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Survey of Dark Energy models

a) Some review articles:

1) Varun Sahni, astro-ph/0403324

2) M. Trodden & S. Carroll, astro-ph/0401547

b) Some important early papers:

1) "Cosmological consequences of a rolling homogeneous scalar field"

B. Ratra and P. J. E. Peebles, Phys. Rev. D37, 3406 (1988)

2) "Cosmology with a time-variable cosmological "constant""

P. J. E. Peebles and B. Ratra, Astro. J. 325, L17 (1988)

3) "Quintessence, Cosmic coincidence, and the cosmological Constant", I. Zlatev, L. Wang, and P. Steinhardt, PRL 82, 896 (1999)

4) "Cosmological tracking solutions", P. Steinhardt, L. Wang and I. Zlatev, Phys Rev D 59, 123504 (1999).

- Acceleration of present universe $\longrightarrow$ mysterious form of "spatially uniform" dark energy with an equation of state:

$p = w\rho$ with $w \simeq -1$.

- Latest constraint on w:

Supernova Legacy Survey (SNLS).

71 High redshift type Ia supernovae + Sloan Digital Sky Survey measurement of baryon acoustic oscillations

$\Rightarrow w = -1.023 \pm 0.090\,(\text{stat}) \pm 0.054\,(\text{syst.})$

$\Omega_M = 0.263 \pm 0.042\,(\text{stat}) \pm 0.032\,(\text{syst.})$ ΛCDM

$\Omega_M = 0.271 \pm 0.021\,(\text{stat}) \pm 0.007\,(\text{syst})$ w = constant

$\Rightarrow \Omega_{D.E.} \simeq 0.7$

Consistent with ΛCDM up to $z = 1$.

What models describe $w \simeq -1$ at least up to $z = 1$?

Quintessence

For many years after the discovery of the acceleration of the universe, Quintessence has been the most popular idea for the Dark Energy.

- What is Quintessence?

Energy associated with a scalar field Q "slowly evolving down" its potential $V(Q)$.

- What does it do?

It is supposed to describe a spatially homogeneous D.E. with <u>negative</u> <u>pressure</u>.

- How?

Homogenous Q:

$$\mathcal{L} = \frac{1}{2}\dot{Q}^2 - V(Q)$$

Energy momentum tensor:

$$T_{\mu\nu} = \partial_\mu Q \, \partial_\nu Q - g_{\mu\nu} \mathcal{L}$$

For Q spatially homogeneous $\Rightarrow \partial_i Q \sim 0$

$$\rho = T_{00} = \frac{1}{2}\dot{Q}^2 + V(Q)$$

$$p = T_{ii} = \frac{1}{2}\dot{Q}^2 - V(Q)$$

$\Rightarrow$ Equation of state:

$$\boxed{w = \frac{p}{\rho} = \frac{\frac{1}{2}\dot{Q}^2 - V(Q)}{\frac{1}{2}\dot{Q}^2 + V(Q)}}$$

$p < 0$ when kinetic $\frac{1}{2}\dot{Q}^2 \ll$ potential $V(Q)$

If, for some dynamical reasons, $\frac{1}{2}\dot{Q}^2 \ll V(Q)$

$\Rightarrow \boxed{w \simeq -1}$ mimic a cosmological constant.

A digression:

Why is $w=-1$ for a cosmological constant?

Energy-momentum tensor for a "perfect fluid":

$$T_{\mu\nu} = (\rho + p) U_\mu U_\nu + p\, g_{\mu\nu}$$

$$U^\mu = (1, 0, 0, 0) \qquad \Rightarrow g_{\mu\nu} = \begin{pmatrix} -1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix}$$

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 \\ 0 & p\, g_{ij} \end{pmatrix}$$

Cosmological constant: $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$

$$\Rightarrow T^{c.c.}_{\mu\nu} = -\frac{\Lambda}{8\pi G} g_{\mu\nu}$$

$$\Rightarrow \rho_\Lambda = \frac{\Lambda}{8\pi G}$$

$$p_\Lambda = -\frac{\Lambda}{8\pi G} = -\rho_\Lambda \quad \Rightarrow \quad w = -1$$

Recall: $$w = \frac{\frac{1}{2}\dot{Q}^2 - V(Q)}{\frac{1}{2}\dot{Q}^2 + V(Q)}$$

Questions :

1) What could $V(Q)$ be ?

