A SYNTHETIC ANTENNA WHICH MAY MEET THE LFST REQUIREMENTS

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April 1967

LFST/9

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I. INTRODUCTION

A filled-aperture, single pencil-beam antenna is ideal for observations of small patches of sky of a few beam areas in extent and for studies of phenomena which vary rapidly with time. If, however, it were agreed that these latter problems, of which lunar occultations and source scintillations are examples, were to be excluded, then most of the other tasks of the LFST could be met with a synthetic antenna. This present paper is intended to outline one such antenna and to describe its properties and limitations. The outline here presented is not an optimum design. It is intended to give a performance approximately the same as that of a 200-meter filled-aperture antenna. This is achieved by using a much smaller collecting area. However, the use of synthesis permits the easy realization of the equivalent of a multi-beam telescope, so that for problems of mapping an area of sky much greater than the synthetic beam the observing time for this and for a single-beam 200-meter are about the same. The requirements for a wide variety of line work can be met provided the number of correlated pairs in the synthetic antenna is small. In the example proposed, there are nine such pairs.

In brief, the antenna described here is a line of five dishes connected as nine interferometer pairs. The line is about 225 meters long, and the line is rotated in azimuth through 180° in one hour. Observations of an area of sky are continuous during this hour, and the end product is a map of this area with a resolution and sensitivity equivalent to that obtainable with a 200-meter dish.

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* Operated by Associated Universities, Inc., under contract with the National Science Foundation.

A considerable amount of elementary analysis has led to the following suggested design parameters. Again, it must be understood that no optimization studies have yet been made and that, if the idea is sufficiently attractive to potential users, these studies will follow. They may well result in changes in the system design.

A. System Parameters

1. Individual dishes

Two dishes about 25 meters diameter

Three dishes about 40 meters diameter

all on alt-az mounts. Capable of being moved on tracks, each of four dishes moves over its own semi-circular path, one dish is fixed at the center. Each dish should be good to 3 cm wavelength.

2. Control system

Alignment and control of the dishes needs study, but no great problems appear in view. The locations of the phase centers of the dishes should always be good to a few millimeters (say 1 mm if 3 cm is chosen as the shortwave limit). Angular pointing control of each dish should be to about 1/20 of the HPBW, and so would be in the 10" to 20" of arc range. The whole control should be well within the capacity of a small specially programmed computer.

3. Electronics

Well-designed feeds and low noise front-ends for five antennas are needed for each frequency used. For line work, nine digital autocorrelation systems are required. Phase stability at each antenna to a few electrical degrees is needed. Again, with present experience plus VLA design, none of this appears to be too difficult or expensive.

4. Observing time

For the purposes of this discussion, it has been assumed that one hour would be used for each sky area observed.

II. THE SENSITIVITY OF SUCH AN ANTENNA

It is obvious to see first whether such an antenna will give adequate sensitivity for the study of sources and of extended sky areas. The following simple discussions deal with these two aspects of sensitivity.

1. The minimum detectable flux density

Consider a single interferometer pair made up of two 40-meter dishes. The r.m.s. noise at the correlator output in degrees K of antenna temperature ΔT:

$$\Delta T = 0.707\ T_R\ (Bt)^{-1/2} \qquad (1)$$

where

T_R is the system noise temperature

B is the IF bandwidth

t is the observing time.

If q_n is the acceptable signal/noise ratio for detection, then the "minimum detectable flux" from a point source S_m is

$$S_m = \frac{2k}{\varepsilon A}\ q_n\ \Delta T \qquad (2)$$

where

k is Boltzmann's constant

ε is the aperture efficiency of one antenna

A is the area of one antenna.

Thus,

$$S_m = 1.2 \cdot 10^{-25}\ T_R\ (Bt)^{-1/2} \qquad (3)$$

where we have used $q_n = 5$ and $\varepsilon = 0.65$ for a 40-meter dish. Thus, for only one pair of elements, with t = 1 hour, B = 1 MHz, and $T_R = 100°K$, we get

$$S_m = 2 \cdot 10^{-28}\ \text{watts (meter)}^{-2}\ (\text{Hz})^{-1} \qquad (4)$$

As we shall see later, the array is two or three times more sensitive than a single pair, but this sensitivity is already enough to give a resolution limited (75 beam areas per source) instrument at 21 cm.

2. The minimum detectable brightness temperature

We assume that an area of an extended source equal to one beam area of the synthesized beam is brighter than its surroundings by T_{Bm}. This minimum detectable brightness temperature we will choose by the condition that the difference in output of the synthesized antenna should be five times the noise when the beam moves on and off T_{Bm}. If $\Delta\Omega$ is the synthesized antenna beam solid angle, the flux from T_{Bm} is

$$\Delta S = \frac{2k}{\lambda^2} \Delta\Omega \cdot T_{Bm} \tag{5}$$

If we collect this flux by a single pair of 40-meter antennas, and set the resulting signal to be five times the radiometer noise, we see that

$$T_{Bm} = \frac{\lambda^2}{\Delta\Omega} \cdot \frac{5}{\varepsilon A} \cdot 0.707\ T_R\ (Bt)^{-1/2} \tag{6}$$

or, using our previous numerical values of $\varepsilon = 0.65$ and $T_R = 100°K$,

$$T_{Bm} = 0.433 \frac{\lambda^2}{\Delta\Omega} (Bt)^{-1/2} \tag{7}$$

Since $\Delta\Omega$ and λ are related approximately

$$\Delta\Omega = \left(1.27 \frac{\lambda}{L}\right)^2 \tag{8}$$

where L is the overall antenna length (in our case 225 meters), we can reduce (7) to

$$T_{Bm} = 1.36 \cdot 10^4 (Bt)^{-1/2} \tag{9}$$

Using again t = 1 hour and B = 10 kHz (to correspond to a quite stringent line case), we see that

$$T_{Bm} = 2.3°K \tag{10}$$

We shall have a factor of the order of two or three to include the effects of all the antenna pairs, so that it seems evident that the minimum brightness temperature will be less than 1°K for a 10 kHz bandwidth.

