ARS

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Dear Alan,

Here are my answers to your questions:

1. Equation (5) in the draft was miswritten, it should be

$$\frac{E_\Delta}{t_j} = \frac{I}{2R}$$

where I is supposed to be the radiation intensity on the jet axis. ~~This~~ The error is purely ~~a~~ "typo" ~~error~~ and it was not carried into later derivations. More accurate, I can be expressed as

$$I_\Delta = 2\int_0^R \frac{E}{t_j}\, dr.$$

Indeed, the relationship ~~bett~~ between R and the radio HWHM depends on the particle and field configurations. However, in the absence of detailed observational information about these factors, one has to make assumptions concerning the cross-sectional distributions of various quantities to simplify the problem. The assumption needed here is that a self-similar profile exists across the jet. Then the above equation can be written as

$$I_\Delta = \frac{2R}{t_j}\int_0^1 E_\Delta(x)\, dx$$

where $x \equiv \frac{r}{R}$ is the independent variable for the self-similar profile. There is another constant ~~fudge~~ fudge factor of order

P.T.O.

unity required to write this equation into the form of Eq(5). If you think that it is necessary to carry this ~~fudge~~ fudge factor ~~explicity~~ explicitly in the derivation of later equations, it is O.K. and easy.

2. I can be normalized (made dimensionless) by scaling it with ~~same~~ the constant I at some fixed location on the jet axis. Other quantities (e.g. R) can also be normalized with the fixed values of these quantities at this location as well. The observed I is of course resolution dependent. In my draft, I assume that the resolution is much better than the jet width ~~eq~~ everywhere, then there is no problem. However, if the resolution is comparable to the jet width, a correction factor has to be put in to guess the theoretical intensity from the observed value. If you want, this ~~ge~~ geometric factor can be estimated, but further assumption probably has to be introduced.

Whether another ~~area~~ jet shows a similar correlation between the polarization and the "entraintment" would be a ~~as~~ crucial test to the present approach. I hope to ~~hea~~ hear from you soon.

Kiing

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Dear Kuring.

I have two questions about your draft of the "entrainment" paper, regarding Eqn. (5) on p.4

$$E_\Delta = \frac{1}{t_j}\left(\frac{I}{2R}\right)$$

1. Why do you normalise by the jet radius R? The relation between R and the radio HWHM depends on the particle distribution and field configuration.

2. How is I normalised? (It is resolution-dependent).

I have new NGC 6251 T_b - radius data but would like to feel happier about calculation of E_Δ before going ahead to try your analysis with that.

Alan

Entrainment and the jet environment — a case study ~~of the northern jet~~ of 3C31

I. Introduction

GC315 .Willis et al. C31-Fomalont et al. GC6251 aunders et .Willis et al. 1982 321+319 anti et al. 1982

Recent observations (e.g. Burch 1979, Perley et. al. 1979) on radio jets have accumulated evidence that in-situ particle acceleration is required to explain the slow decline of radio intensities along the jets. It is also commonly believed that the randomization of the large scale motion of the jet can supply energy to the relativistic electrons. Many theories have been proposed to describe the microphysics involved in the electron acceleration process (e.g. Pacholczyk and Scott 1976, Lacombe 1977, Bell 1978, Ferrari et. al. 1979, Henriksen et. al. 1981) and the in-situ production of magnetic fields (De Young 1980). In this paper we explore the macroscopic dynamics and energetics of radio jets by looking at their observable characteristics.

If (a) steady-state (b) constant v_z.

Hinted by the observational results of 3C31 (Burch 1979, Fomalont et. al. 1980, Bridle et. al. 1980), we develop some methods to analyse the following two problems: (i) particle energy budget — What is the quantitative relationship between the "slow" decline of the radio intensity and the amount of energy replenishment required to sustain the radiating particles (§ II)? (ii) dynamics of entrainment — if there is continuous dissipation of the kinetic energy of the large scale motion to replenish the relativistic electrons, what are the effects ~~of~~ on the dynamical developement of the jet (§ III)? In studying these problems we ~~try to~~ limit ourselves to ~~making only~~ simple and common assumptions. ~~When the above two aspects of the jet are~~ The above aspects of a jet are related. We ~~shall~~ show that when a model of energy conversion between the fluid and the particles is supplied, information~~s~~ about the density, pressure, and temperature distribution of the environment around the jet can be deduced.

