An Analysis of the Smoothness of Line Profiles

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1 Introduction

It is now generally agreed that molecular clouds are clumpy, i.e. they exhibit large point-to-point density fluctuations. This property must be taken into account when we seek to understand the physical processes which take place within molecular clouds, such as heating and cooling, radiative transfer, cloud support and breakup. Clearly, clumpiness must be of fundamental importance to the process of star formation itself. However, quantifying this property has proven to be a difficult task, and basic questions remain unanswered. High angular resolution mm-wave interferometry has been making some inroads into this problem by sam pling very small fields of view in selected molecular clouds. However, it is still not practical to survey large regions of molecular emission with this technique, especially with the high signal-to-noise ratios which would ideally be required. In addition, recent studies indicate that clumpiness exists at all levels down to the highest angular resolution available today; it may thus be futile to try to count clumps at any given level.

Because individual clumps are observed to emit lines which are narrower than those emitted by their ensemble (the cloud), they must leave an imprint as velocity structure superimposed on ensemble profiles. Thus, an alternative route to explore clumpiness is to use single dish telescopes to observe emission lines with very high signal-to-noise ratios and very high velocity resolution, analyze the fluctuations in brightness temperature present on the line shape, and infer from them the properties of the clumps present in the beam. The advantage of this technique is that since a relatively large field of view is sampled, the statistical properties of clumps are obtained directly, without having to go through the process of detecting and mapping each clump as is done with interferometry.

This technique was developed recently and applied to the case of Orion (Tauber, Goldsmith and Dickman 1991, Ap.J., 375, 635), using CO and ^{13}CO lines as tracers. However, in that paper the clumps were assumed to be "black disks", and the beam area filling factor was assumed to be small. We have now generalized the technique to include clumps with an arbitrary central opacity and radial distribution, using as a framework the clumpy cloud model of Martin, Sanders and Hills (1984, MNRAS, 208, 35; hereafter MSH). With this model, the smoothness of the observed profiles can be used to constrain the parameter space of the unresolved clumps.

2 The Model

The essential assumptions of the MSH model are that:

- the number of fragments is large
- each fragment has the same radially-symmetric density structure
- each fragment has an intrinsic gaussian velocity profile of width v_0
- all fragments have identical and radially constant excitation tempera tures
- the central velocities of the fragments have a gaussian distribution of width σ around the velocity of the ensemble (assumed to be null here)
- the cloud properties are uniform within the beam area

The main properties of the clumps are:

- the clump "column density" N (i.e. the number of clumps per unit area)

- the clump effective radius r_0 and central opacity τ_0
- the ratio of the intrinsic clump velocity width to the ensemble velocity distribution ($\frac{v_0}{\sigma}$).

We distinguish three basic situations:

1. **Clumps in emission:**

   The ensemble emission profile can be written as:

$$T_R(v_z) = T_E(1 - e^{-\tau_{ef}(v_z)}) \tag{1}$$

   where T_E is the excitation temperature for any clump, and τ_{ef} is an "effective" optical depth. If $\frac{v_0}{\sigma} < 0.3$, the latter is:

$$\tau_{ef} = N r_0^2 \frac{v_0}{\sigma} A(\tau_0) e^{-\pi(\frac{v_z}{\sigma})^2} \tag{2}$$

   where $A(\tau_0)$ is a constant which depends only on the assumed geometry of the clump and its central optical depth. We define the smoothness of the line as the typical fractional variation of T_R within a velocity bin of width vo near v_z=0. It can be expressed as:

$$\frac{\Delta T_R}{T_R} = \frac{A(\tau_0) r_0^2/B}{e^{\tau_{ef}} - 1} \sqrt{N_{tot}\frac{v_0}{\sigma}} \simeq \begin{cases} 0 & if\ \tau_{ef} \gg 1 \\ \frac{1}{\sqrt{N_{tot}\frac{v_0}{\sigma}}} & if\ \tau_{ef} \ll 1 \end{cases} \tag{3}$$

   where B is the area of the beam, and N_{tot} is the total number of clumps in the beam (at all velocities).