3. Comparison with a 200-meter dish

The antenna will map an area of about (21 x 21) minutes of arc at 21 cm with a beam of 4.0 minutes HPBW in one hour. The brightness sensitivity with the parameters assumed above is 1°K.

A 200-meter antenna would have the same HPBW. To give the same accuracy of brightness temperature with the same radiometer would need an observing time per beam of 150 seconds. To cover the 28 beam areas in the map would need 28 x 150 seconds = 70 minutes. Thus, as we expect, the instrument is equivalent to a 200-meter dish for mapping areas much greater than the primary beam area.

III. ECONOMICAL USE OF DISHES

The "Arsac" array of four dishes gives all interferometer spacings up to six units of spacing. To get adequate sensitivity and to keep the individual dish size reasonable, one dish was added to an "Arsac" type of line. Such an arrangement, shown in Fig. 1, gives all spacings up to nine units with one (three units) appearing twice. In our application, correlators are only used once for each spacing, so that one of the three unit spacings is not used.

Since the telescope is derived from an "Arsac" array by rotation, it is called a Rotating Arsac Telescope (RAT).

IV. A POSSIBLE DESIGN FOR RAT

1. General design

The following design appears to be a good one for a first fairly detailed analysis. The array would consist of two 25-meter dishes and three 40-meter dishes, arranged as shown in Fig. 1. The spacing unit D would be 25-meters. The first two dishes would suffer some shadowing at some positions, but this is probably acceptable. The line array would be rotated through 180° in a time of about one hour. Correlators would be used between the pairs of dishes shown in Fig. 1. The following table shows the building of the synthetic antenna.

TABLE I

Properties of the Correlator Pairs
in the Antenna of Fig. 1

Element Spacing	Dishes Used	Output Weight	Antenna Pattern	Weights Used in Calculations	
				Linear	$(\text{Cosine})^2$
0	40 m	W_0	$(\Lambda_2)^2_\ell$	8.71	0.795
D	2 - 25 m	W_1	$(\Lambda_2)^2_s$	18	1.000
2D	2 - 40 m	W_2	$(\Lambda_2)^2_\ell$	16	0.970
3D	2 - 40 m	W_3	$(\Lambda_2)^2_\ell$	14	0.884
4D	25 m,40 m	W_4	$(\Lambda_2)_s(\Lambda_2)_\ell$	12	0.750
5D	2 - 40 m	W_5	$(\Lambda_2)^2_\ell$	10	0.587
6D	25 m,40 m	W_6	$(\Lambda_2)_s(\Lambda_2)_\ell$	8	0.413
7D	25 m,40 m	W_7	"	6	0.250
8D	25 m,40 m	W_8	"	4	0.117
9D	25 m,40 m	W_9	"	2	0.030

2. Choice of illumination

For convenience we assume that the dishes are all illuminated in the same way, i.e., the distribution of field as a function of radial distance from the center (ρ) is given by

$$E_r = E_o\ (1 - r^2) \tag{11}$$

where $r = \rho/a$ and a is the dish radius. This gives an antenna pattern of the form of the Λ_2 Bessel functions tabulated in Jahnke and Emde, page 182.

These patterns are plotted in Fig. 2 for the three types of interferometer pairs. Fig. 2 is calculated for D = 25 meters and a wavelength of 21 cm. The first zero of the large dish pattern occurs at 29.5 minutes of arc half-angle. The HPBW of this antenna is 23 minutes of arc at 21 cm.

This HPBW, of course, is the factor which determines the useful area of sky which can be mapped in one observational period.

The pattern $(\Lambda)_\ell \times (\Lambda)_s$ for the pairs of different sized elements is also shown in Fig. 2. Note that this pattern is negative over the range θ = 30-46' of arc, where $(\Lambda)_s$ is still positive but $(\Lambda)_\ell$ is negative.

The use of the Λ functions is a fair approximation to the behaviour of a dish with tapered illumination. For instance, the following values for HPBW and side lobe levels are given by the Λ_2 function

HPBW = 1.27 λ/D

First side lobe = 24.6 dB below main beam.

These are quite typical of the values obtained for NRAO illuminated dishes.

3. Pattern calculation

The response of the synthetic linear array to a point source in the zenith can be written

$$R(\theta) = W_o \Lambda_\ell^2 + W_1 \Lambda_s^2 \cos u + W_2 \Lambda_\ell^2 \cos 2u + W_3 \Lambda_\ell^2 \cos 3u +$$

$$+ W_4 \Lambda_s \Lambda_\ell \cos 4u + W_5 \Lambda_\ell^2 \cos 5u + \ldots W_9 \Lambda_s \Lambda_\ell \cos 9u \quad (12)$$

where

$$u = \frac{2\pi D \sin \theta}{\lambda} \quad (13)$$

and Λ_s, Λ_ℓ are the pattern functions for the small and large dishes respectively.

(a) Some simple examples

The power in the main beam R(0) is given by

$$R_o = \sum_n W_i \quad (14)$$

since at θ = 0 all the cosines and both Λ's are unity.

