve

$$\frac{\gamma(2n, b_n)}{\Gamma(2n)} = \frac{1}{2}$$

or

$$P(2n, b_n) = \frac{1}{2}$$

Solve this using Brent's method, starting from the approximation

$$b_n \approx 1.9992n - 0.3271$$

which is valid for $0.5 < n < 10$. Implemented as a first step in `deproject`.

2.2.2 Numerical deprojection

This is done in `deproject`. The luminosity density is:

$$L(r) = \frac{I_e b_n}{n\pi} \int_r^\infty \left(\frac{R}{R_e}\right)^{1/n} \exp[-b_n[(R/R_e)^{1/n} - 1]] \frac{dR}{R(R^2 - r^2)^{1/2}}$$

We use the dimensionless luminosity density $\nu(s)$, where $s = r/R_e$ and $L(r) = (I_0/R_e)\nu(s)$ (Ciotti 1991) and make the substitution $\cos\theta = r/R$, to get:

$$\nu(s) = \frac{b_n}{n\pi} s^{1/n-1} \int_0^{\pi/2} (\sec\theta)^{1/n} \exp[-b_n(s\sec\theta)^{1/n}] d\theta$$

The derived luminosity density agrees with that plotted in Fig. 2 of Ciotti (1991) for values of n between 2 and 10. The values quoted by Mazure & Capelato (2001), which are claimed to be consistent with Young (1976), have been "normalised by a factor $\exp(b_4)\pi 8!/b_4^8$)", but they also use a different notation and definition of $\nu(s)$ (their I_0 is our I_e). Their numbers therefore appear to have been divided by $\pi 8!/b_4^8) = 1.05839366 \times 10^{-2}$ compared with ours. `deproject` agrees well with their Table 1 in the range s = 0.01 – 10 for the de Vaucouleurs case.