2. **Clumps in absorption:**

   The ensemble profile can be written as:

$$T_R = T_C e^{-\tau_{ef}(v_z)} \tag{4}$$

   where T_C corresponds to the intensity of the background continuum. In this case the smoothness is defined as $\frac{\Delta T_R}{T_R - T_C}$, and can be expressed identically to the case of emission above.

3. **Emission and Absorption:**

   The ensemble profile in this case is:

$$T_R(v_z) = T_E(1 - e^{-\tau_{ef}(v_z)}) + T_I(1 - e^{-\tau_i(v_z)})e^{-\tau_{ef}(v_z)} \tag{5}$$

   This equation corresponds to a situation where a clumpy medium is located in front of a homogeneous background of opacity τ_i emitting at a temperature T_I. Normally one expects clumps to be embedded in (rather than in front of) a warm "interclump medium": to model this situation more accurately a first order correction can be made to Eq. 5 by replacing τ_i with $\gamma\tau_i$, and adding a term of the form $T_I(1 - e^{-(1-\gamma)\tau_i(v_z)})$ to its r.h.s.; γ represents the fractional length of the interclump medium which is *on the average* located behind the clumps. In the simpler case of Eq. 5, the smoothness can be written as:

$$\frac{\Delta T_R}{T_R} = \frac{A(\tau_0)\frac{r_0{}^2}{B}}{(e^{\tau_{ef}} - 1)} \frac{1 - (\frac{T_I}{T_E})(1 - e^{-\tau_i})}{1 + (\frac{T_I}{T_E})\frac{(1-e^{-\tau_i})}{(e^{\tau_{ef}}-1)}} \sqrt{N_{tot}\frac{v_0}{\sigma}} \simeq \begin{cases} 0 & if\ \tau_{ef} \gg 1 \\ (\frac{T_E}{T_I(1-e^{-\tau_i})} - 1)A(\tau_0)\frac{r_0{}^2}{B}\sqrt{N_{tot}\frac{v_0}{\sigma}} & if\ \tau_{ef} \ll (\frac{T_I}{T_E})(1 - e^{-\tau_i}) \end{cases} \tag{6}$$

In order to estimate the smoothness of an observed profile, a smooth function should be fitted to it, and the residuals binned to the assumed v_0. The choice of smooth function depends on the overall shape of the line and the typical width of the variations; but in general will be a gaussian.

The case we analyze is that in which at least two lines have been observed, and their relative optical depth is known (e.g. ^{12}CO and ^{13}CO). The quantities that can be used for analysis are the brightness ratio, the width ratio, and the smoothness ratio of the observed profiles. Fig. 1 illustrates the behavior of these quantities for a typical model cloud; the bottom panel shows the potential power of the smoothness ratio (as compared to e.g. the width ratio) as a model discriminator, especially for pairs of lines with high τ_0 ratio. However, this figure also clearly shows that very high instrumental S/N ratios in the observed profiles are required to arrive at meaningful results. Fig. 2 is a typical example of a plot useful for analysis, which can be used to derive, from the measured parameters, the central optical depth of the clumps, and their filling factor.