It may be noted here that the values of the Λ functions perhaps should not have been normalized to unity since the collecting areas of the different pairs are different. We can, however, deal with normalized Λ functions so long as we remember to adjust the weighting factors, W_i, suitably.

The power in the first grating side lobe is given by the case

$$D \sin \theta = \lambda \tag{15}$$

For our 21 cm case this occurs at $\theta = \sin^{-1} 0.008400$ = 28.9 minutes of arc. The response R(FGL) at this first grating lobe is

$$R(FGL) = (W_0 + W_2 + W_3 + W_5)\Lambda_\ell^2 + W_1 \Lambda_s^2 + + (W_4 + W_6 + W_7 + W_8 + W_9) \Lambda_s \Lambda_\ell \tag{16}$$

Although we have not chosen the values of W_i, it is interesting to use the values for a ten element linear array.

$$W_0 = 10 \quad W_1 = 18 \quad W_r = 2(10 - r).$$

Then

$$R(FGL) = 50 (\Lambda_\ell)^2 + 18 (\Lambda_s)^2 + 32 (\Lambda_\ell \Lambda_s)$$

while R(0) = 100.

From Fig. 2

$$(\Lambda_\ell)^2 = 8.5 \cdot 10^{-5}$$
$$(\Lambda_s)^2 = 0.156$$
$$(\Lambda_\ell)(\Lambda_s) = 4.5 \cdot 10^{-3}$$

Hence $\frac{R(FGL)}{R(0)}$ = 2.96%, i.e., the first grating lobe is 15.3 dB down on the main beam.

(b) Choice of weighting functions

It is clear that the response of the synthetic antenna to a point source (its pattern) can be adjusted by our choice of the weighting factors W_i.

This choice, besides affecting the pattern, will also change the signal/noise in the system, and it will be necessary to examine signal/noise ratios for any system for which we regard the pattern as satisfactory.

A full study of the patterns needs a set of computations of the expression (12) for $R(\theta)$. However, we can see some points fairly easily.

(i) The first grating lobe in the zenith. As our example showed, the main contribution to this was from $W_1(\Lambda_s)^2$. As small as possible a value for W_1 is thus desirable, but it seems unlikely that, whatever this choice is, the first grating lobe would go as low as 20 dB below the main beam. Remember that we are still discussing the pattern of the linear array, and not of the synthesized circular aperture.

(ii) The first side lobe. The side lobe nearest the main beam is called the first side lobe. It is a fair approximation to say that this occurs when $u = \frac{3\pi}{9}$. See Fig. 3 for diagrams illustrating the expression (12) for $R(\theta)$ at various values of u. Fig. 3(c) shows the fairly simple graphical solution for $R(\theta)$ at $u = 3\pi/9$, assuming all the Λ_2 functions are still unity. A simple grouping of terms in (12) when $u = \pi/3$ and the Λ's are set equal to unity is

$$R(u = \pi/3) = W_0 - W_3 + W_6 = W_9 + \frac{1}{2} \{W_1 - W_2 - W_4 + W_5 + W_7 - W_8\} \qquad (17)$$

This, for the weights of the uniform 10-element array, gives $R(u = \pi/3) = 3\%$, i.e., the first side lobe is 15.2 dB down.

(iii) Operation away from the zenith. All the discussion so far has been of the performance in the zenith. Most observations will be made away from the zenith. For these, of course, the antenna spacing becomes D cos Z (zenith angle Z). This reduces the angular resolution of the array, but, more important for our present study, it reduces the first grating side lobe. At $Z = 30°$ and the weights used earlier ($W_0 = 10$ $W_r = 20-2r$), we find

R(FGL) = 1.26% or 19 dB down.

(c) Trial patterns

Some trial calculations have been made by hand to see the form of the pattern of the synthesized array in the zenith. Two sets of weighting functions, given in Table 1, were used. One was linear; one was (cosine)^2. The value allotted to W_0 was chosen to make the first minimum of the pattern a zero.

Fig. 4 shows the weighting functions, and Fig. 5 shows the results of the calculations. The level of the first grating lobe is, of course, not much influenced by the weighting functions and was 15.2 dB below the main lobe for the linear weights and 15.5 dB below for the (cosine)^2 weights.

(d) The behaviour of the array when rotated

So far we have studied the pattern of the synthetic array as a line of antennas giving a fan-beam. However, in use this line will be rotated through 180°, and observations of an area of sky will be made continuously during rotation.

The effective response of the antenna to a point source will thus be different from that already discussed for the array. A simple example shows this. Consider a point source S at such an angle from the main beam (θ in Fig. 6) that it will lie in the first grating lobe of the array. Let the half-width of this lobe be α. As the fan-beam pattern is rotated through 180°, the source is within the HPBW of the grating lobe for an array rotation of 2ϕ (Fig. 6), and simple geometry gives

$$\phi = \cos^{-1} \frac{\theta-\alpha}{\theta}$$

If the power of the side lobe with respect to the main lobe is ρ, the effective power ratio in the rotated array will be

$$\rho_o = k\rho \cdot \frac{2\phi}{180} \quad \text{when k is about 0.8.}$$

For the array we are considering, θ is 31 minutes of arc and α is 4 minutes, so that

$$\rho_o/\rho = 0.267$$

Thus, the effect of rotation will be to decrease the first grating lobe a further 5.7 dB below the main lobe, making it about 19 dB down.

