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Dynamical Models of Circumstellar Dust Shells around Long–period Variables: The effect of Multiperiodicity

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We present dynamical models of circumstellar dust shells around carbon–rich Long-period variables which include time-dependent hydrodynamics and a detailed treatment of dust formation, growth and evaporation as well as a treatment of chemistry and radiative transfer.

Important effects due to the complex, nonlinear interaction between the dynamics of the pulsating atmosphere and the dust complex are demonstrated. It is shown that in a certain range of the parameter space the dust shell develops its own time scale which is different from the time scale of the interior pulsation. The key parameter which tunes this eigenfrequency of the dust shell is the overabundance of carbon to oxygen. For specific values of the carbon overabundance the eigenfrequency of the dust shell is an integer multiple of the pulsational period, for other values irregular or chaotic behavior results from the interaction of the pulsation and the circumstellar shell.

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MOTIVATION AND MODELING METHOD

Reliable theoretical models of circumstellar dust shells (CDS) around Long–period Variables (LPVs) require the simultaneous treatment of time–dependent hydrodynamics, dust formation, radiative transfer, and chemistry in order to account for the various interactions among the different physical processes.

As described in detail in Fleischer et al. (1992), Fleischer (1994) our approach comprises

- an explicit solution of the hydro–equations (Lagrangian coordinates in spherical symmetry) according to the difference scheme of Richtmyer & Morton (1967),
- a detailed treatment of the formation, growth, and evaporation of **carbon** grains by means of a moment method Gauger et al. (1990),
- the solution of the radiative transfer problem adopting the Eddington approximation for a grey, spherically symmetric atmosphere acc. to Lucy (1971,1976),
- a treatment of postshock thermal relaxation of the gas by an LTE–approximation for the source function in the energy equation,
- an equilibrium chemistry of the gas phase where the abundances of the elements are assumed to be solar except for the carbon abundance which is treated as a free parameter to account for a carbon–rich mixture,
- the simulation of the interior pulsation by a sinusodially moving inner boundary (*piston*).

⟶ In this paper we briefly demonstrate that the complex interaction between pulsation and dust formation might lead to a periodic behaviour of the CDS on a time scale different from the pulsational period P (*multiperiodicity*).

⟶ Observable consequences of the effect of *multiperiodicity* are shown by Winters et al. (1995).

FIGURES

The model is completely determined by the prescription of the following six parameters: stellar mass M_*, effective temperature T_*, stellar luminosity L_*, overabundance of carbon to oxygen ϵ_C/ϵ_O, period P, and amplitude of the piston Δu_P

As a demonstrative example of a *multiperiodic* model we present a calculation with the following parameter:

$$M_* = 1.0 M_\odot, T_* = 2600K, L_* = 1.0 \cdot 10^4 L_\odot,$$
$$\epsilon_C/\epsilon_O = 1.50, P = 650d, \text{and} \ \Delta u_P = 2kms^{-1}$$

Figures 1, 4, 5, and 7 show a time sequence of radial structures of the model. The **upper row** shows the hydrodynamical velocity **u** (solid line, l.h.s. ordinate) and the radiative acceleration due to radiation pressure on dust $\boldsymbol{\alpha}$ in units of the gravitational acceleration (dashed line, r.h.s. ordinate). The **second row** shows the logarithm of the density $\boldsymbol{\rho}$ (solid line, l.h.s. ordinate) and the degree of condensation $\mathbf{f}_c$ (dashed line, r.h.s. ordinate). The **third row** shows the logarithm of the nucleation rate $\mathbf{J}_*$ (solid line, l.h.s. ordinate) and the logarithm of the number of dust particles per H–atom $\boldsymbol{n}_d/\boldsymbol{n}_{<H>}$ (dashed line, r.h.s. ordinate). The **lower row** shows the temperature of the gas $\mathbf{T}^{\mathrm{g}}$ (solid line, l.h.s. ordinate) and the dust temperature $\mathbf{T}^d$ (dashed line, r.h.s. ordinate).

Figure 6 shows the radial course of the velocity u (top) and the degree of condensation f_c (bottom) at eight successive instants of time (each at the beginning of the hydrodynamical cycle) atop of each other. *"Even"* periods are shown in blue, *"odd"* periods are shown in red.

Figure 2 shows the degree of condensation f_c, i.e. the amount of condensable material that is actually condensed into grains, as a function of radius and time. The color code for this figure is given in **Figure 3**.

RESULTS

- The radial structure of this model repeats after **two** pulsational cycles (cf. Figure 1), i.e. the *multiperiodicity number* of the model is $n_P = 2$ (compare also 1^{st} column of Figure 1 and 2^{nd} column of Figure 7).

- Especially in the inner part (around 5 R_*) of Figure 6 it can be seen that besides $n_P = 2$ other, longer frequencies are present. These "hyper–periods" show up as a long–term drift of the magnitude in theoretical lightcurves Winters et al. (1994).

$\longrightarrow$ The quantity n_P strongly depends on the overabundance of carbon to oxygen ϵ_C/ϵ_O Fleischer et al. (1995).

- Effective nucleation of dust particles is possible in a broad time interval were the gas temperature is low enough (less than 1300 K, cf. Figures 4,5).

- The nucleation phase is followed by a phase of effective dust growth caused by the compression of the seed bearing material by a wave resulting from the interior pulsation (cf. Figures 5, 7).

- Radiation pressure on newly formed dust particles re–accelerates the material (cf. Figure 5) and causes the dominant shock waves which traverse the atmosphere.

$\longrightarrow$ Resulting quantities of this model

  - mass loss rate $\dot{M} = 8.9 \cdot 10^{-6} M_\odot\ \mathrm{yr}^{-1}$
  - final outflow velocity $u_\infty = 16.5\ \mathrm{km\ s}^{-1}$
  - dust–to–gas–ratio $\rho_d/\rho_g = 2.6 \cdot 10^{-3}$.

$\longrightarrow$ Pulsation *and* CDS have to be seen both as oscillating systems: they form a *coupled pendulum*:

- If the coupling strength is negligible, i.e. in the case where no pulsational driving is present, the CDS behaves on its intrinsic time scale, shock waves are produced by an *exterior κ–mechanism*. This case is discussed in Fleischer et al. (1995).
- If the coupling strength is maximal, i.e. pulsation is dominating, the CDS is forced to oscillate on the time scale of the interior period. This case is discussed in Fleischer et al. (1992).
- An intermediate coupling strength leads to the behaviour discussed here. In the *resonance case* between pulsational period and the characteristic time scale of the CDS *multiperiodic behaviour* with *integer–valued multiperiodicity numbers* results. Otherwise the CDS shows an irregular or even chaotic behaviour.

$\longrightarrow$ The drift in time either of the averaged magnitude or the amplitude found in observed *lightcurves* of LPVs can be naturally explained by respective theoretical models Winters et al. (1994) as a consequence of a multiperiodic behaviour of the CDS.

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