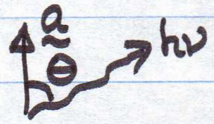
Thomson Scattering by free electrons (MKS units)

EM theory → power radiated by accelerated charge

$$\frac{dP(\theta)}{d\Omega} = \frac{q^2 a^2 \sin^2\theta}{16\pi^2 \epsilon_0 c^3} \text{ watts/steradian}$$



$$P = \int_{4\pi} dP(\theta)\, d\Omega = \frac{q^2 a^2}{6\pi\epsilon_0 c^3} \text{ watts}$$ (LARMOR'S FORMULA)

Consider free electron exposed to incident $\underset{\sim}{E}$-field from em wave

$|B| = \frac{1}{c}|E|$ so $\underset{\sim}{v} \times \underset{\sim}{B} \sim \frac{v}{c}|E|$ can be neglected for $v \ll c$

acceleration $a = qE/m$. Take $h\nu \ll m_0 c^2$ so wave approx. o.k.

So scattered power $\langle P \rangle = \dfrac{q^4 \langle E^2 \rangle}{6\pi\epsilon_0 m^2 c^3}$ watts $\langle\ \rangle$ - time average

If average power density (w/m^2) in incident wave is $\langle I \rangle$ then

$$\langle I \rangle = \epsilon_0 c \langle E^2 \rangle$$

Define σ — scattering cross-section of free electron — as area of incident rad^n containing scattered power, i.e.

$$\langle P \rangle = \sigma \langle I \rangle$$

then

$$\sigma = \sigma_T = \frac{q^4}{6\pi\epsilon_0^2 m^2 c^4}$$

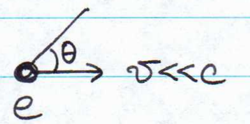
$$\sigma_T = \frac{8\pi}{3} r_0^2$$

THOMSON CROSS-SECTION FOR ELECTRON-"PHOTON" SCATTERING

$6.652 \times 10^{-29}\ m^2$

where r_0 is "classical electron radius" defined by $mc^2 = \dfrac{q^2}{4\pi\epsilon_0 r_0}$

Radiation Drag (Thomson drag)



Radiation striking electron from angle θ appears to have brightness temperature

$$T(\theta) = T_\perp\left(1 + \frac{v}{c}\cos\theta\right)$$

Energy flux / steradian $= u(\theta) \cdot \frac{c}{4\pi} = aT^4(\theta) \cdot \frac{c}{4\pi}$ [Watts/m^2/ster]

Momentum flux / steradian $= \frac{E(\theta)}{c} = \frac{aT^4(\theta)}{4\pi} = \frac{a}{4\pi} T_\perp^4 \left(1 + \frac{v}{c}\cos\theta\right)^4$

Electron reradiates; in its own frame $\sin^2\theta$ dependence $\rightarrow$ no net momentum radiated

So, net momentum absorbed per unit time from all directions in black-body radiation field is

$$F_{drag} = -\int_{4\pi} \sigma_T \cdot \frac{a}{4\pi} T_\perp^4 \left(1 + \frac{v}{c}\cos\theta\right)^4 \cdot \cos\theta \, d\Omega$$

$$= -\frac{4}{3}\sigma_T \cdot \frac{v}{c} \cdot aT^4$$

per electron

F_{drag}/volume $= -\frac{4}{3}\sigma_T \cdot n_e \cdot \frac{v}{c} aT^4$ [n_e = # density of electrons]

$$= -\frac{4}{3}\sigma_T \cdot n_e \cdot \frac{v}{c} \cdot \rho_r c^2$$

$$= -\frac{4}{3}\sigma_T \cdot n_e \cdot \rho_r v c$$

Can self-gravity of region overcome drag forces and "turn around" Hubble flow?

Consider region density ρ, scale radius L

$$F_{grav/vol} \sim \frac{GM^2}{R^2 V} \sim \frac{G.\left[\frac{4}{3}\pi L^3 \rho\right]^2}{L^2 . \frac{4}{3}\pi L^3} \sim \frac{4}{3}\pi G L \rho^2$$

For differential velocities ~ Hubble flow we need $v_{rel} = HL$ [turn-around]

at which $F_{drag} \sim \frac{4}{3}\sigma_T . n_e . HL \rho_r c$

To get local collapse we need $F_{grav} > F_{drag}$ when $v \sim HL$

$$\frac{4}{3}\pi G \cancel{L} \rho^2 > \frac{4}{3}\sigma_T . n_e . H\cancel{L} \rho_r c \qquad (L \text{ cancels})$$

Fully-ionised matter, $\rho \sim n_e m_p$ $\qquad m_p$ = mass of proton

So need $\qquad \rho^2 > \sigma_T \frac{\rho}{m_p} . \frac{H \rho_r c}{\pi G}$, i.e. $\rho > \left(\frac{\sigma_T H c}{\pi G m_p}\right)\rho_r$

Note that $H(t)$ is Hubble parameter at the time t in model that we want turn-around to occur.

In radiation era $\quad R(t) = A\sqrt{t}$, $\quad \dot{R}(t) = \frac{1}{2}At^{-1/2}$, $\quad H = \dot{R}/R = 1/2t$

i.e. we need $\rho(t) > \left(\frac{\sigma_T c}{2\pi G m_p t}\right)\rho_r(t)$

$$\rho(t) > \frac{6.652\times10^{-29} \times 2.9979\times10^{8}}{2\times\pi\times 6.67\times10^{-11}\times 1.6726\times10^{-27}} \frac{\rho_r(t)}{t_{sec}}$$

$$\rho(t) > 2.84\times10^{16} \frac{\rho_r(t)}{t_{sec}}$$

ρ [gravitating] $\qquad \rho_r$ [radiative].

Towards the end of the plasma era, $t \sim$ few $\times 10^{13}$ sec $\quad$ ($T=3500$ at $t=1.88\times10^{13}$ sec)

$\rightarrow$ need $\rho_{gravitating} \sim 1500\ \rho_r(t)$ for turn-around. $\quad$ $\rho_{grav} \sim 1510\ \rho_r$(rec)

But this is before end of radiation era, so $\bar{\rho}_m < \rho_r$ in models at this time

— i.e. only gross overdensities $\rho_{grav}/\bar{\rho}$ could turn around in the presence of Thomson drag.

Must also be overdensities involving ρ_m only, not ρ_r (otherwise f_{drag} increases too)

As $\bar{\rho}_m < 0.15\rho_r$ for most of rad^n. era, would need $\rho_m/\bar{\rho}_m \gtrsim 10^4$.

$\longrightarrow$ Thomson drag prohibits growth of reasonable gravitational fluctuations throughout the plasma era.

At recombination $n_e \rightarrow 0$, $f_{drag} \rightarrow 0$. and gravitational fluctuations now have a chance to turn around against Hubble flow.

Photon diffusion in plasma era

Photon mean free path $\lambda = \dfrac{1}{n_e \sigma_T} = \dfrac{m_p}{\rho_m \sigma_T}$

$= \dfrac{25}{\rho_m}$ metres.

In elapsed time t, photons diffuse $\sqrt{N}\lambda$ where N = # of scatterings (RANDOM WALK)

# of scatterings $= ct/\lambda$ on average

So distance diffused by photon through plasma, $L = \sqrt{\dfrac{ct}{\lambda}}\cdot\lambda$

ADIABATIC FLUCTUATIONS ($n_\gamma/n_b \sim$ constant): $= \sqrt{ct\lambda}$

Volume "smeared out" by photon diffusion within one expansion time $(\sim t$ for $R(t) \sim \sqrt{t})$ is

$$\sim \frac{4}{3}\pi L^3 \sim \frac{4}{3}\pi (ct\lambda)^{3/2}$$

Mass "smeared out" within one expansion time is $\sim \frac{4}{3}\pi \rho_m L^3$

$$M_s \sim \frac{4}{3}\pi\rho_m (ct\lambda)^{3/2} \sim \frac{4}{3}\pi\rho_m c^{3/2} t^{3/2}\left(\frac{25}{\rho_m}\right)^{3/2}$$

$$\sim \underline{\underline{2.7\times10^{15}\, t^{3/2}\, \rho_m^{-1/2}}}$$

By end of plasma era $(\lambda \to \infty)$, $t \sim 1.8\times10^{13}$ s

$\rho_m \sim 10^{-18}$ kg/m^3 [9.8×10^{-19}]

$\overline{M}_s \sim 2.08\times10^{44}$ kg or $\sim 10^{14}\,M_\odot$ (typical cluster mass)

Only the most massive scales in the adiabatic fluctuation spectrum will survive through the plasma era without being attenuated due to matter-photon coupling.

Once the matter recombines $\lambda \to \infty$, there is no photon damping.

- $\bar{M}_S$ will be mass of region over which radiation begins to smear out adiabatic fluctuations.

If take region over which optical depth = 1, $M_{\tau=1} \sim \rho_m \lambda^3 \cdot \frac{4}{3}\pi$

$\sim 6.5\times10^{4}/\rho_m^2$ kg $\rightarrow$ 6.8×10^{40} kg at $t = t_{rec}$ ($\rho_m = \sim 9.8\times10^{-19}$ kg/m³)

$\rightarrow$ $3.4\times10^{10} M_\odot$ (typical small galaxy mass)

Note that at $t = t_{rec}$, $L/\lambda = \sqrt{(\#\text{ of scatterings})} = \sqrt{\frac{ct}{\lambda}} = \sqrt{\frac{ct\rho_m}{25}} = \underline{7.2}$

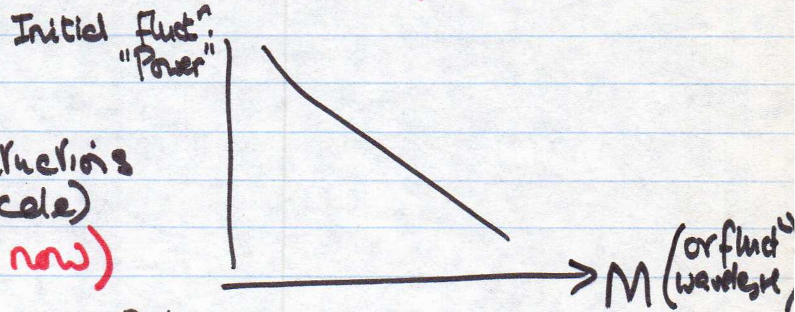
(~ 52 scatterings in that time)

→ EXPECT ADIABATIC FLUCTUATIONS IN RANGE

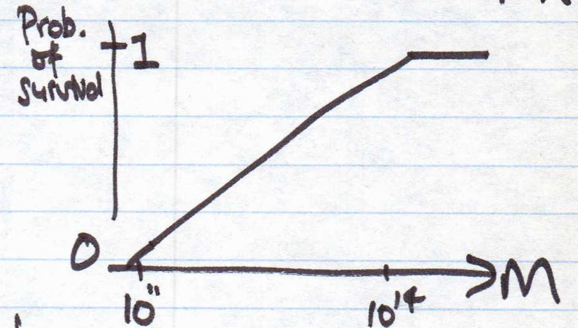
$\sim 3\times10^{10} M_\odot$ – $10^{14} M_\odot$ to "survive", and no damping of > few

$\times10^{14} M_\odot$.