A more general way of seeing the same result may be got from the fact that we know the transfer function of the array in the (u,v) plane. This function is sketched in Fig. 7 and is, of course, a series of semi-circular zones. The value of the function is the same in each zone and is given by the weights assigned in Equation 12. The power radiation pattern is the Fourier transform of the transfer function. Each zone, of weight W_r and radius in the (u,v) plane of r (measured in wavelengths), gives a term in the transform of the form

$$W_r\ J_o\ (2\pi r \sin\theta). \tag{18}$$

Thus, we can estimate the magnitude of a side lobe by evaluating and adding terms of the form of (18). However, since (18) does not include the effects of the primary antenna patterns, we find when these are considered that only the smallest antennas at the closest spacing contribute to the transform. Thus only the term $W_1\ J_o\ (2\pi)$ is needed. This, when combined with the antenna pattern, shows the first grating lobe to be 22 dB below the main beam for observations in the zenith.

(e) Signal/Noise in the RAT

Let a given source provide a signal/noise ratio R at the output of the 40 meter single dish. Assume all radiometers are identical. Then the signal/noise ratios at the outputs of the various interferometer pairs are given by

Separation	Signal/Noise
D	$R\cdot \frac{A_{25}}{A_{40}} = 0.3906\ R$
0, 2D, 3D, 5D	R
4D, 6D, 7D, 8D, 9D	$R\cdot \frac{\sqrt{A_{25}\cdot A_{40}}}{A_{40}} = 0.625\ R$

In this table A_{25} and A_{40} represent the collecting areas of the dishes.

When these radiometer outputs are combined with various weights, the signals add, but the noise adds as the square root of the sum of the squares.

Thus the signal/noise ratio in the output of the synthetic antenna, R_T, is given by

$$\frac{R_T}{R} = \frac{1}{\sqrt{\Sigma W_r^2}} \left\{ 0.3906\, W_1 + 0.625 \sum_{\substack{r = 4,6,7,\\ 8,9}} W_r + \sum_{\substack{r = 0,2,\\ 3,5}} W_r \right\} \tag{19}$$

For a simple example, we might choose the weights $W_0 = 10$, $W_1 = 18$, $W_r = 20-2r$, and we could get

$$R_T/R = \frac{1}{\sqrt{1240}} \{18 \cdot 0.3906 + 32 \cdot 0.625 + 50\}$$

$$R_T/R = 2.19$$

Thus, it seems reasonable to get about a 2/1 signal/noise improvement over the result from a single 40 meter dish.

V. FUTURE STUDIES

The concept appears interesting enough to warrant a fairly detailed study. It would meet many of the LFST needs over a large part of the sky. The cost might easily be below $10 million. The performance is good, although not outstanding.

The following program will be followed:

1. Computer studies of beam and side lobe patterns at various positions in the sky. Choice of weighting functions, antenna sizes, etc.
2. Investigate critical areas in the mechanical and electrical needs:
   - (a) Mechanical accuracy of driving antennas, station keeping, pointing accuracy, etc.
   - (b) Phase accuracy attainable and various means of achieving it.