2) How does Q evolve ?

* 3) Why does $\Omega_Q \simeq 0.7 = O(\Omega_M \simeq 0.3)$ Now ?

Cosmic coincidence

To get (3), one has to extremely fine tune $V(Q)$ and set the initial conditions very carefully :

cosmic coincidence

- Is there a way out of this fine-tuning problem?

"Tracker" solutions : Wide range of initial conditions $\Rightarrow$ Rapid convergence to a common $\rho_Q(t)$ and $w_Q(t)$.

- What does "tracking" mean?

ρ_Q remains close to ρ_M and ρ_R for most of the cosmological history and dominates over ρ_M now. .

- Conditions? $\Gamma = 1 + \frac{w_B - w_Q}{2(1+w_Q)} - \frac{1+w_B-2w_Q}{2(1+w_Q)}\frac{\dot{x}}{6+\dot{x}} - \frac{2}{1+w_Q}\frac{\ddot{x}}{(6+\dot{x})}$

$\Gamma = V''V/(V')^2 > 1$ $\quad x = \frac{1+w_Q}{1-w_Q} = \frac{1}{2}\dot{Q}^2/V$

↑ Determines whether or not tracker solutions exist.

- Models?

An early model was by Ratra and Peebles (1988).

$$V(Q) = \frac{V_0}{Q^\alpha}$$

In this model: $\rho_Q = \frac{1}{2}\dot{Q}^2 + \frac{V_0}{Q^\alpha}$

For $a(t) \propto t^n$ and $\rho_Q \ll \rho_B$ ← to avoid messing up with B.B. nucleosynthesis

$$\frac{\rho_Q}{\rho_B} = \left(\frac{a(t)}{a_1}\right)^{\frac{4}{n(\alpha+2)}} \propto t^{\frac{4}{\alpha+2}}$$

$\alpha > 0 \Rightarrow \frac{\rho_Q}{\rho_B}$ increases until $\rho_Q > \rho_B$!

- Other models: $(\phi \equiv Q)$

$V_0 \, e^{-\lambda\phi}$

$m^2\phi^2, \; \lambda\phi^4$

$V_0 \, \dfrac{e^{\lambda\phi^2}}{\phi^\alpha}$

$V_0 \, (\cosh\lambda\phi - 1)^p$ Sahni, Wang

$V_0 \, (e^{M_{pl}/\phi} - 1)$ Zlatev, Wang, Steinhardt

⋮

- The graph 1 shows the model of Zlatev, Wang, Steinhardt with tracking solutions of ρ_Q vs $z+1$
- Graph 2 shows w_Q vs $z+1$
- Graph 3 shows ρ_Q v.s. $z+1$ for $V_Q = \dfrac{M^4}{Q^6}$
- Graph 4 shows w_Q v.s. $z+1$ for $V_Q = \dfrac{M^4}{Q^6}$
- Graph 5 shows Ω_Q vs. $z+1$ " " "

- Graph 6 shows W_Q v.s. $\Omega_M = 1 - \Omega_Q$ for $V \sim 1/Q^6$, $\exp(1/Q)$, $1/Q$

Since $\Omega_M \simeq 0.3 \Rightarrow W_Q \simeq -0.7$ to -0.3.
Since $w \simeq -1$, it appears to rule out many of these models.

Critiques of "tracking" models:

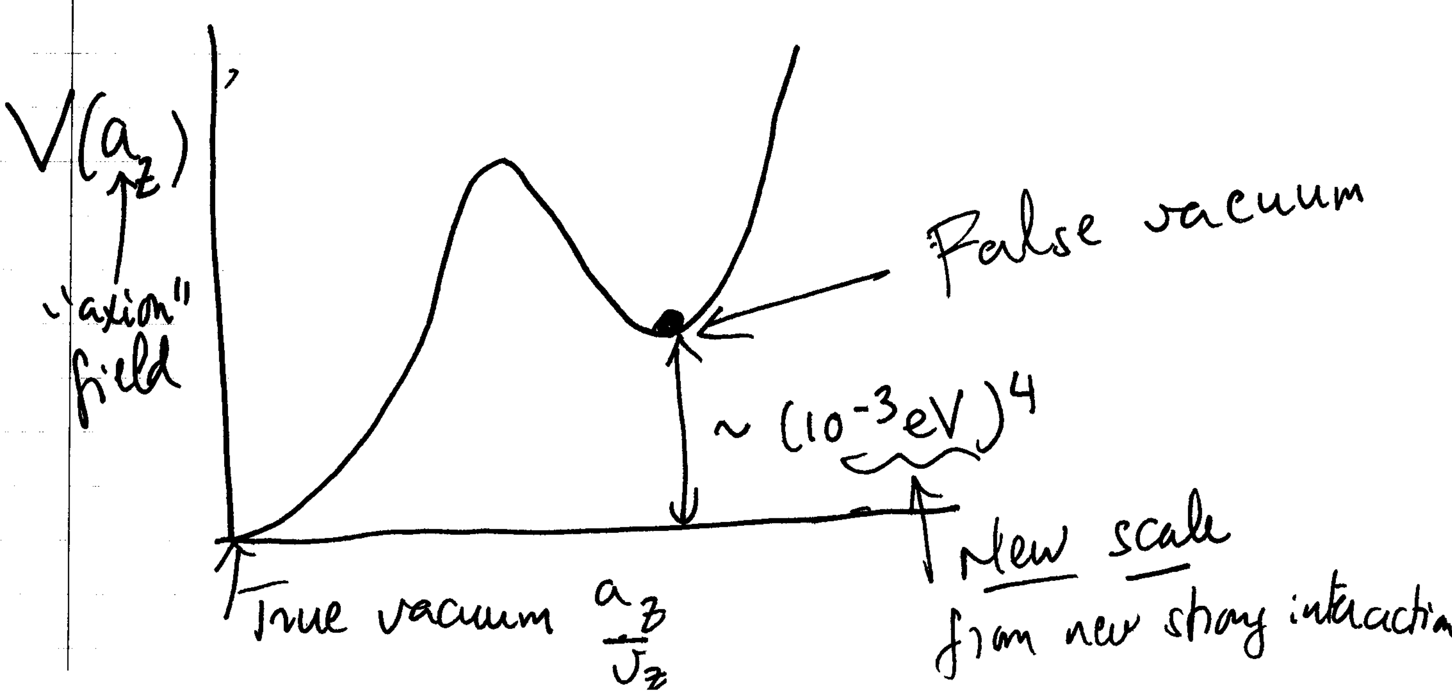
1) What's Q? How can we test whether it has any relevance to other physical phenomena?
2) No unique model
3) Even if one can have widely different initial conditions which converge to a common tracking solution, why $\rho_Q \sim (10^{-3} eV)^4$?
4) What about SNLS recent constraint $w \simeq -1$?
A large class of tracker models do not give that value.

A common feature of tracker models:

$\boxed{w \text{ varies with } z}$ ⟸ Crucial test.

The latest SNLS results ⟹ $w \simeq -1$ for a "large" range of z ⟹ consistent with ΛCDM i.e. a universe dominated by a cosmological constant Λ and cold dark matter (CDM).

Can one construct a quintessence model that can "mimic" ΛCDM? P&H



- How long does the universe get trapped in the false vacuum?

$\tau \gg$ age of universe.

$\Rightarrow$ The universe will undergo a late inflationary stage i.e $a(t) \sim e^{Ht}$

- When did this potential arise?

$t \sim 125$ Myr.

- Where does the new scale $\sim 10^{-3}$ eV come from?

Some new gauge group ($SU(2)_z$) which grows strong at that scale starting from a coupling at $\sim 10^{16}$ GeV having a value comparable to that of the Standard Model couplings.

- Other cosmological consequences:
- WIMP Cold Dark matter.
- Leptogenesis $\Rightarrow$ Baryogenesis
- Laboratory detection of some of these particles.

- Many other models:

e.g. Braneworld models where $w \lesssim -1$

Chaplygin gas where $p = -\frac{A}{\rho}$ and

where $w : \underset{a \ll 1}{0} \longrightarrow \underset{a \gg 1}{-1}$

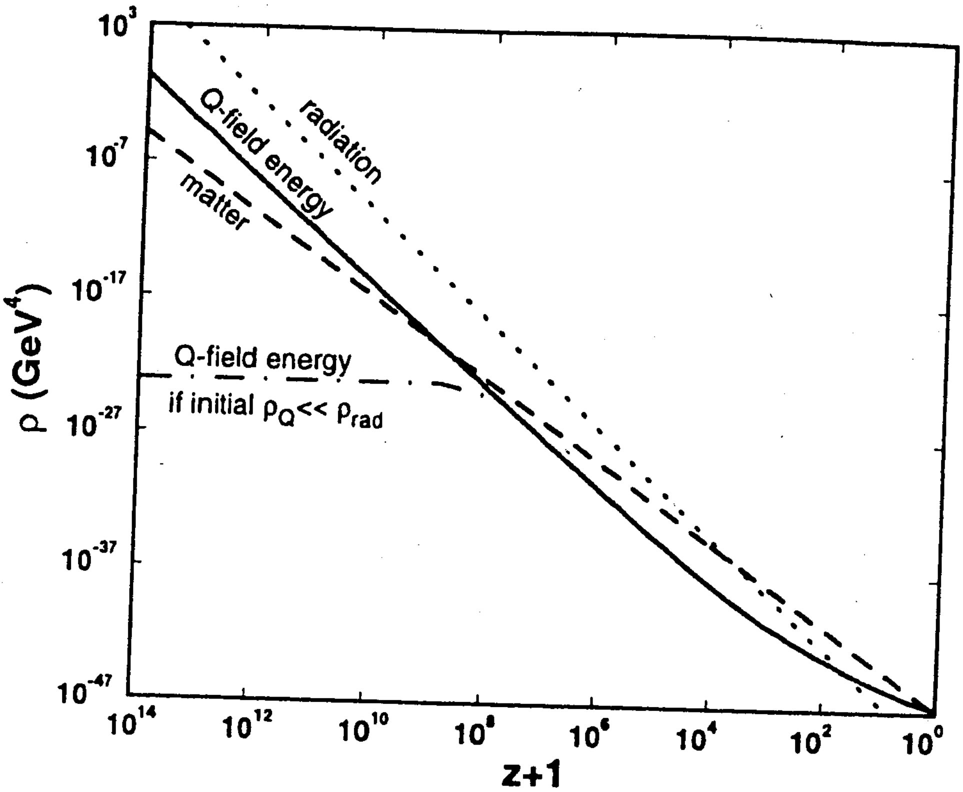


FIG. 1. The evolution of the energy densities for a quintessence component with $V(Q) = M^4[\exp(M_p/Q) - 1]$ potential. The solid line is where ρ_Q is initially comparable to the radiation density and immediately evolves according to tracker solution. The dot-dashed curve is if, for some reason, ρ_Q begins at a much smaller value. The field is frozen and ρ_Q is constant until the dot-dashed curve runs into tracker solution, leading to the same cosmology today: $\Omega_m = 0.4$ and $w_Q = -0.65$.

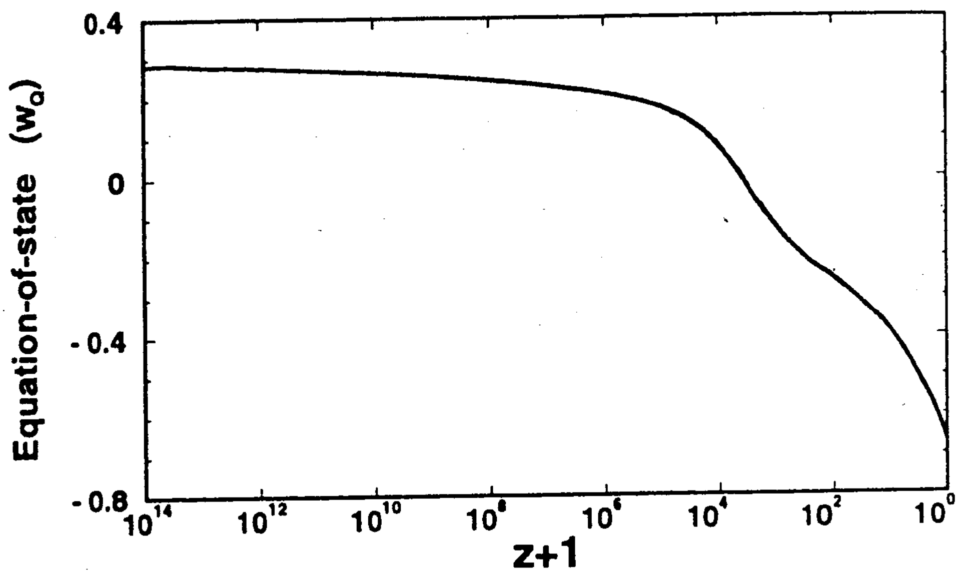


FIG. 2. w_Q vs z for the model in Fig. 1. During the radiation-dominated epoch (large z), $w_Q \approx 1/3$ and the Q energy density tracks the radiation background. During the matter-dominated epoch, w_Q becomes somewhat negative (dipping down to $w_Q \approx -0.2$ beginning at $z = 10^4$) until ρ_Q overtakes the matter density; then, w_Q plummets towards -1 and the Universe begins to accelerate.