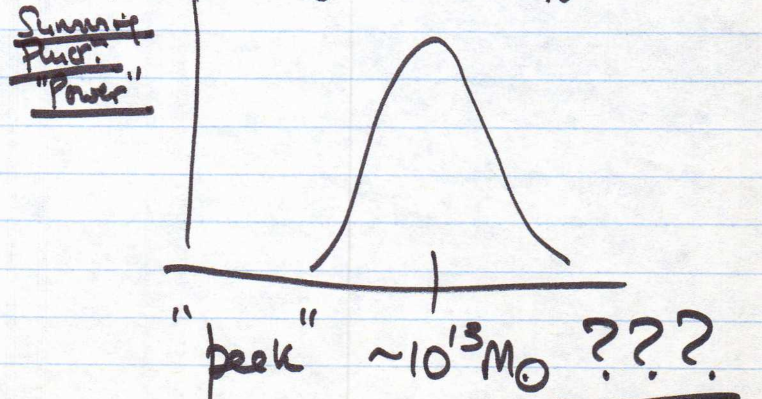
- Presumably initial "spectrum" of fluctuations decreases towards high mass (large-scale) fluctuations (→ large-scale UNIFORMITY now)



Survival of damping by radiation would "filter" this initial spectrum so that low-mass end is "erased"



Combination at $t \sim t_{rec}$ →



"peak" $\sim10^{13} M_\odot$???

- in surviving adiabatic fluct. spectrum

ISOTHERMAL FLUCTUATIONS (n_γ = constant)

are not ironed out by radiation pressure, and would survive <u>undamped</u> to t = trec (but prevented from self-gravitating by radiation drag). (<u>Photon diffusion</u> does not iron out isothermal fluctuations). This → emergence of "all" isothermal perturbⁿ. scales t > trec.

<u>Catch</u>: How does Universe <u>make</u> an isothermal fluctuation, given the coupling between n_γ and n_b originally? Varying matter/antimatter <u>excesses</u>? (Remember that only ~ 1 in 10^9 is supposed to survive from "original" particles).

Probably both adiabatic and isothermal perturbⁿˢ. are idealised <u>extremes.</u> General perturbⁿ. affects ρ_m <u>and</u> n_γ/n_b (entropy per baryon).

Then → some surviving "isothermal" component of <u>all</u> original perturbations, possibly <u>some</u> preference for masses ~ $10^{13} M_\odot$? because of "adiabatic" cpt.

<u>What mass scales become characteristic at t > trec ?</u>

Derivation of the Jeans Length Criterion for stability of gravitating fluid.

MASS CONSERVATION $\quad D\rho/Dt = -\rho\underset{\sim}{\nabla}.\underset{\sim}{v} \Rightarrow \dfrac{\partial\rho}{\partial t} + (\underset{\sim}{v}.\underset{\sim}{\nabla})\rho = -\rho\underset{\sim}{\nabla}.\underset{\sim}{v}$

$$\Rightarrow \boxed{\frac{\partial\rho}{\partial t} = -(\underset{\sim}{v}.\underset{\sim}{\nabla})\rho - \rho\underset{\sim}{\nabla}.\underset{\sim}{v}} \qquad (1)$$

GRAVITATION $\quad \boxed{\underset{\sim}{\nabla}.\underset{\sim}{F} = -4\pi G\rho} \qquad (2)$

EULER MOMENTUM EQUATION $\quad D\underset{\sim}{v}/Dt = \underset{\sim}{F} - \frac{1}{\rho}\underset{\sim}{\nabla}p$

$$\Downarrow$$

$$\boxed{\frac{\partial\underset{\sim}{v}}{\partial t} + (\underset{\sim}{v}.\underset{\sim}{\nabla})\underset{\sim}{v} = \underset{\sim}{F} - \frac{1}{\rho}\underset{\sim}{\nabla}p} \qquad (3)$$

Use subscript 0 for equilibrium values of p, ρ, v
1 for perturbations from equilibrium

To first order in perturbation, $\dfrac{\partial v}{\partial t} = \dfrac{\partial\underset{\sim}{v}_0}{\partial t} + \dfrac{\partial\underset{\sim}{v}_1}{\partial t}$

$$(\underset{\sim}{v}.\underset{\sim}{\nabla})\underset{\sim}{v} \rightarrow ((\underset{\sim}{v}_0+\underset{\sim}{v}_1).\underset{\sim}{\nabla})(\underset{\sim}{v}_0+\underset{\sim}{v}_1) \rightarrow (\underset{\sim}{v}_0.\underset{\sim}{\nabla})\underset{\sim}{v}_0 + (\underset{\sim}{v}_0.\underset{\sim}{\nabla})\underset{\sim}{v}_1 + (\underset{\sim}{v}_1.\underset{\sim}{\nabla})\underset{\sim}{v}_0$$

$$\underset{\sim}{F} = \underset{\sim}{F}_0 + \underset{\sim}{f}_1$$

$$\frac{1}{\rho}\underset{\sim}{\nabla}p \rightarrow \frac{1}{\rho_0+\rho_1}\underset{\sim}{\nabla}(p_0+p_1) \quad \text{and} \quad p_1 \sim \rho_1\frac{dp}{d\rho} = c_s^2\rho_1$$

As $\dfrac{\partial\underset{\sim}{v}_0}{\partial t} + (\underset{\sim}{v}_0.\underset{\sim}{\nabla})\underset{\sim}{v}_0 = \underset{\sim}{F}_0 - \dfrac{1}{\rho_0}\underset{\sim}{\nabla}p_0$, the perturbation must satisfy, to first order:

$$\boxed{\frac{\partial\underset{\sim}{v}_1}{\partial t} + (\underset{\sim}{v}_0.\underset{\sim}{\nabla})\underset{\sim}{v}_1 + (\underset{\sim}{v}_1.\underset{\sim}{\nabla})\underset{\sim}{v}_0 = \underset{\sim}{f}_1 - c_s^2\underset{\sim}{\nabla}\left(\rho_1/\rho_0\right)} \qquad (4)$$

Normally, ρ_0 and $\underset{\sim}{v}_0$ will be fns. of position $\rightarrow$ difficult to generalise solutions

Jeans Approximation   Assume stationary background of constant density
i.e. assume $\underset{\sim}{v}_0 = 0$, $\rho_0 = \text{constant}$

Then $\dfrac{\partial v_1}{\partial t} \Rightarrow F_1 - c_s^2 \underset{\sim}{\nabla}\left(\rho_1/\rho_0\right)$ from (4)

$$\frac{\partial}{\partial t}(\underset{\sim}{\nabla}.\underset{\sim}{v}_1) = \underset{\sim}{\nabla}.\underset{\sim}{F}_1 - c_s^2 \nabla^2(\rho_1/\rho_0)$$

But $\underset{\sim}{\nabla}.\underset{\sim}{F}_1 = -4\pi G\rho_1$ so $\boxed{\dfrac{\partial}{\partial t}(\underset{\sim}{\nabla}.\underset{\sim}{v}_1) = -4\pi G\rho_1 - c_s^2\nabla^2(\rho_1/\rho_0)}$ (5)

from (1) $\dfrac{\partial \rho_0}{\partial t} + \dfrac{\partial \rho_1}{\partial t} = -\left((\underset{\sim}{v}_0 + \underset{\sim}{v}_1).\underset{\sim}{\nabla}\right)(\rho_0 + \rho_1) - (\rho_0 + \rho_1)\underset{\sim}{\nabla}.(\underset{\sim}{v}_0 + \underset{\sim}{v}_1)$

With Jeans Approx, $\dfrac{\partial \rho_1}{\partial t} \Rightarrow -\rho_0(\underset{\sim}{\nabla}.\underset{\sim}{v}_1)$ are only surviving terms

So we can put $\dfrac{\partial^2 \rho_1}{\partial t^2} \Rightarrow -\rho_0 \dfrac{\partial}{\partial t}(\underset{\sim}{\nabla}.\underset{\sim}{v}_1)$ (6)

Substitute for $\partial/\partial t(\underset{\sim}{\nabla}.\underset{\sim}{v}_1)$ from (6) into (5) and define <u>fluctuation</u> $f = \rho_1/\rho_0$

$$\Rightarrow \quad \frac{\partial^2 f}{\partial t^2} = 4\pi G\rho_0 f + c_s^2\nabla^2 f \qquad (7)$$

Consider <u>plane density wave</u>, wavelength λ propagating along x-axis

$$f = A(t)\cos kx \qquad (k = 2\pi/\lambda) \quad \begin{matrix} \nearrow \ddot{f} = \ddot{A}(t)\cos kx \\ \searrow \nabla^2 f = -k^2 A(t)\cos kx \end{matrix}$$

Then (7) $\rightarrow$ $\boxed{\ddot{A}(t) = (4\pi G\rho_0 - k^2 c_s^2)\,A(t)}$

Amplitude $A(t)$ <u>oscillates</u> (sound wave) if $4\pi G\rho_0 < k^2 c_s^2$
<u>grows exponentially</u> (unstable) if $4\pi G\rho_0 > k^2 c_s^2$

i.e. waves are unstably amplified by their self-gravity if

$$k^2 < \frac{4\pi G\rho_0}{c_s^2}, \quad \text{i.e. } \lambda > \sqrt{\frac{\pi}{G\rho_0}}\, c_s$$

JEANS CRITERION

Jeans Mass M_J.