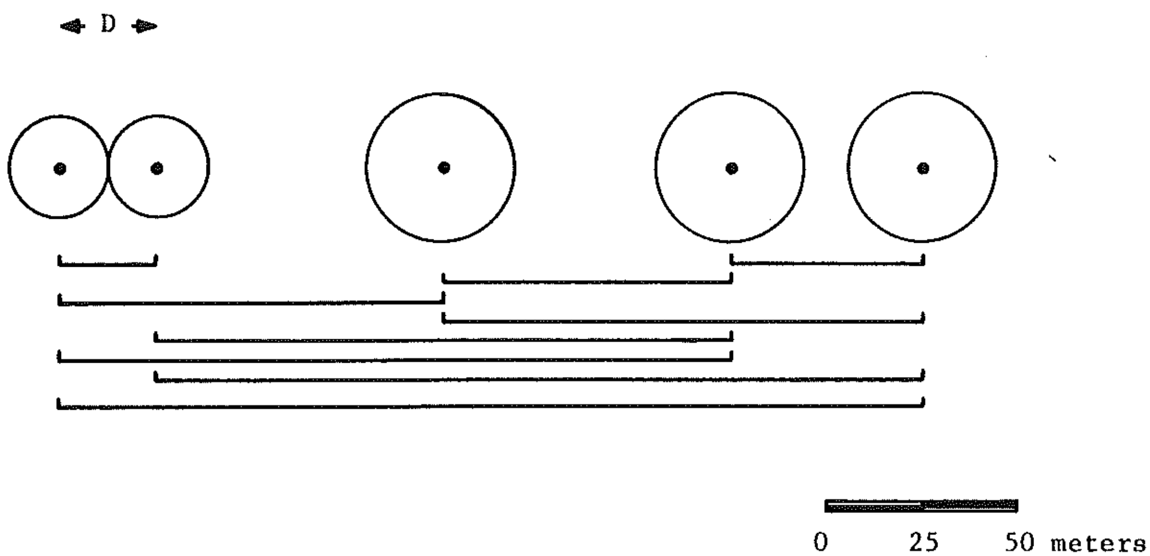


Figure 1

Locations of Dishes and Correlated Pairs

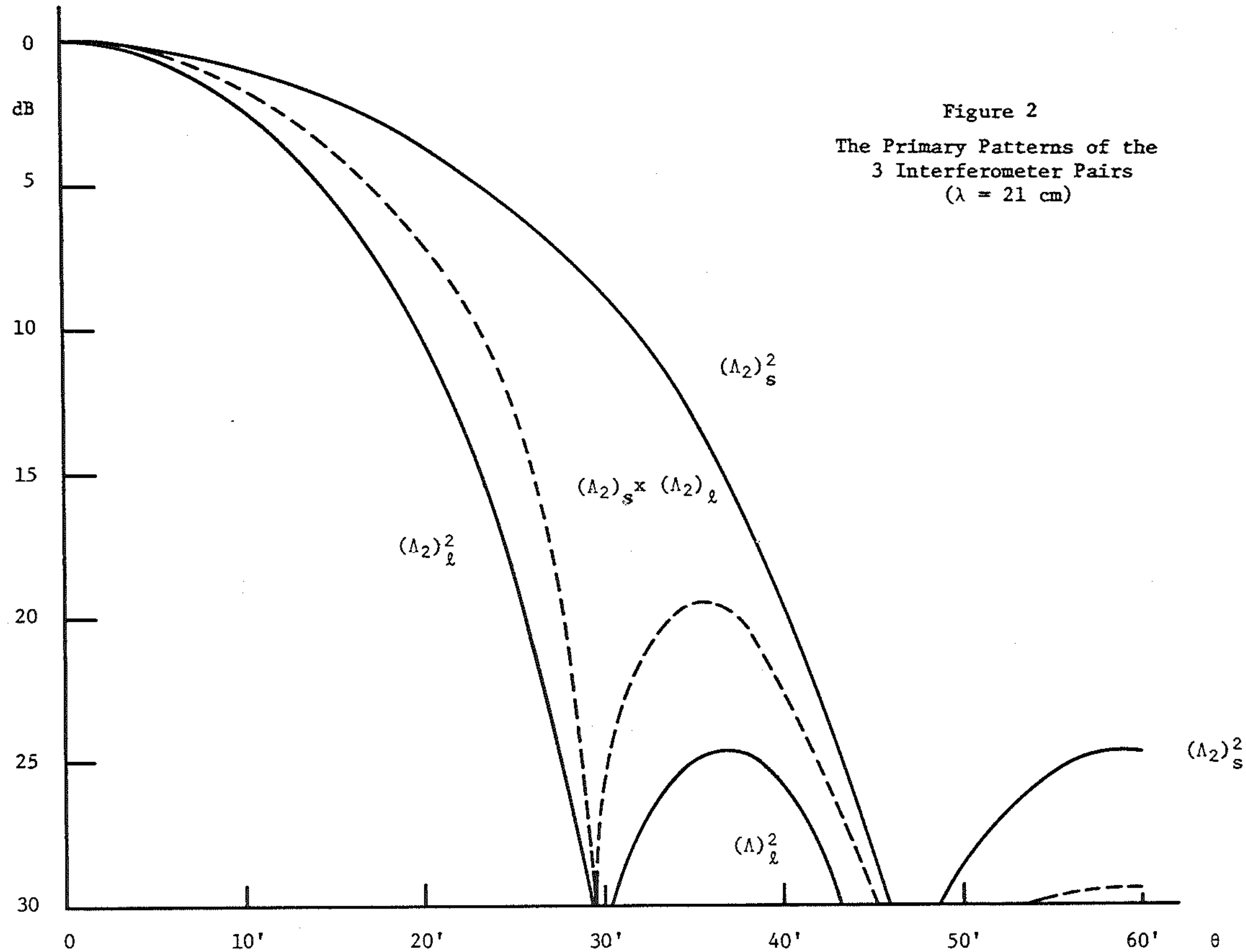


Figure 2

The Primary Patterns of the
3 Interferometer Pairs
(λ = 21 cm)