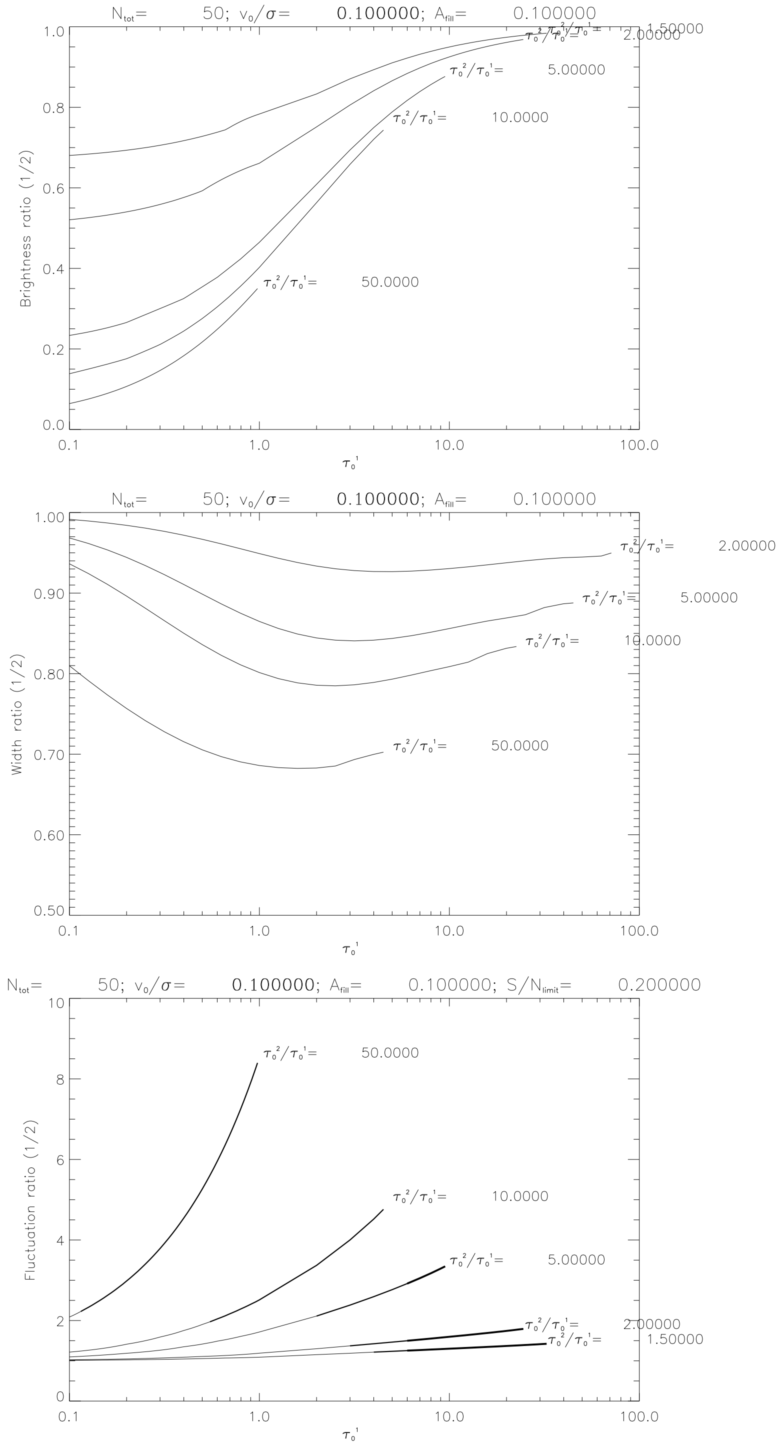

Figure 1: The top panel shows the peak brightness ratio (BR) of two lines as a function of the optical depth of the thinnest of the pair. Each line corresponds to a given ratio of the clump intrinsic center optical depth (marked). The calculations correspond to a model with N_{tot}=50, $\frac{v_0}{\sigma}$=0.1, and the beam filling factor of a single clump $\frac{{r_0}^2}{B}$= 0.1. The middle and bottom panels show the "width ratio" (WR) and the "smoothness ratio" (SR) respectively, for the same model. In the bottom panel, the lines are drawn in double thickness when the $\frac{\Delta T_R}{T_R}$ fluctuation level of the weaker (optically thinner) line is lower than 0.2, and in quadruple thickness when the fluctuation level of the brighter line goes below the same level. Thus, if the peak instrumental S/N level is ∼15, SR can be determined where the lines are drawn in normal thickness, an upper limit on SR can be determined where the lines are drawn in double thickness, and the S/N is not adequate where the lines are drawn in quadruple thickness. In this figure calculations have been stopped when τ_0^2 reaches a value of 50.