If $p \ll \rho$ pressure is not gravitationally important, but pressure gradients may stabilise small-scale fluctuations.

Matter has some p - ρ relation (equation of state), then pressure-stabilised fluctuations oscillate like sound waves of characteristic velocity $c_s = \sqrt{dp/d\rho}$.

Jeans' Criterion $\to M_J$ $\quad$ $M < M_J$ can be stabilised by pressure (oscillatory)

$M > M_J$ collapses under gravity.

Take our fluid equations from model:

Mass Conservation $\quad D\rho/Dt = -\rho \underline{\nabla}\cdot\underline{v} \quad \Rightarrow \dfrac{\partial\rho}{\partial t} + (\underline{v}\cdot\underline{\nabla})\rho = -\rho\underline{\nabla}\cdot\underline{v}$

$$\Rightarrow \dot{\rho} = -(\underline{v}\cdot\underline{\nabla})\rho - \rho\underline{\nabla}\cdot\underline{v}$$

Gravitation: $\quad \underline{\nabla}\cdot\underline{F} = -4\pi G\rho$

Euler Momentum Equation: $\quad D\underline{v}/Dt = \underline{F} - \frac{1}{\rho}\underline{\nabla}p$

$$\frac{\partial\underline{v}}{\partial t} + (\underline{v}\cdot\underline{\nabla})\underline{v} = \underline{F} - \frac{1}{\rho}\underline{\nabla}p$$

Now use subscript 0 for equilibrium value of p, ρ, v

subscript 1 — perturbation

Consider "static" case (expansion of model & fluctuation growth)

Then first-order linear eqns. for perturbation:

$$\frac{\partial \underset{\sim}{v}}{\partial t} \rightarrow \frac{\partial \underset{\sim}{v}_1}{\partial t} + \frac{\partial \underset{\sim}{v}_0}{\partial t}$$

$$(\underset{\sim}{v}\cdot\underset{\sim}{\nabla})\underset{\sim}{v} \rightarrow ((\underset{\sim}{v}_0+\underset{\sim}{v}_1)\cdot\underset{\sim}{\nabla})(\underset{\sim}{v}_0+\underset{\sim}{v}_1) \rightarrow (\underset{\sim}{v}_0\cdot\underset{\sim}{\nabla})\underset{\sim}{v}_0 + (\underset{\sim}{v}_0\cdot\underset{\sim}{\nabla})\underset{\sim}{v}_1 + (\underset{\sim}{v}_1\cdot\underset{\sim}{\nabla})\underset{\sim}{v}_0$$

$$\underset{\sim}{F} \rightarrow \underset{\sim}{F}_0 + \underset{\sim}{f}_1$$

$$\frac{1}{\rho}\underset{\sim}{\nabla} p \rightarrow \frac{1}{\rho_0+\rho_1}\underset{\sim}{\nabla}(p_0+p_1) \qquad p_1 \sim \rho_1\frac{dp}{d\rho} \sim c_s^2\rho_1$$

As $\displaystyle \frac{\partial \underset{\sim}{v}_0}{\partial t} + (\underset{\sim}{v}_0\cdot\underset{\sim}{\nabla})\underset{\sim}{v}_0 = \underset{\sim}{F}_0 - \frac{1}{\rho_0}\underset{\sim}{\nabla} p_0$

the perturbation satisfies (to first order)

$$\frac{\partial \underset{\sim}{v}_1}{\partial t} + (\underset{\sim}{v}_0\cdot\underset{\sim}{\nabla})\underset{\sim}{v}_1 + (\underset{\sim}{v}_1\cdot\underset{\sim}{\nabla})\underset{\sim}{v}_0 = +\underset{\sim}{F}_1 - c_s^2\underset{\sim}{\nabla}(\rho_1/\rho_0)$$

NORMALLY ρ_0 and v_0 will be fns. of position → DIFFICULT CENTRAL PROBLEM

$\boxed{\text{JEANS APPROX.} \quad \underset{\sim}{v}_0 \sim 0 \quad \rho_0 \sim \text{constant}}$

Stationary background of constant density

$$\rightarrow \quad \frac{\partial \underset{\sim}{v}_1}{\partial t} = +\underset{\sim}{F}_1 - c_s^2\underset{\sim}{\nabla}(\rho_1/\rho_0)$$

But $\underset{\sim}{\nabla}\cdot\underset{\sim}{F}_1 \sim -4\pi G\rho_1$ so $\boxed{\dfrac{\partial}{\partial t}(\underset{\sim}{\nabla}\cdot\underset{\sim}{v}_1) = -4\pi G\rho_1 - c_s^2\nabla^2(\rho_1/\rho_0)}$

Now consider $\dot\rho = -\rho\underset{\sim}{\nabla}\cdot\underset{\sim}{v} - (\underset{\sim}{v}\cdot\underset{\sim}{\nabla})\rho$ from $\frac{\partial \rho}{\partial t} + (\underset{\sim}{v}\cdot\underset{\sim}{\nabla})\rho = -\rho\underset{\sim}{\nabla}\cdot\underset{\sim}{v}$

$$\dot\rho_0 + \dot\rho_1 = -(\rho_0+\rho_1)\underset{\sim}{\nabla}\cdot(\underset{\sim}{v}_0+\underset{\sim}{v}_1) - ((\underset{\sim}{v}_0+\underset{\sim}{v}_1)\cdot\underset{\sim}{\nabla})(\rho_0+\rho_1)$$

$$\rightarrow \quad \dot\rho_1 = -\rho_1\underset{\sim}{\nabla}\cdot\underset{\sim}{v}_0 - \rho_0\underset{\sim}{\nabla}\cdot\underset{\sim}{v}_1 - (\underset{\sim}{v}_0\cdot\underset{\sim}{\nabla})\rho_1 - (\underset{\sim}{v}_1\cdot\underset{\sim}{\nabla})\rho_0$$

$$\dot\rho_1 \sim -\rho_0(\underset{\sim}{\nabla}\cdot\underset{\sim}{v}_1) \quad \text{with J.A.}$$

- So $\ddot{\rho}_1 = -\rho_0 \frac{\partial}{\partial t}(\underset{\sim}{\nabla} \cdot \underset{\sim}{v}_1)$

and we have in terms of <u>fluctuation</u> $f = \rho_1/\rho_0$

$$\underline{\underline{\ddot{f} = 4\pi G \rho_0 f + c_s^2 \nabla^2 f}}$$

Consider plane density wave of wavelength λ propagating ∥ to x

$$f = A(t)\cos kx \qquad k = 2\pi/\lambda$$

$$\ddot{f} = \ddot{A}(t)\cos kx$$

$$\nabla^2 f = \frac{d^2 f}{dx^2} = -k^2 A(t)\cos kx$$

- So $\ddot{A}(t) = (4\pi G\rho_0 - k^2 c_s^2)\, A(t)$

i.e. A $\left.\begin{matrix}\text{oscillates} \\ \underline{\text{grows exponentially}}\end{matrix}\right\}$ if $4\pi G \rho_0 \lessgtr k^2 c_s^2 \}$

i.e. waves are <u>unstably amplified</u> by self-gravity if

$$\underline{\underline{k^2 < \frac{4\pi G\rho_0}{c_s^2} \qquad \lambda > \sqrt{\frac{\pi}{G\rho_0}}\, c_s .}}$$

N.B. this is a <u>kludge</u> because $\underset{\sim}{v}_0 = 0 \Rightarrow \underset{\sim}{F} = \frac{1}{\rho}\underset{\sim}{\nabla} p$

and if $\rho = $ const, $p =$ const so we need $\underset{\sim}{F} = 0$!! (no gravity). (or <u>infinite</u> system)

i.e. JEANS ASSUMPTION DOES NOT SATISFY BASIC EQUATIONS !

for <u>finite</u> masses such as we are interested in $\to$ galaxies.

Nonetheless, more precise treatments $\to$ <u>essential validity</u> of result as a

- stability criterion (<u>even in Hubble-law expanding Universe!</u>) See Bonnor, M.N., <u>117</u>, 104 (1957)

Gribbin, Ch. 7

$$\lambda_J = \sqrt{\frac{\pi}{G\rho_0}}\, c_s \qquad \underline{\underline{M_J \sim \tfrac{4}{3}\pi \rho_m (\lambda_J/2)^3}} \sim 3 c_s^3 / G^{3/2} \rho_m^{1/2}$$

- Before $t = t_{rec}$, what is matter temperature T_m?

Basically, Thomson scattering holds the matter at $T_m = T_r$ until the time between scatterings ~ $t_{expansion}$. As the time between scatterings is

$$t_s \sim \frac{1}{\sigma_T c n_\gamma}$$

at $t = t_{rec}$, $n_\gamma \sim 2\times10^7\, T^3 \sim 10^{18}/m^3$

$t_s \sim 46$ seconds $\ll t_{rec}$, which is 1.8×10^{13} sec.

So $T_m = T_r$ for $t \leq t_{rec}$.

Beyond $t = t_{rec}$, $T_m \neq T_r$.

The black-body background thus stems from THE LAST SCATTERING at $z \sim 1300$. $\left(\begin{smallmatrix} T = 2.72 \\ T_{rec} = 3500 \end{smallmatrix}\right)$

- The ~0.1% ISOTROPY OF THE BLACK-BODY BACKGROUND SHOWS THAT THE ADIABATIC FLUCTUATIONS MUST STILL HAVE BEEN VERY SMALL AT $t = t_{rec}$, ESPECIALLY ON LARGE SCALES.