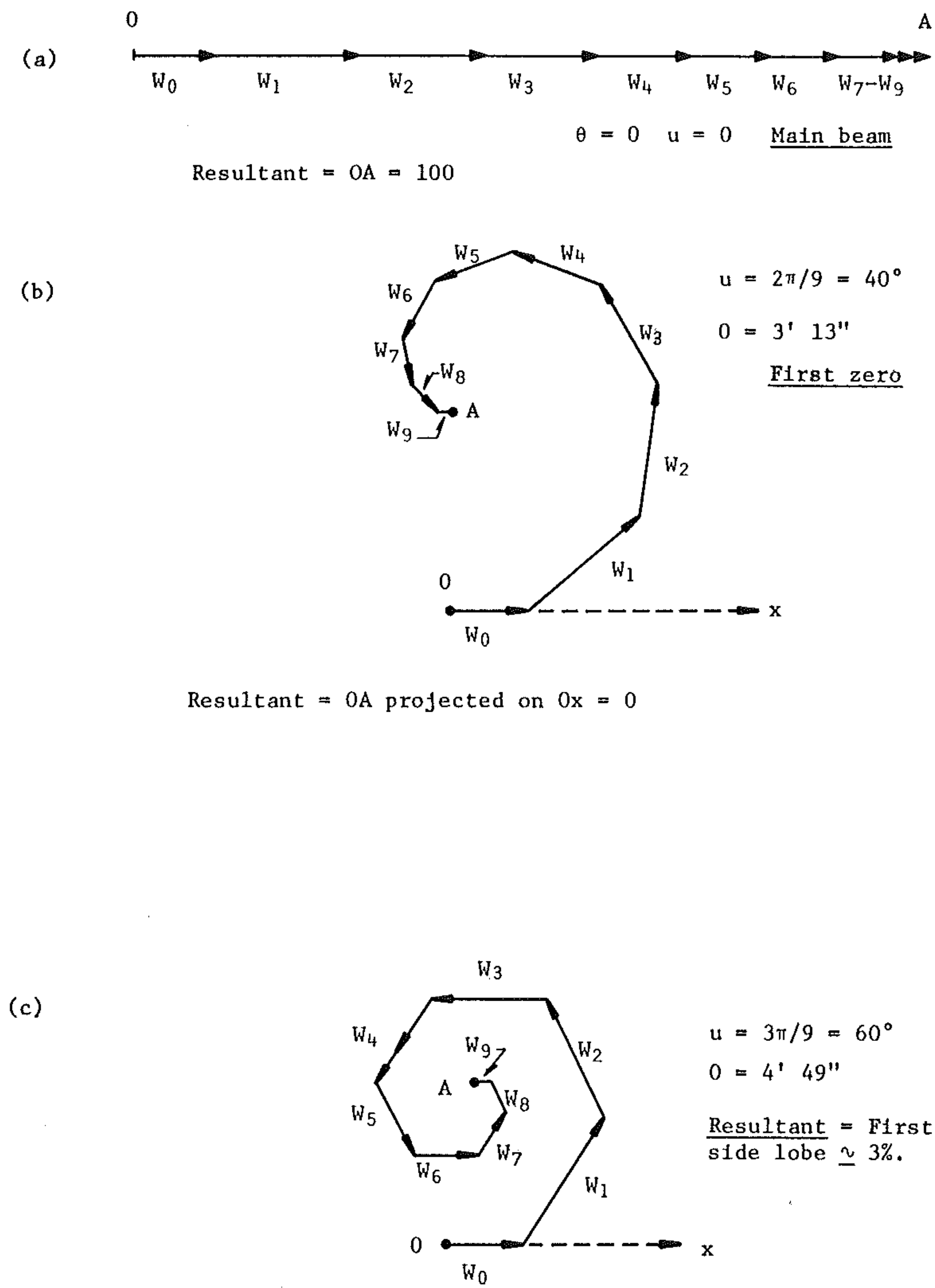
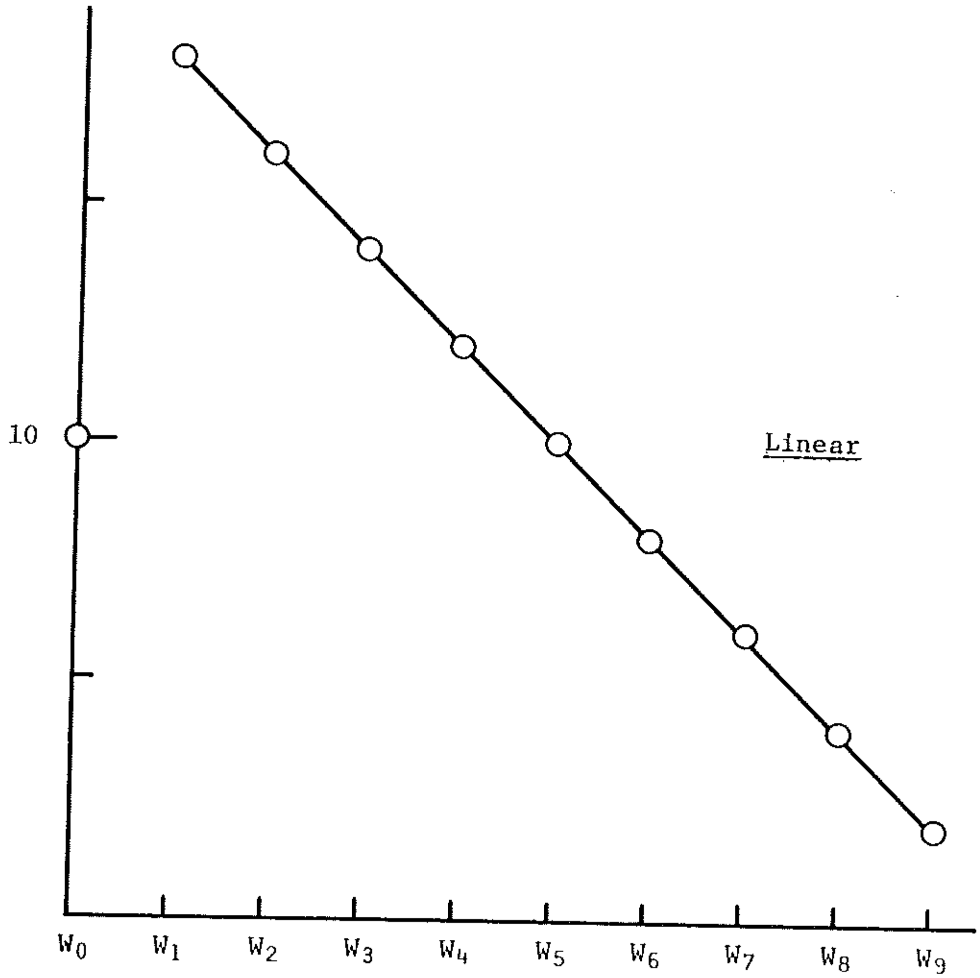