II. Energy budget of the radiating particles

For a locally isotropic distribution of relativistic electrons which has a power spectrum with index $-n$, the evolution of the distribution function $N(\gamma)$ is described by

$$\frac{d}{dt} N + N \, \underset{\sim}{\nabla} \cdot \underset{\sim}{u} = q - \dot{\gamma} \frac{\partial}{\partial \gamma} N \qquad (1)$$

where γ is the Lorentz factor, $\underset{\sim}{u}$ is the large scale velocity, q is the source term, and all other symbols have their standard meanings. The decay rate

? $\dot{\gamma}$ in energy space is composed of two parts: (i) adiabatic loss: $\dot{\gamma}_{ad} = -\gamma\, \underset{\sim}{\nabla} \cdot \underset{\sim}{u}/3$, (ii) synchrotron loss $\dot{\gamma}_{sy} = -\gamma/t_{sy}$ where (Tucker 1975)

$$t_{sy} = 7.7 \times 10^{8}\, \gamma^{-1} B^{-2} \text{ sec} . \tag{2}$$

units?

B is the strength of the magnetic field. Let $E_\Delta \equiv \int \gamma m_e c^2 N\, d\gamma$ be the integrated relativistic energy contained within a small interval between γ and $\gamma + \Delta\gamma$ and $Q_\Delta \equiv \int \gamma m_e c^2 q\, d\gamma$ be the energy production rate in this same interval; Eq (1) can be transformed into an equation for E_Δ.

$$\frac{d}{dt} E_\Delta + \left(1+\frac{n}{3}\right)(\underset{\sim}{\nabla} \cdot \underset{\sim}{u}) E_\Delta + \frac{n}{t_{sy}} E_\Delta = Q_\Delta . \tag{3}$$

An important fact is that E_Δ can be directly related to the radio intensity I observed in a frequency interval between ν and $\nu + \Delta\nu$ where

$$\nu = 4.2 \times 10^{6} B \gamma^{2} \text{ Hz}. \tag{4}$$

The relationship can be expressed as

$$E_{s} = \frac{1}{t_j}\left(\frac{I}{2R}\right) \tag{5}$$

where R is the radius of the jet,

$$t_j = \frac{2}{27\,a(n)}\, t_{sy}\ , \tag{6}$$

and $a(n)$ is a constant of order 0.1 whose exact value depends on n (Tucker 1975).

Now, let us assume that the jet flow is stationary and the velocity $\underset{\sim}{u}$ is approximately uniform across the flow, then Eqs (3) and (5) can be combined to yield a formula for Q_s

$$\frac{t_u}{t_j}\left(\frac{2RQ_s}{I}\right) = \frac{d}{dz''}\ln\left[t_{sy}\left(\frac{I}{2R}\right)\left(R^2 u_z\right)^{1+\frac{n}{3}}\right] + n\,\frac{t_u}{t_{sy}} \tag{7}$$

in which $z'' = z/L$, $t_u = L/u_z$, z is the projected distance from the core (perpendicular to the line of sight), u_z is the projected velocity, and L is the projected distance of one arc second. The ~~sa~~ evaluation of the ratio

$$\frac{t_u}{t_{sy}} = 0.11\, L_{kpc}\, U_{z8}^{-1}\, B_{-5}^{3/2}\, \lambda^{-1/2} \tag{8}$$

,

which describes the relative importance of synchrotron loss, requires a knowledg of B and u_z. In later discussion, we ~~shall~~ approximate B by the equipartition value $[\propto (I/2R)^{2/7}]$. Furthermore, we ~~shall~~ take u_z to be approximately constant in the region of interest (see §II). Then, using Eq (7), one can find the rate of energy injection to E_s from observed developement of I and R.