Once decoupled from the radiation, the matter expands adiabatically:

$$T_m V^{\gamma-1} = \text{constant}$$

and the specific heat ratio $\gamma = 5/3$ for monatomic hydrogen, $T_m V^{2/3} = \text{const}$

so now $$T_m(t) = T_r(t_{rec})\left(\frac{\rho_m(t)}{\rho_m(t_{rec})}\right)^{2/3} \quad \text{for } \gamma = 5/3$$

The equation of state of the matter is $p \doteq \rho k T_m / m_H$

So $c_s = \sqrt{\dfrac{dp}{d\rho}} = \sqrt{\dfrac{kT_m}{m_H}}$ can be found at any $t > t_{rec}$.

- Immediately after recombination:

$$M_J = 3\left(\frac{kT_m}{m_H}\right)^{3/2} \Big/ G^{3/2}\rho_m^{1/2} \qquad \left(3c_s^2 / G^{3/2}\rho_m^{1/2}\right)$$

$\sim 8.7\times10^{35}$ kg or $\sim 4.4\times10^{5}\ M_\odot$ for $T_m = 3500$, $\rho_m = 9.8\times10^{-19}$

This is the mass of a GLOBULAR STAR CLUSTER.

So, immediately after recombination, all perturbation scales > masses of GLOBULAR CLUSTERS can collapse (if they can turn-around the Hubble flow locally)

The first condensations to separate out from the flow would be those for which $\Delta\rho_m/\rho_m$ was greatest just after recombination.

- If the initial fluctuations were STRICTLY ADIABATIC, the first "structures" to appear would have masses ~ MASSIVE GALAXIES or CLUSTERS, as the low-mass end of fluct^n spectrum would have been lost

If the initial fluctuations were STRICTLY ISOTHERMAL, the first "structures" would likely have masses ~ GLOBULAR CLUSTERS, if fluctuation spectrum has power ~ M^{-x}. (no ab initio argument for any particular fluct^n spectrum!)

Data consistent with adiabatic $\Delta\rho/\rho \ll 1$ at $t \sim t_{rec}$, so that small adiabatic PERTURBATION approach is probably ~ O.K.

LIMITS TO TURN-AROUND TIMES

- If bound systems condensed by gravitational instability from expanding background

$$\bar{\rho}_{cond}(\text{now}) > \bar{\rho}_m(\text{turnaround}) \text{ in Universe}$$

i.e. condensations must be denser now than they were at turnaround.
Einstein-de Sitter needs $\bar{\rho}_{cond} > 5.5\bar{\rho}$ to turnaround (xs > for $q_0 < 1/2$)
This → UPPER LIMIT on red shift at which systems of given $\bar{\rho}$(now) came into local equilibrium. (upper limit for E-deS density contrast).

Globular Clusters now $\sim 10^6 M_\odot$ in radius $\sim 10^{18}$ m (~30 pc).

$$\rightarrow \bar{\rho}_{cond}(\text{now}) \sim 5\times10^{-19}\ \text{kg/m}^3$$

This is $\sim \rho_m$ at t_{rec}. SO GLOBULARS COULD HAVE FORMED AT ANY TIME $z < 800$ by this constraint

- Galaxies now $\sim 10^{12} M_\odot$ in radius $\sim 10^{21}$ (m) ~ (30 kpc)

$$\rightarrow \bar{\rho}_{cond}(\text{now}) \sim 5\times10^{-22}\ \text{kg/m}^3$$

As $\rho_m \sim (1+z)^3$, this means that galaxies could not have turned around until $z < 80$, much later than earliest time allowed for globular cluster formation.

Clusters $\sim 10^{14} M_\odot$ in radius $\sim 10^{22.3}$ m (~ 600 kpc)

$$\rightarrow \bar{\rho}_{cond}(\text{now}) \sim 5\times10^{-24}$$

could not have turned around until $z < 17$

- Is the hierarchy $\boxed{\text{GL.CL}} \xrightarrow{\text{clump}} \boxed{\text{GAL}} \xrightarrow{\text{clump}} \boxed{\text{CLUSTER}}$ in the order by "gravitational clustering"?

This would be favoured if original perturbations were ISOTHERMAL.

Alternative to gravitational clustering:-

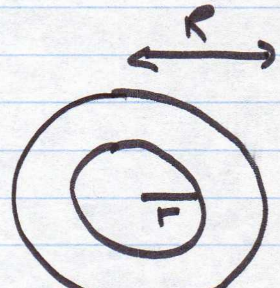
COLLAPSE / FRAGMENTATION

Gal. clusters $\xrightarrow[\text{fragment}]{\text{collapse}}$ Galactic-mass clouds $\xrightarrow[\text{fragm}]{\text{collapse}}$ stars, star clusters

Gas-dynamics dominates at all stages but all early masses are like surviving adiabatic fluctuations in which $M \gg M_J$.

Cluster densities now $\rightarrow$ all this did not get going until $z \ll 30$, so gas dynamics remains valid long after $t \sim t_{rec}$.

<u>Collapse of a sphere</u> (cold, uniform).



<u>Cold</u> sphere, uniform density, at rest initially

$$\ddot{r} = -\frac{GM(r)}{r^2} = -\frac{4\pi G \rho_0(0)\, r_0^3(0)}{3r^2} \leftarrow \text{constant}$$

$r(t=0) = r_0$.

$$\dot{r}\frac{d\dot{r}}{dr} = -\frac{4\pi G \rho_0 r_0^3}{3r^2}$$

$$\frac{1}{2}\dot{r}^2 = -\frac{4\pi G \rho_0 r_0^3}{3r} + \text{const}$$

$\dot{r} = 0$ when $r = r_0$ so $\text{const.} = -\dfrac{4\pi G \rho_0 r_0^2}{3}$

$$\dot{r}^2 = -\frac{8\pi G \rho_0 r_0^3}{3}\left(\frac{1}{r} - \frac{1}{r_0}\right)$$

Put $r/r_0 = \cos^2\beta$, then $\beta + \frac{1}{2}\sin 2\beta = t\sqrt{\dfrac{8\pi G\rho_0}{3}}$

All shells reach centre at <u>same</u> time $(\beta = \pi/2)$ $(r = 0)$

<u>free-fall</u> $$t_{ff} = \sqrt{\frac{3\pi}{32 G\rho_0}} = \frac{4.3\times10^7}{\sqrt{n_H(0)}} \text{ years.}$$

$$= \frac{6.65\times10^4}{\sqrt{\rho}} \text{ seconds}$$ (~ $\cancel{1.3\times10^{14}}$ 7×10^{13} sec for t_{rec} <u>AT REST</u>) $(T_r = 3500,\ \rho_0 = 4.6\times10^{-18})$

Now let gas be cold, but ρ_0 is $\rho_0(r_0)$

Shells will not cross if ρ_0 is decreasing fn. of r_0.

But inner regions collapse first $\quad (t_{ff} \sim 1/\bar{\rho}_0)$
outer ones fall in later

→ very peaked central density distribution

Important for details of collapse; does T of gas rise/fall/stay constant?

⇓

pressure variation → $t_{coll} >> t_{ff}$.

M_J variation → fragmentation?

$[3(kT_m/m_H)^{3/2}/G^{3/2}\rho_m^{1/2}]$

Cooling time-scale $t_{cool} \sim \dfrac{3k\rho T}{2\mu m_H \Lambda(T)} \quad \left(\sim \dfrac{3nkT}{2\Lambda(T)}\right)$

↓

cooling rate/unit volume
usually $\propto n^2$

If $t_{cool} > t_{ff}$, T increase from compression will not radiate away and Mass in clump is forced to contract at COOLING TIME SCALE without fragmentation

If $t_{cool} < t_{ff}$, isothermal collapse (maybe faster if T actually drops due to increased cooling rate).

→ $M_J \sim T^{3/2}/\rho^{1/2}$ decreases with time → fragmentation.

- Eventually ρ must increase to point where OPACITY of cloud fragments becomes significant and we make transition to ADIABATIC COLLAPSE.

Opacity per ~~gram~~ atom/mol is ~ independent of density, so the optical depth of a fragment $\tau \sim nKd \sim \rho\lambda_J$ i.e. $\sim \rho^{1/2}T^{1/2}$ Must eventually go over to "black-body." Then surface can radiate at rate

$$4\pi R^2 \sigma T^4 \quad \rightarrow \text{ cooling rate of volume } \tfrac{4}{3}\pi R^3.$$

whereas gravitational heating rate goes as $\sim \dfrac{GM^2}{R\, t_{ff}}$

- $\rightarrow$ halts fragmentation when $4\pi R^2 \sigma T^4 \sim \dfrac{GM^2}{R\, t_{ff}}$

$\rightarrow M^2 = 1.54\left(\dfrac{k}{\mu m_H}\right)^{9/2} \dfrac{\sqrt{T}}{\sigma G^3}$ (with some effort) $\rightarrow$ $\boxed{M = \dfrac{0.005\, T^{1/4}}{\mu^{9/4}} M_\odot}$ (putting $M = \frac{4\pi}{3}\left(\frac{\lambda_J}{2}\right)^3$)

i.e. "should" get fragmentation down to ~ stellar masses.

$T \sim 10^4$, $\mu \sim 1$ reasonable $\rightarrow 0.05\, M_\odot$, or ~~typical~~ red dwarf star !!

Obviously approximate because we do not put pressure in t_{ff}, consider any non-grav. forces - magnetic or rotation.

But indicates that we $\rightarrow$ stars ultimately, at least stellar masses!

- After opacity sets in, T rises and if γ (sp. ht. ratio) $> 4/3$ adiabatic collapse $\rightarrow$ fragmentation permanently shut off.

In fact find $F(M_*) \sim M_*^{-2.35}$ $\quad 60\, M_\odot > M > 0.029 \pm 0.013\, M_\odot$ (Reddish 1978)

Gravitational Clustering (Press & Schechter, Ap.J., 187, 425 (1974))

"Clusters of stars" ~ $10^6 M_\odot$ form first at $t \sim t_{rec}$.

→ local "particles" → gas dynamics not significant thereafter, but a Newtonian N-body problem. (All systems close to Hubble flow because of the radiation drag at $t < t_{rec}$).

"Clumpiness" grows ∵ gravity enhances positive correlations in space.