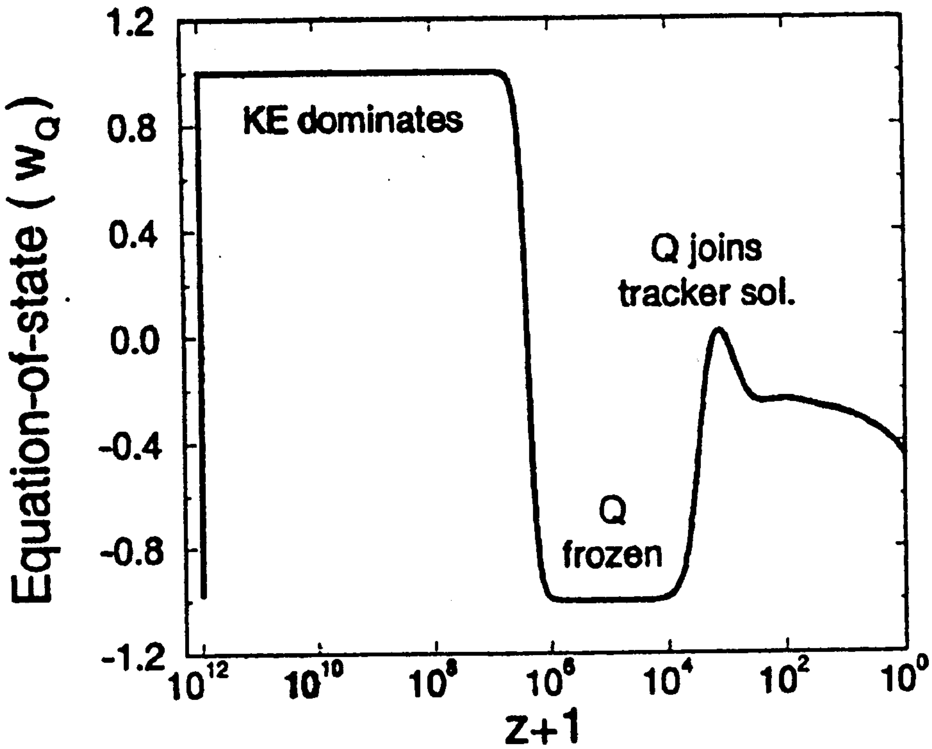


FIG. 3. A plot of w_Q vs redshift for the overshoot solution shown in Fig. 1. w_Q rushes immediately towards $+1$ and Q becomes kinetic energy dominated. The field freezes and w_Q rushes towards -1. Finally, when Q rejoins the tracker solution, w_Q increases, briefly oscillates and settles into the tracker value.