Figure 3

Graphical Solutions of R(θ) for Three Important Cases for the Array



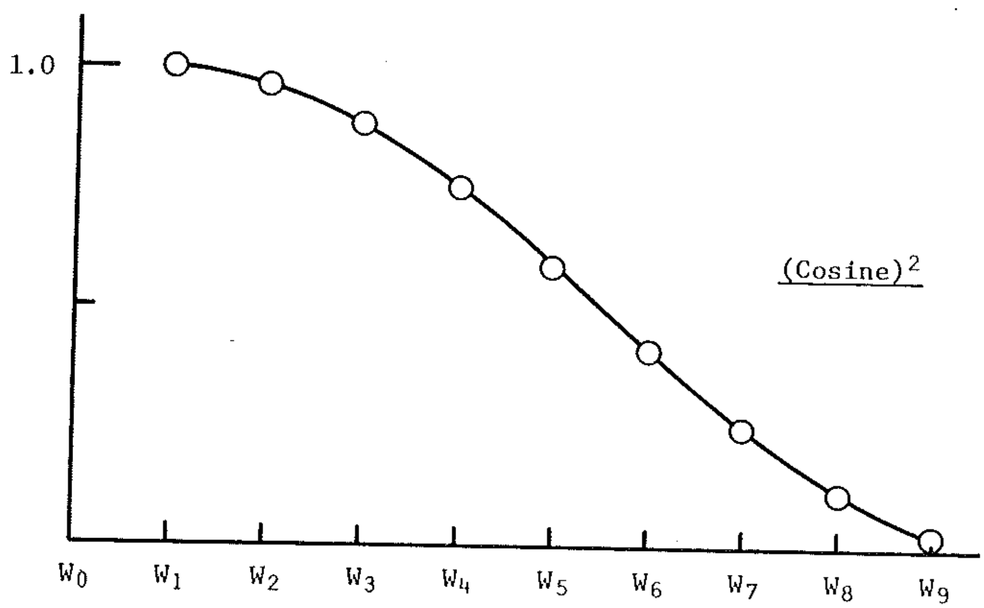


Figure 4

Weighting Functions - W_i used for
Trial Pattern Calculations

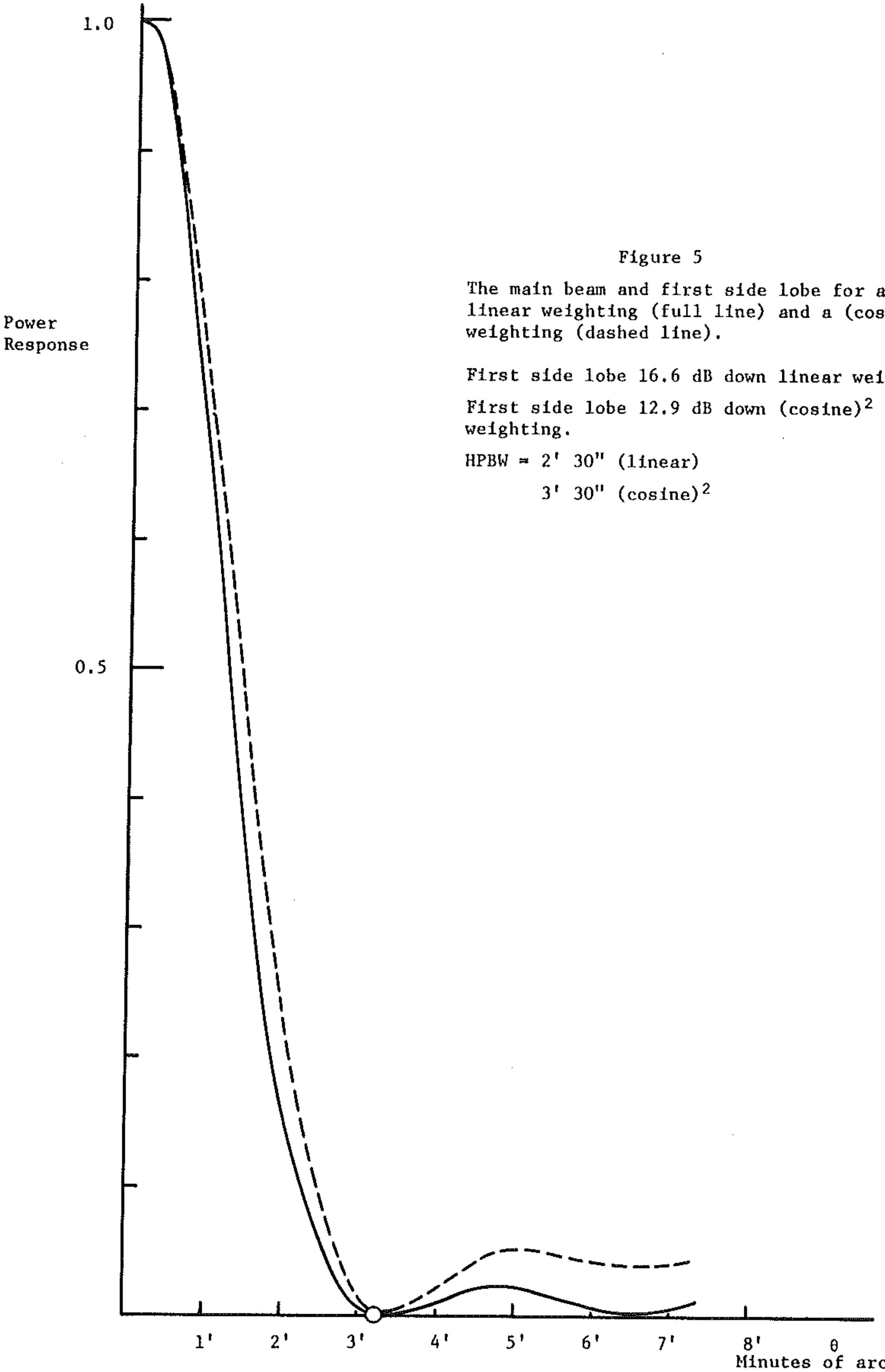


Figure 5

The main beam and first side lobe for a linear weighting (full line) and a (cosine)^2 weighting (dashed line).