Simulations with randomly-distributed initial "particles" investigated in computer code. Mass variance in volume V

$$\Sigma^2(V) = \langle M \rangle^2 - \langle M^2 \rangle$$

$\langle\ \rangle$ - ensemble averaging over different volumes. $M = \iint m\, n(m)\, dm\, dV$

$n(m)$: distrib^n of particles' mass m.

All particles randomly distrib → $\boxed{\Sigma^2(V) \propto V}$ (A)

have "lattice site" and displace only on scales << lattice dist

$\boxed{\Sigma^2(V) \propto V^{2/3}}$ (B)

Cases (A) and (B) expanded and numerically followed. ($q = 1/2$ used) all models start $q \sim 1/2$

Condensations > 10 × replaced by new massive "particles"

(A) → typical condensation mass $M \propto R^2(t) \sim \left(\frac{1}{1+z}\right)^2$

i.e. since $z \sim 1400$ could grow $10^6 M_\odot \to 2 \times 10^{12} M_\odot$

(B) starts as $M \propto R^{3/2}(t)$ but evolves to $M \propto R^2(t)$ later

growth factor ~ 6×10^4 because it's slower at first.

i.e. could accumulate glob. cl. scale up to galaxy scale in time available but could not → gal. clusters $10^{14} - 10^{15} M_\odot$.

Can't explain clusters unless "seed masses" ~ $3 \times 10^7 M_\odot$ (A)

→ $3 \times 10^9 M_\odot$ (B)

Grav! clustering → no real characteristic scales   Peebles correl^n. fns.?

How lumpy Universe "looks" increases with time → masses of clumps depend on when you look. "Most common mass scale" varies (increases) with time.

Galaxies $\sim 10^{12} M_\odot$ are just the scale on which clumpiness is now becoming important.

Clusters of galaxies would not be bound yet, but will be later, and will evolve into single giant cD galaxies. (SUPERGALAXIES)

Predicts cumulative total mass $\mathcal{M}(<M) \propto M^{1/2}$ (Case A)

$\propto M^{2/3}$ (Case B)

This is consistent ($M^{1/2}$) with mass-function in rich cluster like Coma.

But this "pure clustering" model would say that we cannot have well defined scales of GALAXIES and CLUSTERS.

Need better studies of statistics of galaxy masses / clusters
→ are we just looking at M^{-x}, i.e. $\mathcal{M}(<M) \propto M^{1-x}$ and picking out examples of different scales by selection effects?

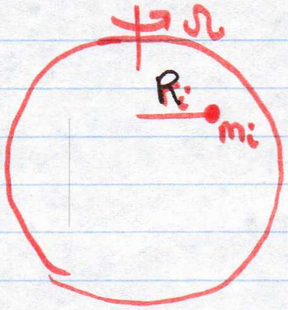
More detailed problems:

Angular momentum } prevent approach to free fall
Magnetic flux

Perturbation approach to homogeneous sphere collapse → not clear that smallest condensations can contract fast enough to avoid coalescence with each other as entire cloud contracts toward centre (Layzer, Ap. J., 137, 351 (1963))

∡r mtm. Isolated cloud J = total ∡r mtm = constant

Suppose initial solid-body rotⁿ ∡r vely Ω (obviously only an approx.)



$$J = \sum m_i R_i v_i = \sum m_i R_i^2 \Omega$$
$$= \frac{2}{5} M R^2 \Omega \quad \text{(sphere)}$$

Inwards force on equatorial particle $= \dfrac{GMm_i}{R_i^2} - m_i \Omega^2 R_i$

$$= \left(\frac{4}{3}\pi G\rho - \Omega^2\right) R_i m_i$$

Evidently $\Omega(t) R^2(t)$ = constant so inwards force → 0 when

$$\frac{4}{3}\pi G\rho = \Omega_0^2 \frac{R_0^4}{R_i^4(t)} \Rightarrow R_i^4(t) = \frac{3\Omega_0^2 R_0^4}{4\pi G\rho}$$

Collapse cannot be homologous (sphere → disk).

If preserve J for each particle, $J_i = m_i v_i r_i = m_i r_i^2 \Omega$

$[\Omega_0^2 R_0^4 = J_{0i}/m_i = J_i/m_i]$

→ particle can reach radius $\boxed{R_i^4 = \dfrac{3J_i^2}{4\pi G\rho m_i^2}}$

→ Sorting by angular momentum

low J_i/m_i material → centre of system (SLOWLY ROTATING CORE)

high J_i/m_i - - - - → "halo"

Polar material can reach centre more easily → flattening system?

If no star formation occurred (gas dynamical collapse without fragmentation)

→ GAS PRESSURE PREVENTS "INSTANT FLATTENING"
BUT COLLISIONS IN GAS → K.E. OF INFALL RADIATED AWAY
EVENTUALLY → FLATTENED GAS DISK, CONC. TO CENTRE.

Time scale depends on whether RADIAL VELOCITIES due to COLLAPSE exceed RANDOM VELOCITIES due to TURBULENCE.

Eventually → non turbulent disk, size governed by $\measuredangle^r$ m/m.
central condensation det. by $\frac{\text{collapse}}{\text{streaming}}$

But when stars form → MASSIVE "PARTICLES" WITH LONG MEAN FREE PATHS (long $\tau_{collision}$)

→ DISSIPATIVE PROCESSES DO NOT ACT ON STARS.

→ STARS RETAIN MOTIONS OF GAS FROM WHICH THEY FORMED BUT DO NOT EVOLVE ORBITAL SHAPES TO FOLLOW SUBSEQUENT CHANGES IN GAS SHAPE.

Galaxy Forms:

Spheroidal
Elliptical
Disks (e.g. S0)
Spiral disks

Model with:
Increasing $\measuredangle^r$ m/m
Increasing ratio (late/early) star formation
?
Increasing gas now

"Normal model"

opacity → adiabatic core → T_{rise} → nuclear fusion

① Early production of a few massive stars (rapid evolution $L_* \sim M_*^4$)
or of a massive core (QSO ?)

→ "rapid" few hundredths % of H/He → "metals"

via $3He^4 \rightarrow C^{12}$ above 10^8 K
→ O^{16}, Ne^{20} $1-2.5\times10^8$ K
→ up to Ca^{40} at $\sim 10^9$ K
} "α-favoured."

Equilibrium buildup to Fe^{56} (e-processes) at ~ 4×10^9 K

S process n-addition → Bi
R process → U, Th, transuranium elements.

→ heavy-element "enrichment" to → obs. comp. of old globular clusters.

② First major star formation while shape of gal. still not v. flattened.

→ Spheroidal / elliptical population.

SPHEROIDAL / ELLIPTICAL GALAXIES MUST HAVE BEEN EFFICIENT STAR-FORMERS AT THIS STAGE, AS THIS IS ALL THEY HAVE.

→ Gas formed by stellar evolⁿ either blown out (low-mass)
(SNRs, PNEBS)

or dissipated into core (high-mass)

} E's don't even have gas that should be there by * evolⁿ!

$10^8 - 10^{10}$ M☉ dwarf ellipticals → ageing population of original stars

$10^{10} - 10^{12}$ M☉ ellipticals → new stars in inner regions (metal-rich)
→ accretes onto massive core (BH)?
→ ACTIVITY, PATHOLOGY ??? → emission-line systems.

Accretion luminosity of core $\dfrac{G M_{core} \dot{M}}{R}$

$\boxed{\text{LOW-MASS ELLIPTICALS → NO METALLICITY GRADIENTS; HI-MASS ——— → "METALS" DECLINE WITH RADIUS}}$

Audouze & Tinsley, Ann. Rev. 14, 43 (1976)

DISK GALAXIES WERE INEFFICIENT AT THIS STAGE. Why??

AS RANDOM TURBULENCE DISSIPATED GAS → DISK

→ LATER STAR FORMATION IN DISK, ENRICHED WITH HEAVIER ELEMENTS FROM EVOLUTION OF "SPHEROIDAL CPT." STARS

→ Sϕ's and SPIRALS.

Why the variations in efficiency? Not really known. Linked to ϕr mtm, turbulence, mass of galaxy (core formation reheats gas??)

Spiral Disks — Density pattern, velocity $\Omega_{pattern} \neq \Omega_{Keplerian}$ (Like traffic bunches on a freeway)

Triggers * formation in shock as material drifts through.

These have much more gas (5-50% by mass) remaining than do Sϕ's. (disk but no sp.)

Spirals in Clusters (Spiral-"rich") / (Spiral-"poor" – all E/Sϕ gals)

1. Never in the cluster cores.
2. If at all, in the outer parts of clusters.

} E/Sϕ in cores stripped of gas by friction w. cluster atm or with central giants?

Giant E's prefer rich clusters

Giant Spirals prefer poor groups, and there ARE NO DWARF SPIRALS.

Nuclei of "active" spirals show QSO character → Starlike
Nonthermal light
Compact variable radio sources
Emission lines.

5·22

Review: Trimble, Earth Extr. Sci., 4, 109 (1978)

Big Problems in Galaxy formation / Evolution

① WHAT DO ELLIPTICALS DO WITH THEIR GAS?

Giant E's in clusters — eat it up or have it stripped
Dwarf E's near big field spirals → lose it to spiral → its disk?
Dwarf E's in clusters — stripped out as they move through cluster?

- do they lose it early on by massive core formation
  multiple SN, PNEB explosions
- then keep losing it later as it's released by stellar evolution? ? ?

XRAYS FROM CLUSTERS → Fe LINES/CONTINUUM suggest "NORMAL" Fe/H (0.1–0.8 of solar)

② WHERE DO SPIRALS GET THEIRS?

- are they ellipticals that manage to accrete themselves disks?
  - from cluster environs? (late on) (edges)
  - from i/galactic medium?

③ WHAT HAPPENS TO GALAXY CORES?

→ Very luminous objects — QSO's? $\boxed{\text{blow off gas}}$? - burst of * formation in shock above an opaque core?
→ Black Holes?
→ fragment into stars?

recollapse it later ⇒ spiral?