We employ the data on 3C31 in Fomalont et.al. (1980) and Bridle et.al. (1980) to estimate the in-situ relativistic energy injection. I is taken to be the 6 cm ridge-line intensity at 2.5"x 5.0" resolution. ~~and R is taken from the combined~~ The spectral index α is taken to be 0.5 so that $n=2$. The projected distance of 1" is taken to be 480 pc. (z= , H_0=) Then,

$$\frac{t_u}{E_{sy}} = 0.02\, u_{z8}^{-1}\, B_{-5}^{3/2} \qquad (9)$$

which is proportional to $(I/2R)^{3/7}$ for equipartition field.

As the data always contain noise and is not smooth, the calculation of the derivative term in Eq(7) presents some difficulty. We use central differencing with a spacing of one ~~arce~~ arc second between data points and then take averages over intervals of 5'' to ~~t~~ obtain smoothed values for this term. However, we note that this procedure is not vitally important to our later results. As it ~~tex~~ turns out, this ~~te~~ derivative term becomes negative in a region between $z'' = 25$ and 30. This is caused by a fast drop in I and a later dip in R. The minimum value is -0.01 at $z'' = 27$. The existence of negative values in this term imposes restriction on the minimum size of the second term. Since the source term on the LHS of Eq(7) cannot be negative, the second term must be large enough to keep the sum on the RHS non-negative. This means that the synchrotron loss must be large enough to account for the fast drop of intensity. Using Eq(9), one obtains the following limitation on the factor $u_{z8}^{-1} B_{-5}^{3/2}$ at $z'' = 27$.

$$u_{z8}^{-1} B_{-5}^{3/2}(27) \gtrsim 0.25 \,. \qquad (10)$$

Taking u_{z8} to be 1, $B_{-5}(27)$ must be $\gtrsim 0.25$. We adopt a value of 1 for $B_{-5}(27)$ in our sample calculation; this yields an average value of $\simeq 1.4$ for B_{-5} in the region between $z'' = 5 - 35$. The resulting values of the dimensionless source term $t_n 2RQ_\delta / t_j I$ and the total production rate $\pi R^2 Q_\delta$ are plotted in Fig. 1 which also shows the percentage of polarization

? ~~pos~~ in inverted direction. The existence of an anticorrelation between the energy injection and the polarization shows up clearly in the region with $z'' > 10$ for z'' below 10, the situation is complicated by the increased importance

?? B_0 is greater at greater distances?

of the organized magnetic field whose strength does not follow the equipartition value. For example, at $z'' = 9$, the polarization goes through a minimum which corresponds to the transition of the organized field from parallel to perpendicular directions (w.r.t. the jet axis). To test the effect of the organized field on our results, we made several calculations with an added component of B proportional to $(1 + R(9)/R)/R$. We found that when the averaged size of the organized field is smaller ~~a~~ or comparable to the equipartition field strength ~~as~~ we have previously used, there is no qualitative difference between the results.

The existence of the anticorrelation between Q_δ and the polarization suggests that the injection of energy is associated with some kind of "turbulent" process. We shall discuss a turbulent entrainment model and its consequences in next section

III. Entraintment and the jet environment

Assuming that a ~~stationar~~ quasi-stationary jet flow has self-similar cross-sectional profiles, its dynamics is described by the following conservation laws:

(mass) $$\frac{d}{dz}(mu) = \dot{m} \qquad (11)$$

(momentum) $$mu^2 = \mathcal{F} \qquad (12)$$

(energy) $$\frac{d}{dz}\left(\frac{1}{2}mu^3\right) = -\dot{E} \qquad (13)$$

where $m = \pi R^2 \rho_j$, ρ_j is the mass density inside the jet, $\mathcal{F}$ (the momentum flux) is a constant (ignoring gravitational effects), $\dot{m}$ is the mass entraintment rate, and $\dot{E}$ is the mechanical energy dissipation rate (transformed into heat, relativistic particle energy ... etc.). For such a mechanical system, all variables can be related to the mass entraintment rate :

$$u = \left(\frac{\mathcal{F}}{m}\right)^{1/2} , \qquad (14)$$

$$\frac{d}{dz} m = 2 \mathcal{F}^{-1/2} m^{1/2} \dot{m} , \qquad (15)$$

and

$$\mathring{E} = \frac{1}{2} \mathcal{F} \frac{\dot{m}}{m} . \qquad (16)$$

Eq (10) directly links the dissipation of the mechanical energy to the entraintment of mass. A useful conclusion can then be made : if there is no entraintment, no bulk mechanical energy can be transformed into relativistic energy.

If $\dot{m}$ is known, all other variables can be determined. We need a model for $\dot{m}$. We shall assume that it has the form

$$\dot{m} = \rho_e \pi R u' \qquad (17)$$

where ρ_e is the density of the external meduim and u' is some kind of velocity. For a turbulent jet, u' is proportional to the turbulent velocity. For jets under laboratory conditions (uniform gas), the turbulent velocity is about 10% of the mean velocity of the jet. However, when both the external and the jet densities are non-uniform, the exact relationship is completely unknown. Even so, we make the assumption that u' is proportional to u; this is consistent with the assumption of self-similarity in the cross-sectional profile.

Now, let us look at some of the consequences of such a simple model of entraintment. Suppose that ρ_e is proportional to Z^{-K} and $R/Z \approx$ a constant $\ll 1$, Eq (15) gives

$$m = m_0 + \left[\frac{2u'}{u}\right]\left(\rho_{e0}\pi R_0^2\right) \times \begin{cases} \frac{1}{2-K}\left[\left(\frac{Z^2 \rho_e}{Z_0^2 \rho_{e0}}\right) - 1\right] & (K \neq 2) \\ \ln\left(\frac{Z}{Z_0}\right) & (K=2) \end{cases} \quad (1$$

~~where~~ in which the subscript o indicates that ~~the quantities~~ ~~are evaluated at some reference~~ the original values of the quantities. ~~locations the reference location Z_0s~~ If ρ_e drops faster than Z^{-2}, both m and u approach a finite value. On the other hand, if $K \leq 2$, m increases without limit and u approaches 0, implying that the jet can only travel a limited distance from the core. Furthermore, entraintment and dissipation of mechanical energy is important only in regions where ρ_e drops slower than Z^{-2}.

~~Observationally~~, The pc scale jets (VLBI) are believed to have a speed closed to c ~~relativistic speed~~; on the other hand, the Kpc scale jets are believed to have speeds around $10^{-2}c$. Therefore, m ~~must~~ must have increased 10^4 times between the two scales (total mass flux increased by 10^2 times). This suggests the following scenario for the "cascade" of the jet between the two scales. When the jet is in relativistic speed, its density is low compared to that of the external medium so that m increases very quickly near the core. ρ_j can then catch up with ρ_e very quickly. However, as the jet ~~continues~~ continues outward, its density may become higher than that of the environment. Then, the speed of the jet would not drop as quickly and the jet can travel a long distance.

evidence?

I shall add a discussion on why jets start radiating away from the core here.

Our next step is to introduce a relationship between the mechanical energy dissipated by the fluid and the relativistic energy received by the radiating electrons. Then the variation of the jet intensity will be able to provide information about the jet environment. We shall adopt the simplest model here, namely

$$\pi R^2 Q_\Delta \propto \dot{E} \tag{19}$$

Then, given Q_Δ, the distribution of ρ_e can be found by Eqs. (16) and (17).