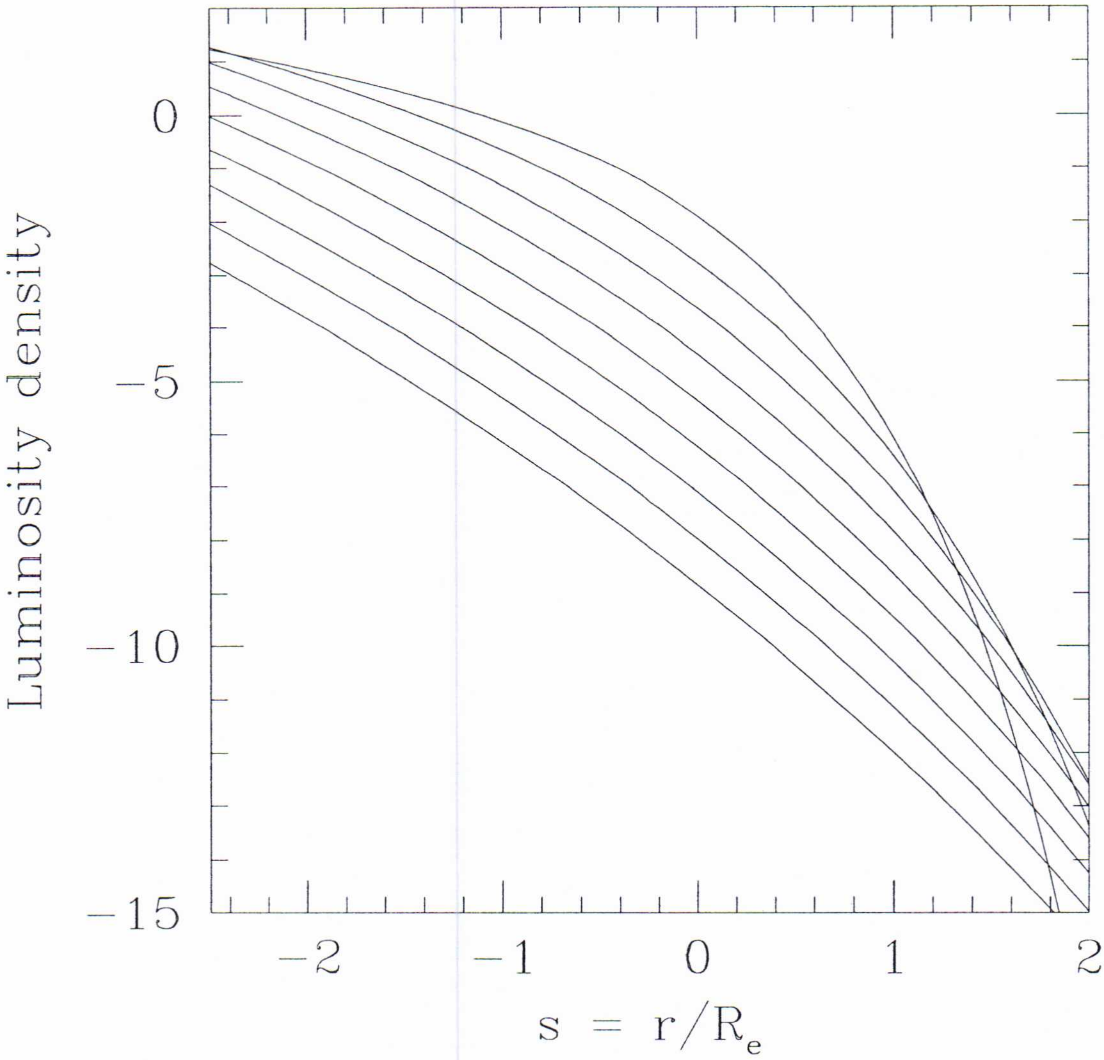

Figure 1: The dimensionless luminosity density $\nu(s)$ for the Sérsic profile, for comparison with Ciotti (1991) Fig. 2.

2.2.3 Mellier & Mathez method

I do not understand the normalization. Fortunately, this is not a problem for the published curve in Laing & Bridle (2002b), which was normalized to and aperture magnitude.

3 Stellar luminosity and mass-loss rate

3.1 Solar colours and absolute magnitude in Kron-Cousins system

Solar absolute magnitudes in the Johnson system (AQ) are: $M_{R\odot} = +4.30$, $M_{V\odot} = +4.82$ and $M_{B\odot} = +5.48$ (AQ). Fernie (1983) gives the relation between $V - R$ in the Kron-Cousins and Johnson systems:

$$(V - R)_{\rm KC} = -0.024 + 0.730(V - R)_{\rm J}$$

Both systems use the same V band, so $M_{R\odot} = +4.46$ in the Kron-Cousins system. We will also need the colour $B - R = 1.18$ (Johnson).

3.2 Relation between mass-loss rate and luminosity

We started with the mass loss rate predicted by Faber et al. (1976) for an elliptical galaxy stellar population as a function of the blue luminosity, L_B, in solar units,

$$(\dot{M}/M_\odot\,{\rm yr}^{-1}) = 0.015(L_B/10^9 L_{B\odot})$$

which is consistent with the estimate from infrared observations by Knapp et al. (1992):

$$(\dot{M}/M_\odot\,{\rm yr}^{-1}) = 0.0021(L_K/10^9 L_{K\odot})$$

(scaling from the B-band value would give 0.0026).

The B-band value needs to be scaled to the right wavelength for the observations using the extinction-corrected magnitudes for the Sun and the radio galaxy. [We could use the K-band value directly for NGC 315.] The scaling constant is $0.015 \times 10^{[(B-R)_\odot - (B-R)_{\rm gal}]/2.5}$.

4 Application to individual sources

4.1 3C 31

4.1.1 General

1. We started with the R-band CCD photometry of Owen & Laing (1989). A galactic extinction of $A_R = 0.189$ (Schlegel et al. 1998) was removed, a K-correction was applied, assuming a flat spectrum as in Owen & Laing (1989):

$$\frac{\sigma_{\rm rest}}{\sigma_{\rm obs}} \approx (1+z)^4$$

Table 1: Magnitudes and extinctions for NGC 383.

Band	m	A	m - A
B	13.28	0.305	12.98
R	11.26	0.189	11.07

giving a correction of -0.072 mag for $z = 0.0167$ and the fit was converted to absolute magnitude (note the change of Hubble Constant to $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ from that used by Owen & Laing 1989). This conversion assumed a luminosity distance of $cz/H_0 = 71.52\,\mathrm{Mpc}$ for $z = 0.0167$. In the original calculation, we subtracted 34.533 mag to convert from apparent to rest-frame absolute mag arcsec^{-2}.

2. Next, convert to solar luminosities assuming an absolute magnitude of 4.46 for the Sun in the Kron-Cousins R band (Astrophysical Quantities, Fernie).