→ radiation pressure if you reionize above the shock

④ FLATTENING OF ELLIPTICALS

Ellipticals do rotate slower than spirals (less J/M)
but too slowly to → their flattening! (factors 2–10)

→ Primordial flattened fluctuations?
→ Protogalactic "pancakes" fragment → flattened protogalaxies?

⑤ WHAT DOES EFFICIENCY OF * FORMATION DEPEND ON?

ρ? T? B-field? Turbulence?

Active Galaxies — general review by Brecher in "Frontiers of Astrophysics" (on reserve)
Van den Bergh, TRASC, 69, 105 (1975)

"Normal" galaxies $L_0 \sim$ few $\times 10^{36}$ watts for brighter spirals, ellipticals in optical
$L_{IR} \sim$ few $\times 10^{35}$ watts
$L_{radio} \sim$ few $\times 10^{31}$ watts

Bulk of light is starlight — thermalised Planck curve continuum at few 1000 K
Morphology and velocity field indicate gross gravitational equilibrium.

"Activity" → significant nonthermal (non-Planckian) radiation
→ rapid variability ("jets")
→ disequilibrium motions on large-scale or disequilibrium morphology

Galaxies with Optical Activity

1) Seyfert Galaxies — review by Weedman, Ann. Rev., 15, 69 (1977)

Seyfert (1943) noted spiral galaxies with unusually compact, bright nuclei.
→ short exposures show virtually stellar "light spike" in most cases.

Spectroscopy → Continuum peculiarities
Strong IR radiation typically few $\times 10^{36}$ watts
UV excess over BB spectrum.
See Brecher F.O.A. diagram
Nuclei may show optical power-law spectrum $L(\nu) \propto \nu^{-\alpha}$
Indication of severe reddening in nucleus → dust concentration?
→ Line peculiarities
Strong emission lines superimposed on stellar population
Mostly from in and around galactic nucleus.
Large ($\equiv$ 1000 km/sec typically) emission line widths.
Type 1 — only H lines are broadened
Type 2 — all element lines broadened.

Most of these peculiarities are shared by "N galaxies", except IR excesses.

Original Seyfert group is virtually spiral galaxies by definition.
The clearly spiral Seyferts (90 to 95%) are generally weak radio sources. ~75% of their emission comes from the spiral disk. Nuclear sources may be variable, ~10 x enhanced over normal spiral nuclei.

[This is main contrast with N galaxies, which tend to be ellipticals with much enhanced radio emission (see later)]

About 1 spiral in 200 is probably classificable as a Seyfert.

A typical case: NGC 1068 $z = 0.00364$

Infrared spectrum: Telesco et al. (1976) ApJ, 203, L53 peaks at 100μ

$L_{IR} \sim 4 \times 10^{36}$ watts.

Jones and Stein (1975), ApJ, 197, 297 model this as dust reradiation of an intense nuclear UV flux. Needs $\sim 10^3 M_\odot$ of dust in ~ 50 pc region

Emission-lines: Walker (1968), ApJ, 151, 71

Total widths ~ 3000 km/s

Spatially separated clouds $\sim 10^{6-7} M_\odot$, moving at $\gtrsim$ escape velocity from nucleus as estimated from nuclear mass given by velocity field (Burbidge et al. (1959), ApJ, 130, 26.) $\sim 5 \times 10^9 M_\odot$ in < 500 pc.

Variability ~ 4^h $\sim \Delta m = 0^m.3$

→ several % of nuclear mass being ejected, replenishment timescale $\sim 10^{2-3}$ years
→ strong nuclear activity, dust concentration.

2) N Galaxies

Bright nuclei, almost starlike — rest of galaxy v. difficult to detect
Strong emission lines, UV excesses. Emission lines broad (up to 10,000 km/s!)
→ many are high-redshift systems ($z > 0.1$), $L_{opt} \sim 10^{37}$ watts or more
→ strong radio sources $\sim 10^{36-37}$ watts ($\sim 10^{4-5}$ enhancement)
Deep plates show them predominantly spirals
see e.g., Morgan, IAU Symposium #44, p. 97 (1970)

Markarian Galaxies

Markarian in mid-1960's made objective-prism survey with 40-in Schmidt at Byurakan Observatory → catalog of emission-line and uv excess objects

→ QSO's, Seyferts, N galaxies, ~~galaxies~~ galaxies with very young stellar populations (v. blue spiral arms). A grab-bag not all of which are particularly active or unusual. ~5% of all galaxies

Zwicky compacts (hi surface brightness, often v. blue), many are Markarians.

4 Radio galaxies — review chapter by Kellermann, in Verschuur/K book.

Rough definition → galaxy with radio emission enhanced by $\gtrsim 10^2$ times over "normal" emission of stellar/gas populations, so that $L_{rad} \to L_{opt}$, or even greater.

1) Invariably elliptical galaxies — only possible exceptions are Seyfert nuclei.
2) Very commonly dominant galaxies in clusters, particularly cD galaxies.
3) Gross radio spectral shape is similar to that in optical continuum of active nuclei $\sim \nu^{-\alpha}$.
4) Sometimes show optical emission lines, but not always.
5) Sometimes show jets or plumes, but not always.

} early optical-radio ID's emphasised these characteristics ∵ of poor positions → only "spectacular" ID's were possible.

Realised from study of nearby systems that bulk of radio activity occurs outside the optical object — e.g. Centaurus A system.

Linear polarisation up to ~20% (integrated flux) → nonthermal emission

$\nu^{-\alpha}$ → nonthermal emission

Lunar occultations showed some only ~1" in size → nonthermal emission (hi-T_b).

— more about structures and detailed forms later

— can prepare by reading Fomalont/Wright chapter in K-V.

Lacertids (BL Lacertae Objects) Good review by Stein et al. ARev. 14, 173 (1976)

- original object VRO 42.22.01 (~1968)
  - centimetre-excess rapid (days) radio variable
  - id with "variable star"
  - fuzzy on POSS
  - no spectral lines in optical!
  - ~20% optical linear polarisation
  - IR excess.

- defining properties
  - rapid, violent radio variables (always compact) (days, weeks)
  - optical (up to ~5^m)
  - core light is lineless.
  - usually strong IR excess, Light ~ $\nu^{-1.8}$
  - usually variable 10-30% optical lin. pol^n.
  - sometimes faint fuzz accompanies them

- Some lacertids are definitely in galaxies, from spectra of the fuzz.

AP Libra $z_g = 0.049$ (morphologically was called an N galaxy)
1102+38 $z_g = 0.030$ Markarian 421
1652+39 $z_g = 0.034$ 501

BL Lac itself controversial — Oke/Gunn ApJ, 189, L5 (1974), Egal z = 0.07 fuzz absn. lines
Miller and Hawley, ApJ, 212, L47 (1977) → z = 0.0688, usual weak radio-galaxy lines em/abs
Baldwin et al —, 195, L55 (1975), no lines, fuzz UVXS
Thuan/Oke/Gunn, —, 201, 45 (1975) confirm O/G

- Similarities to N galaxies, but at LOWER redshifts?
  without their emission lines
  without their double radio sources

- N galaxy 3C371 shows rapid optical variability and opt. lin. pol^n.

- Similarities to ~~QSSs~~ QSOs
  - radio properties similar to compact QSS
  - more variable QSO's also have opt. power law ~ $\nu^{-1.8}$

Linelessness
  - continuum swamps emission lines?
  - very wide lines lost on continuum?
  - UV flux has not suff. ionised gas that is there
  - no gas there.

Radio Sources (Radio Gals + QSS)

"Compact"

< 1 kpc

variable

lo-α

dispersed in α-α

QSO's

active galactic nuclei

Some are double or core + jet [one-sided]

"Super-luminal expansions"

Some occur in ext. structures.

Spectra → SSA. $T_{max} \sim 10^{12}$ K.

Ext cores ~ 2kpc α-Prel'n?

"One-sided jets"?

"Extended"

> 10 kpc

$N(\ell) \sim \exp[\ell/300\text{kpc}]$

steady

hi-α

undispersed in α-α

QSO's

ellipt. gals.

"double" (wide jets, trails etc)

Systematics.

Incr. lum → incr α (α-Prel'n)

hot spots

edge loc'n of hot spots

clarity of bifurcation.

[jet one-sidedness]?

Distortions greater in cluster sources

low lum. sources

optically fainter objects.

[Luminous classical doubles are either not in cl., or are dominant cl. members, or QSS]

Gal. Sizes in/out of clusters ~ same (Guindon)

Gal Orient'n prefer minor axes

QSO sizes decrease with cosm epoch? → $\theta_m \sim z^{-1}$.]]??

Reacc'n needed (lack of synchrotron aut'n)

Radio Source Confinement (Expansion $\sim c/\sqrt{3}$ must be stopped)

Gravitational Self-Confinement $\frac{2GM^2}{d} > U$

Cyg A needs $M \sim 10^{10} M_\odot$!

External thermal pressure

$\frac{3}{2} n_{ext} kT \gtrsim U/d^3$

Cyg A hot spots need $nT \sim 10^7\ \mathrm{K\,cm^{-3}}$
Typical extended structures $nT \sim 10^5\ \mathrm{K\,cm^{-3}} - 10^6\ \mathrm{K\,cm^{-3}}$
Typical cluster therm. brems XR $T \sim 10^8\ \mathrm{K}$
$n \sim 10^{-3} - 10^{-2}\ \mathrm{cm^{-3}}$.

External magnetic pressure

$B^2/8\pi \sim U/d^3$

Needs $B \sim 10^{-4}$ gauss (absurd)

Ram pressure $p \sim \rho_{ext} v^2$ $n \sim 10^{-2}$, $v \sim 1000$ km/s significant.

Detailed models: Trails: Pacholczyk & Scott, ApJ, 203, 313 (1976)
Lobes: Christiansen, MN, 145, 327 (1969)

Optical catalogue – Burbidge et al. ApJ. Suppl., 33, 113 (1977)

QUASARS

- originally "starlike object of high redshift identified with a radio source"

- now QSS — the above — "RADIO LOUD"
  QSO — the above, without the radio emission. — "RADIO QUIET"

Sl. — began with 3C273 accurate radio posn (lunar occultn) (Hazard et al., Nature, 197, 1037 (1963))
spectrum interp. (Schmidt, Nature, 197, 1040 (1963))

Sl. — other common properties
- variable light (τ ~ days/yrs)
- UV excess in their rest-frame (needn't look blue)
- broad emission lines (sometimes strong enough to pervert color)
- hi-z QSO's show multiple absorption lines.
- some show infrared excess

— visually similar to luminous Seyfert nuclei or to N galaxy cores
— power-law optical continuum ~ extension of power-law radio.