¶ Using the values of Q_Δ found in the previous section, we calculated the distribution of ρ_e for 3C31; the result is shown in Fig. 2. If we further take the external pressure p_e to be approximately equal to the pressure of the relativistic particles inside the jet ~~which is proportional to~~ $\left[\propto (I/2R)^{4/7}\right]$, all thermodynamic variables of the galactic environment can be determined. The distribution of P_e is also shown in Fig 2. Notice that eventhough the distribution of ρ_e is very bumpy beyond $z'' > 20$, the average slopes of both curves are clearly flatter than -2, indicating that the observed jet is in a plateau of both ρ_e and p_e. In fact, by looking more closely at these curves, one can notice that the curves between $z'' = 10$ to 20 are a little flatter than other portions. These distributions agree qualitatively with the pressure confinement model used by Bridle et. al. (1980) to explain the increased collimation of 3C31.

The apparant dip of ρ_e between $z'' = 20-35$ has to be interpreted carefully. The exact depth of this dip sensitively depends on the size of the second term ~~of Eq(7)~~ on the RHS of Eq(7) ~~and~~ which in turn depends on the chosen value of ~~u_8~~ $u_{28}^{-1} B_{-5}^{3/2}$. Even though it is safe to say that some dip exists, we cannot be certain how deep it really is. If it is really as deep as indicated, the matter distribution in this region of the galaxy is very clumpy (either ~~either~~ filled with hot bubbles or cold clouds). Another feature demonstrated in Fig. 2 is that ρ_e in general declines faster than P_e. This implies that the temperature of the environment increases outward. For the distributions shown in Fig. 2, the temperature in the region below $z'' = 20$ is about 3 times cooler than the region above it. Such a behavior is surprisinly similiar to that of M87 found by X-ray observations (Fabricant, Lecar, and Gorenstein 198

IV Summary and discussion

By assuming that the radiating electrons have an isotropic power law distribution, we have derived a rather general equation which relates the evolution of I and R to ~~be~~ the rate of in-situ relativistic energy production. The application of this equation to interpret observational results can yield useful information on the physical conditions of a jet. The fact that the percentage of polarization is anti-correlated with the rate of energy production tends to support the view that the production ~~re~~ of relativistic energy is associated with some kind of violent process.

Assuming that the jet has ~~a~~ self-similar profiles in the ~~cross-sectional~~ cross stream direction, we studied the entraintment dynamics of the jet and arrived at the conclusion that entraintment is ~~—~~ the necessary condition for converting the fluid energy to the electron energy. This conclusion is independent of the detail process of conversion.

When we ~~used~~ use the word "turbulent" in this paper, we take its most liberal meaning. A "turbulent" jet in a non-uniform medium can be very different from those in uniform medium. For example, if the jet is very ~~close~~ dense compared to the environment, its rectilinear motion may not be affected substantially by entraining. It's ~~cone~~ expansion angle ($2dR/dz$) can be more sensitive to the pressure balance between itself and the environment, ~~rea~~ rather than the lateral spreading effect of the <u>weak</u> turbulence. This serves as a good explanation to why expansion angles of radio jets can be very small compared to those of turbulent jets in laboratory (~~full width~~ half width / length $\approx$ ~~0.2~~ 0.1)

If the radio jets are really turbulent as described above, the pressure confinement model of Chan and Henriksen (1980) and Bridle et. al. (1981) needs to be revised. First, the notion of a sonic point is misleading. For the ~~entraintment~~ entrainted material, eventhough they have to go through the process of being accelerated from subsonic to supersonic speed, the transition is not at a single location. Furthermor the speech of the ~~red~~ jet is not accelerating as in the nozzle model; instead it is decelerating. Secondly, the effect of turbulence on the expansion angle should be included in the ~~equation for the deve~~ dynamical equations. In view of the lack of knowledge in the behavior of turbulent jets in nonuniform medium, this is not going to be ~~easy~~ a easy task.

Better ways to handle the derivatives of R and I required by Eq(7) are useful ~~to~~ for obtaining more definite informations. This problem can be approached from two sides. From the observational side, results with better resolution and less noise will improve the ~~base~~ quality of the basic data. From the technical side, employing more sophisticated technique to find the derivatives of ~~smoot~~ noisy data can also help.

Why not do all this with NGC315 NGC6251?

Smoothed data