$$I(R)/L_{\odot}\mathrm{arcsec}^{-2} = 10^{[38.993-\sigma(R)]/2.5}$$

3. The surface-brightness distribution was then deprojected (details depend on the functional form – see below).

4. Scale the mass loss rate from Faber et al. (1976) to the R band using extinction-corrected colours for NGC 383 (Sandage, Schegel et al. 1998) and the Sun (Astrophysical Quantities). For NGC 383, the values are given in Table 1. so $B - R = 1.91$ (Johnson), corrected for extinction. For the Sun (see above) $B - R = 1.18$ (Johnson). The scaling constant is therefore $0.015 \times 10^{[(B-R)_{\odot}-(B-R)_{\mathrm{gal}}]/2.5} = 0.077$ and

$$(\dot{M}/M_{\odot}\,\mathrm{yr}^{-1}) = 0.0077(L_R/10^9 L_{R\odot})$$

4.1.2 Power-law deprojection

1. The surface-brightness profile is well fitted by a power-law surface-brightness distribution

$$\sigma(R)/\mathrm{mag\,arcsec}^{-2} = 15.53 - 2.5\delta\,\mathrm{lg}(R/\mathrm{arcsec})$$

with $\delta = 1.65$ (index quoted by Owen & Laing 1989; normalization by reading the data off a postscript version of the plot and fitting it). After extinction and K-correction, the rest-frame surface brightness in solar units is

$$
\begin{aligned}
I(R)/L_\odot \mathrm{arcsec}^{-2} &= 10^{[23.463-2.5\delta \lg(R/\mathrm{arcsec})]/2.5} \\
&= 2.428 \times 10^9 (R/\mathrm{arcsec})^{-\delta}
\end{aligned}
$$

2. Convert to distance, assuming 0.34 kpc arcsec^{-2}, giving

$$
I(R)/10^9 L_\odot \mathrm{kpc}^{-2} = 3.54(R/\mathrm{kpc})^{-1.65}
$$

3. The surface-brightness distribution was then deprojected to give the luminosity density as above. For $\delta = 1.65$, $I_0/L_0 = 0.4435$, so

$$
L(r)/10^9 L_\odot \mathrm{kpc}^{-3} = 1.57(r/\mathrm{kpc})^{-2.65}
$$

4. Then use the conversion factor between R-band luminosity in solar units and mass-loss rate to give

$$
\begin{aligned}
(\dot{M}/M_\odot\, \mathrm{yr}^{-1}\mathrm{kpc}^{-3}) &= 1.21 \times 10^{-2}(r/\mathrm{kpc})^{-2.65} \\
(\dot{M}/\mathrm{kgyr}^{-1}\mathrm{pc}^{-3}) &= 2.41 \times 10^{19}(r/\mathrm{kpc})^{-2.65}
\end{aligned}
$$

($\mathrm{M}_\odot = 1.989 \times 10^{30}$ kg).

4.1.3 De Vaucouleurs deprojection

4.2 NGC 315

5 Mass input in the inner jets

$$
\frac{d}{dz}(n_p \Gamma \beta A) = \dot{n}_p A
$$

where $\dot{n}_p$ is the number of protons injected per unit volume and time, whic is a Lorentz invariant. Hence the mass flux is

$$
\Psi = \int_0^{z_{\mathrm{flare}}} \dot{m} dz
$$

5.1 3C 31

Suppose initially that the power-law fit can be extrapolated into the nucleus (this is unreasonable, as the enclosed luminosity does diverges as $r \to 0$ for the measured index, although the luminosity density does not). Then the mass input rate is $2.41 \times 10^{19}(z/\mathrm{kpc}^{-2.65}$ kg pc^{-3} yr^{-1}. For the jet geometry of Laing & Bridle (2002a), the area is $\pi(z \tan \xi_1)^2$ so the mass input rate per unit length of jet is $\dot{m} = 3.34 \times 10^{19}(z/\mathrm{kpc}^{-0.65}\mathrm{kg\,kpc}^{-1}\,\mathrm{s}^{-1}$.

However, the luminosity density must certainly flatten off at small r and indeed is measured to do so for large ellipticals (core galaxies).

6 Mistakes, worries, questions and things to do

For 3C 31 we used z = 0.0167 and a crude value of the luminosity distance (cz/H_0). For consistency, we should adopt z = 0.0169 and a concordance cosmology. Also check that the luminosity distance produced by `flat` and the adopted K-correction are mutually consistent.

We write the mass-loss rate in terms of solar luminosities in the R band using Johnson R, but then calculate the luminosity in the Kron-Cousins system. Is this logically correct? Seems odd to base everything on solar luminosities for an old stellar population: is that what Knapp et al. did?

The plotted stellar mass loss curve in Laing & Bridle (2002b) is actually derived from the de Vaucouleurs fit in Owen & Laing (1989) rather than the power-law fit as it says in the text. Need to confess.

Try using the Knapp et al. K-band mass-loss rate directly for NGC 315.

References

Binney & Merrifield

Ciotti, L, 1991. A&A 249, 99

Magure A & Capelato HV 2002 A&A 383, 384

Mellier Y & Mathez G 1987 A&A 175, 1

Young PJ 1976, AJ 81, 807

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Graham A W & Driver SP, 2005, PASA 22, 118.