Radio and optical luminosities ~100 × powerful radio galaxies if z's cosmological.

Radio quasars (QSS) — Two classes

Compact sources. ($\theta < 1''$)
Self-absorbed spectra
Variability
VLBI → small-scale doubles
often ~ 0.″001
Two subclasses (?):-
~~~ spectra — often weak/no lines, usually variable
∩ — v. strong em lines, quite stable fluxes
Dominant QSO popn. in hi-freq radio surveys.
$\langle V/V_{max} \rangle = 0.52 \pm 0.05$

(often asymmetric)
Extended/doubles. Luminous systems with "hot spots" in lobes and same size distribution as radio galaxies.
>90% have "core" emission
follow α-P relation
% hot spot - P relation
"Transparent" spectra $\nu^{-\alpha}$, hi-α

Bridle, Kesteven & Guindon (1972) Ap Lets 11, 27
Jenkins and McEllin (1977) MN 180, 219

These QSS show $\langle V/V_{max} \rangle$ 0.57 ± 0.02

Dominant QSO popn. in low-freq. radio surveys

Possible deficiency of luminous [large] systems at high redshifts → Compton "snuffing"?
$U_{rad} \sim (1+z)^4$ ~~[illegible]~~
"snuffs" low-B sources rel to high-B
~~(Note this could also remove "hot" sources → observational selection)~~
~~~

	Seyferts	Ngals	BL Lac objects (Lacertids)	QSOs
$z > 1$	✗	✗	✗	✓
$z > 0.1$	✗	✓	✗	✓
Em. lines	✓	✓	✗	✓
Power-law light	✓	✓	✓	✓
Opt. pol^n.	✓	✓	✓	✓
Opt. var.	✓	✓	✓	✓
"Fuzzy"	✓	✓	✓	maybe 2 kinds
Double radio	✗	✓	✗	maybe 2 kinds
Compact radio	✓	✓	✓	✓
Radio var.	✓	✓	✓	✓
"Intervening" abs^n.	✗	✗	✗	✓

Rowan-Robinson model. (ApJ, 213, 635 (1977)) Activity ∝ galaxy luminosity

QSO = active galaxy with v. strong nucleus, outshines the rest of object.

Assumes every active galaxy has compact optical and radio core

- - — E galaxy has an extended double radio source

– then the compact, flat spectrum QSO's ≡ "spiral" QSO's ~ ~~most luminous~~ Seyferts.

extended, steep spectrum QSO's ≡ "elliptical" QSO's ~ N galaxies

Lacertids?

Maybe then the flat-spect QSO's are "local", explains low $\langle V/V_{max} \rangle$

steep-spect — "cosmol". — high $\langle V/V_{max} \rangle$?

Optical aspects ($z_{QSO} > z_N > z_{gal}$)

1) Lack of an overall Hubble relation (wide luminosity f^n.) e.g. Sandage, Vatican Sympos. "Nuclei of Gals" (1971)

2) Brightest QSO's in any magnitude range DO show a Hubble rel^n. Bahcall Woltjer

3) Extended-source QSO's have underlying fuzz (Kristian, ApJ, 179, L61 (1973))
   - but no spectral evidence that fuzz is a galaxy in any hi-z QSO.
   * Wampler et al., ApJ, 198, L49 (1975) → fuzz around 3C48 QSS is emission lines from hot gas.
   - colours of N galaxies ~ elliptical galaxy + a QSO. Sandage, ApJ, 180, 687 (1973)

4) All QSO's with $z > 2$ show absn. lines in spectra, many with multiple-z_{abs}.
   → intervening galaxies? Ionisation states of absn → interstellar lines.
   Few QSO's with $z << 2$ show absn. lines at $z_{abs} << z_{em}$.

5) QSS PKS 2251+11 $z = 0.323$ has an associated cluster of gals $z = 0.32$
   Gunn ApJ Letts, 164, L113 (1971); Oemler et al. ApJ Letts, 176, L47 (1972)

6) See forbidden emission lines → lo-ρ environment for some lines at high z.

7) Some bright variable N galaxies would be called QSO's at maximum light if they were more distant (Cannon et al., MN, 152, 79 (1971)).

* Also Richstone and Oke, ApJ, 213, 8 (1977) 3C 249.1 QSS $z_{em})_{fuzz} = z_{QSO} = 0.3125$

Ward Chapter in F.O.A.

Equilibrium of Circumstellar Gas Disk

star-dominated

Vertical Balance $$0 = \frac{GM_\odot}{r^2}\cdot\left(\frac{z}{r}\right) + \frac{1}{\rho_g}\frac{dp}{dz}$$

if not, $\frac{d\Phi}{dr}$ ← gravl. pott.

Horizontal Balance $$\Omega^2 r = \frac{GM_\odot}{r^2} + \frac{1}{\rho_g}\frac{dp}{dr}$$

Gas Law $$p = \rho kT / \mu m_H = \rho RT/\mu$$

$T(r,z)$ determined by solar input and $\left.\begin{matrix}\text{radiative}\\ \text{convective}\end{matrix}\right]$ transfer

need models for opacity here !
solar-composition, too
→ silicates, iron !

⇓

Model of P.S.N. e.g. $\boxed{\text{Cameron \& Pine}}$ (Numerical models for real disk periods)
Icarus, 18, 377 (1973)

→ ρ, T as fn. of r, z.

Mercury Condensation at $T \sim 1400\,K$

High-temperature condensates

- corundum Al_2O_3
  perovskite $CaTiO_3$
  melilite $Ca_2Al_2SiO_7 - Ca_2MgSi_2O_7$
  spinel $MgAl_2O_4$

* Metallic Iron (Fe, Ni)

* Some magnesium silicates, Mg_2SiO_4, $CaMgSi_2O_6$

* bulk of material because of abundances of constituents.

→ mean atomic weight of condensates ~ 35

Venus Condensation at $T \sim 900\,K$.

The above, plus much more silicate materials, and feldspars. Some incorporation of sulphur into iron sulphide, FeS.

→ mean atomic weight of condensates ~ 26.5 ← less % of iron

Earth Condensation at $T \sim 600\,K$

Incorporation of some water into tremolite

Greater formation of FeS, FeO, $(Fe,Mg)_2SiO_4$ from iron

→ mean atomic weight ~ 27

Mars. Condensation at $T \sim 450\,K$

Much greater incorporation of water → tremolite, FeO

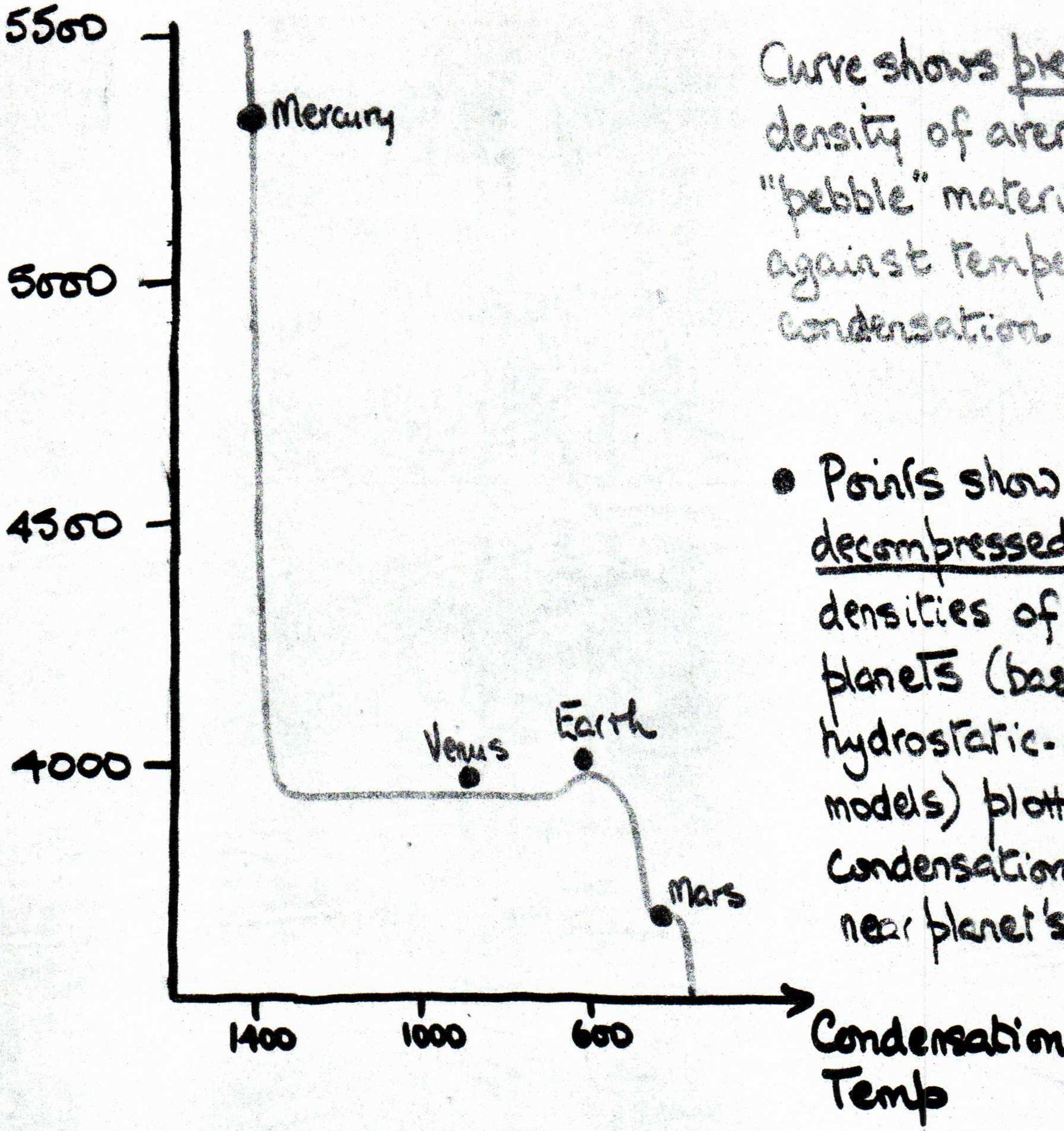
→ mean atomic weight ~ 25

→ more water incorporated in the crystal structure of the "Martian" pebbles than in the "Earthy" pebbles.