First side lobe 16.6 dB down linear weighting.

First side lobe 12.9 dB down (cosine)^2 weighting.

HPBW = 2' 30" (linear)

3' 30" (cosine)^2

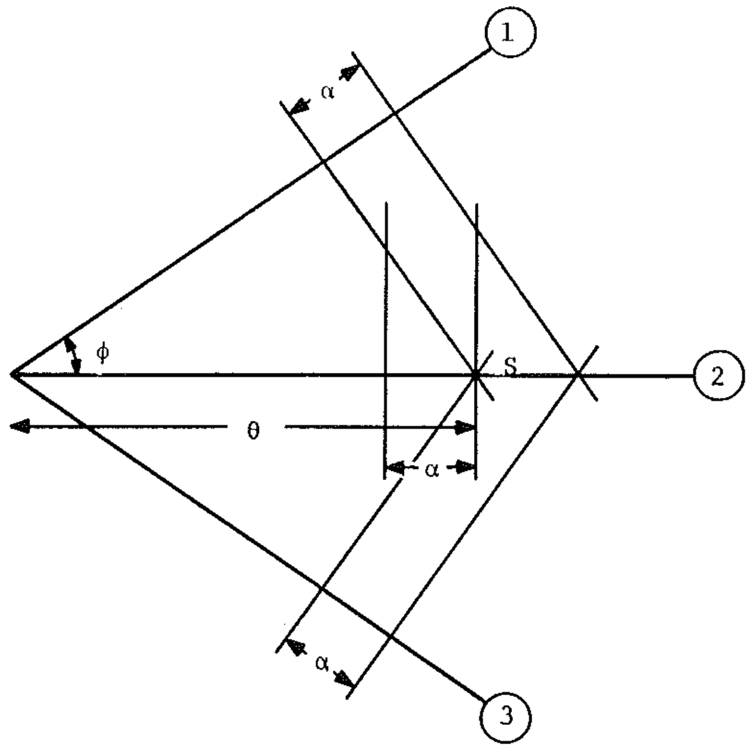


Figure 6

A Source in a Side Lobe

Antenna azimuth when

(1) S enters side lobe

(2) Antenna azimuth at center

(3) Antenna azimuth when S leaves side lobe

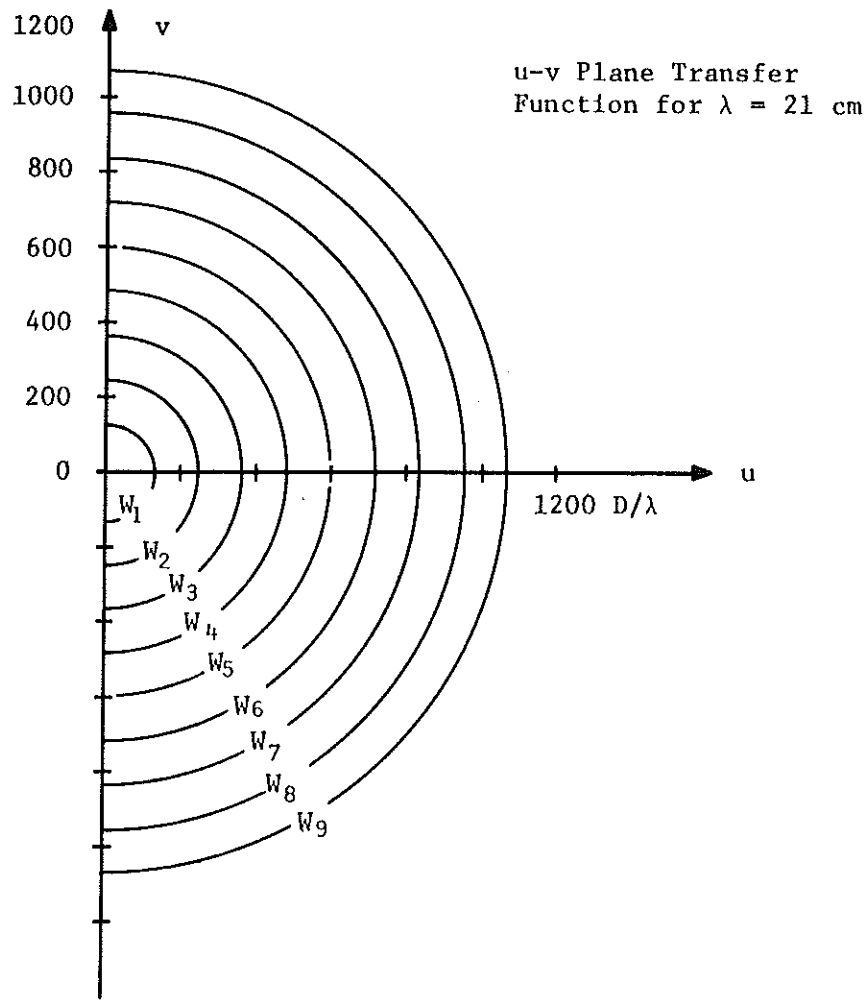

1200
v
u-v Plane Transfer
Function for λ = 21 cm
1000
800
600
400
200
0
u
1200 D/λ
W1
W2
W3
W4
W5
W6
W7
W8
W9

Figure 7