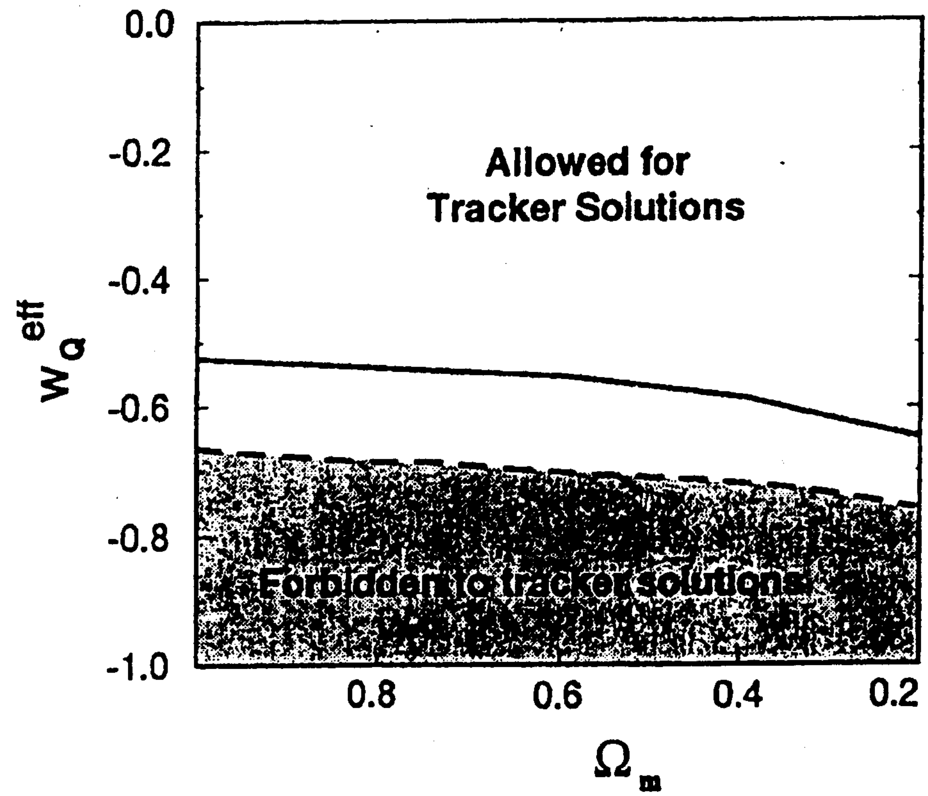


FIG. 9. A plot of w_Q^{eff} versus $\Omega_m = 1 - \Omega_Q$, showing the minimum w_Q^{eff} possible for tracker solutions. The solid line is the lower boundary assuming the constraint that the Q-field begins with equipartition initial conditions and begins rolling before matter-radiation equality. The dashed line is the lower boundary if this condition is relaxed to allow general $V \sim \Sigma c_k / Q^k$. As explained in the text, w_Q^{eff} is the value that would be measured in supernova and microwave background experiments which effectively integrate over a varying w_Q.

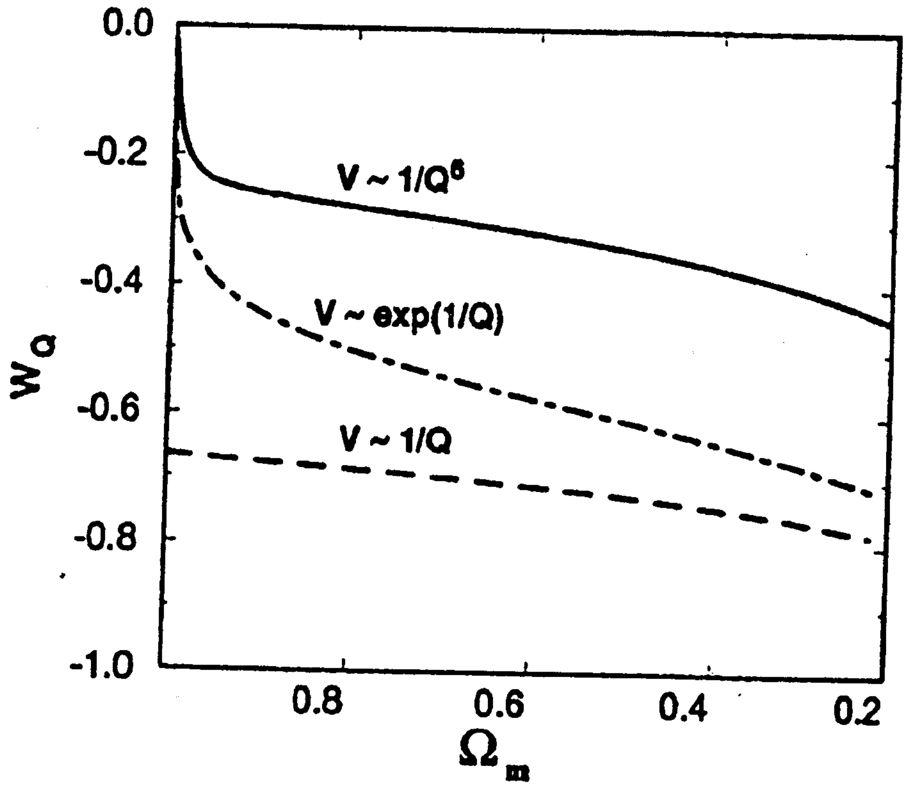


FIG. 8. The Ω_Q-w_Q relation for various potentials assuming a flat universe $\Omega_m = 1 - \Omega_Q$, where w_Q represents the present value of w_Q. The potentials and notation are the same as in Fig. 7.