→ a source of abundant water on Mars later.

→ iron present more as oxide and sulphide in Mars than as free metal.

Overall, free iron content decreases as temperature decreases.



Curve shows predicted density of average "pebble" material against temperature of condensation.

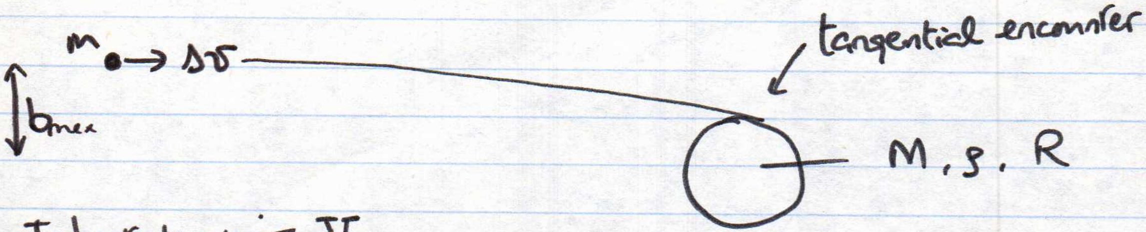
- Points show estimated decompressed mean densities of terrestrial planets (based on the hydrostatic-equilibrium models) plotted against condensation temperature near planet's orbit now.

CONCLUDE : Equilibrium condensation from gas disk will produce pebbles whose composition varies with distance from the Sun exactly as required to explain the bulk compositions of the terrestrial planets.

Also would expect outer terrestrial planets to be more massive than inner ones — greater fraction of material can condense. Explains why Mercury the smallest one.

Q. As more components of gas could condense at distance of Mars, why does Mars contain less matter than Earth?

Circuler-orbit accretion



Impact at velocity V

Cons. of ang. mtm $\qquad m\,\Delta v\, b_{max} = mVR$

Cons. of Energy $\qquad \frac{1}{2}mV^2 = \frac{1}{2}m\,\Delta v^2 + \frac{GMm}{R}$

Velocity from ⊙' orbit $\qquad \Delta v^2 = \frac{b_{max}^2}{4}\cdot\frac{GM_{sun}}{r^3}$

Eliminate $\Delta v, V$ $\rightarrow$ $b_{max}^2 = \frac{1}{2}R^2\left[1+\sqrt{1+\frac{128\pi\rho r^3}{3M_{sun}}}\right]$

$$b_{max} > R.$$

For Earth now, $b_{max} \sim 125R \sim 8\times10^5$ km

→ Material within $\pm b_{max}$ of Earth orbit not enough to make Earth!
(only $\sim 3\times10^{22}$ kg, whereas $M_\oplus = 5.98\times10^{24}$ kg

→ noncircular motions $\left(\Delta v \neq \frac{b_{max}}{2}\sqrt{\frac{GM_\odot}{r^3}}\right)$ are vital in explaining plan. accum.

→ gravitational "stirring" of small particles by big particles in swarm

Weidenschilling, Icarus, 22, 426 (1974) → Maxwellian distribution of velocities
(only an approximation because accretion and inelastic collisions (regolith to "bounce") mean that this is not a "thermal equilibrium" gas!

Low-velocity systems get accreted, so $\bar{v}$ (mean equilibrium relative velocity) increases with time.

Safronov/Weid' $v_e = \theta^{1/2}\bar{v}$ $\rightarrow$ $M(t) = \frac{M_0 e^{t/\tau}}{1+(M_0/M_p)e^{t/\tau}}$ $\qquad M_0 \rightarrow M_p$

$$\tau = \frac{\sqrt{\theta}}{(1+4\theta/\pi)}\frac{1}{\sqrt{G}}\left(\frac{4\rho}{3\sqrt{\pi}}\right)^{2/3}\left(\frac{3}{32\pi\rho}\right)^{1/6}\delta_0^{-1}$$

Space density of matter in feeding zone is $\delta = \delta_0(1-M/M_p)$

Planetesimals at Earth orbit.

$$D \leqslant \frac{8\pi f \sigma_g D_E^3}{M} \rightarrow \frac{8\times\pi\times 0.0035\times 1.5\times10^4\times(1.496\times10^{11})^3}{1.989\times10^{30}}$$

$$= 2.22\times10^6 \text{ m.}$$ Size of collapse region.

Earth orbit

$$M_D = \frac{\pi D^2}{4} \times \underset{f}{0.0035} \times \underset{\sigma_g}{1.5\times10^4}$$

$$= 2.03\times10^{14} \text{ kg.}$$ Planetesimal maximum mass.

$$v' = \frac{D}{4}\sqrt{\frac{GM}{d^3}} = \frac{2.2\times10^6 \text{ m}}{4} \times \sqrt{\frac{6.67\times10^{-11}\times1.989\times10^{30}}{(1.496\times10^{11})^3}}$$

$$= \frac{2.2\times10^6 \text{ m}}{4} \times 1.99\times10^{-7}$$

$$= 0.109 \text{ metres/sec.}$$

Hence planetesimal encounter time $= \frac{2\pi d}{v'} = \frac{2\times\pi\times1.496\times10^{11}}{0.109}$

$$= 8.6\times10^{12} \text{ sec}$$

$$= 2.7\times10^5 \text{ yrs.}$$

Mass in encounter range $= 2\pi d \times D \times f\sigma_g$

$$= 1.1\times10^{20} \text{ kg}$$

Radius of protoplanet $= \left(\frac{3M}{4\pi\rho}\right)^{1/3} = \left(\frac{3.3\times10^{20}}{4\times\pi\times4000}\right)^{1/3} \text{m} = 1.9\times10^5 \text{ m}$

$= 190$ km.

"The Large Numbers"

Bohr radius $\sim \dfrac{4\pi\epsilon_0\hbar^2}{m_e q^2} \sim 5\times10^{-9}$ cm

Gravitational H atom $\sim \dfrac{\hbar^2}{G m_e^2 m_p} \sim 10^{31}$ cm

Ratio (dimensionless) $\sim 10^{39}$ measures rel. strengths of gravity and e/m forces.

$$\boxed{N_1 \sim \frac{q^2}{4\pi\epsilon_0 m_e m_p G}}$$

Another dimensionless ratio (of distances) = $\boxed{\dfrac{cH_0^{-1}}{r_0} \sim 10^{40}\ (N_2)}$ r_0 — classical el. radius

Dirac suggested that $\boxed{N_1 = N_2 \text{ for fundamental reasons}}$?

If microscopic "constants" fixed, as H_0^{-1} increases with time, in N_2 G should decrease with time in N_1.

→ G-varying cosmologies — Solar luminosity + terrestrial orbital evol.^{n} / Orbits of planets (radar ranging)] $<10^{-10}$/yr.

But note: lifetime of MS *, $\boxed{t_* \sim \dfrac{(\text{nuclear energy supply})\times(\text{time for photon diff}^n\text{. out of }*)}{(\text{energy of radiation trapped in }*)}}$

→ $ct_* \sim \eta\left(\dfrac{m_p}{m_e}\right) N_1 r_0$. if diffusion dominated by Thomson scatt.

where η = fraction of rest-energy released in H-burning ($\eta \sim 0.007$)

i.e. $ct_* \sim 12 N_1 r_0$ or $\sim 10^{40} r_0$.

i.e. $\boxed{t_* \sim 10^{40} r_0/c}$ while N_2 has $\boxed{H_0^{-1} = 10^{40} r_0/c}$

Carter, etc → if galaxies form at time t_{gal}

we must around at $t_{gal} < t < t_{gal} + t_*$ or else we're in a dead gal

$\boxed{\text{Must}}$ therefore have $t \sim t_* \sim H_0^{-1}$ unless t_{gal} is very long cf t_*.

The Large-Number Coincidences"

Dimensionless ratio of electromagnetic / gravitational interactions

$$N_1 = \frac{q^2}{4\pi\epsilon_0 G m_e m_p} \sim 10^{39}$$

Dimensionless ratio of electronic to cosmological scales

$$N_2 = \frac{c H_0^{-1}}{r_0} \sim 10^{40}$$

H_0 = Hubble parameter
r_0 = classical electron radius

$N_1 \sim N_2$ "fundamental" [Dirac, et al. ...]

$\rightarrow$ G-varying cosmologies
H-R diagrams, orbits in S.S., solar luminosity ~ $\dot{G}/G \lesssim 10^{-10}/\text{yr}$

Lifetime of main-seq star ~ $\dfrac{\left[\begin{matrix}\text{nuclear energy}\\ \text{supply}\end{matrix}\right] \times \left[\begin{matrix}\text{time for photon}\\ \text{diffusion out of } *\end{matrix}\right]}{[\text{energy of rad}^n \text{ trapped in } *]}$

$$\rightarrow ct_* \sim \eta \left(\frac{m_p}{m_e}\right) N_1 r_0$$

if diffusion dominated by Thomson scattering

η ← efficiency of rest-energy conversion
[~ 0.007 for H → He fusion]

$$\sim 12 N_1 r_0 \text{, or } \sim 10^{40} r_0$$

i.e. $t_* \sim 10^{40} r_0/c$ $\sim H_0^{-1}$

i.e. "the coincidence" is ~ "age of Universe" ~ "age of $*$"

If galaxies form at t_g, <u>we</u> must observe at $t_g < t < t_g + t^*$, to have a $*$ to live by $\Rightarrow$ <u>$t \sim t^*$</u>