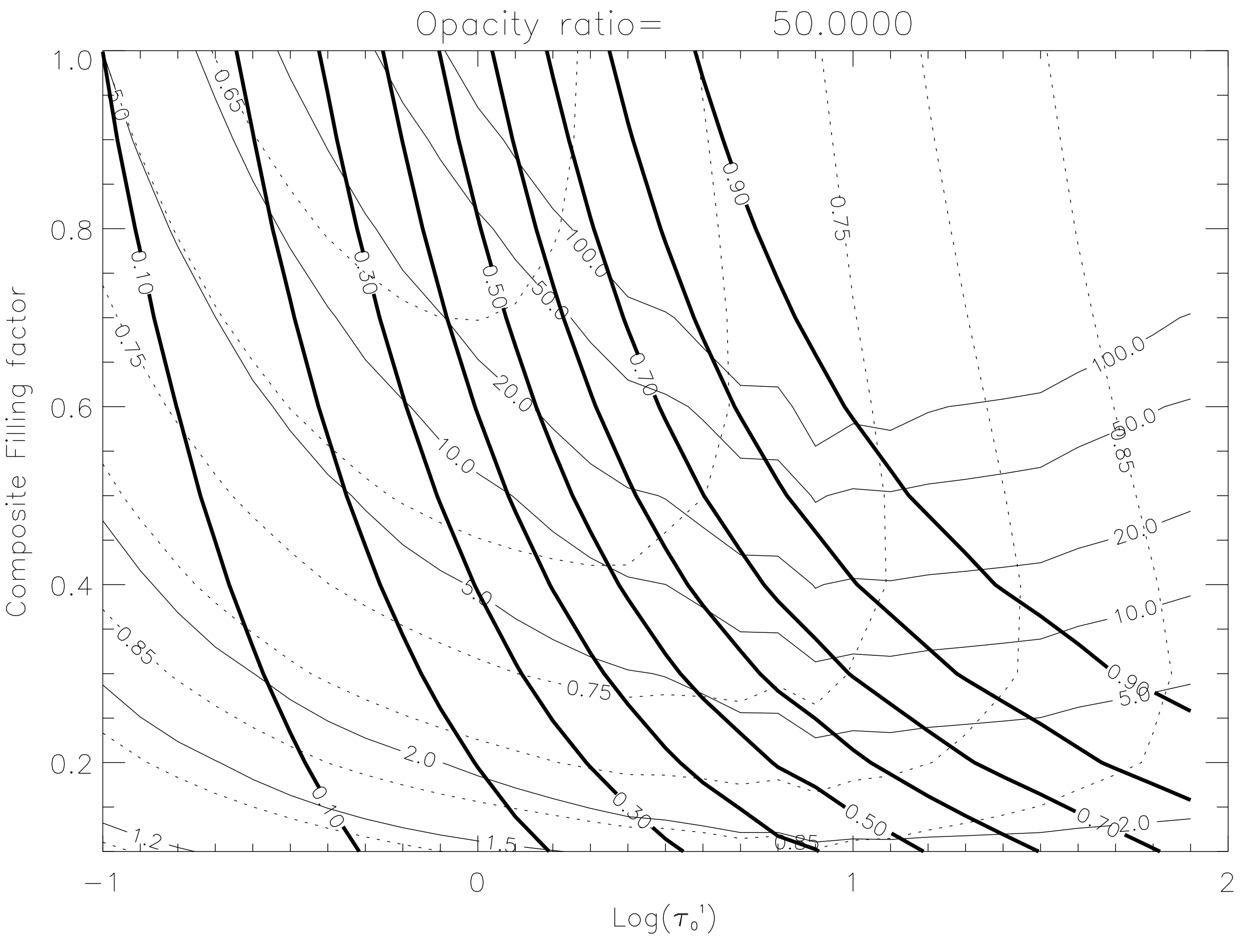

Figure 2: Curves for analysis of a pair of lines with a ratio of the clump intrinsic center optical depth of 50. The clumps have a gaussian density distribution. The horizontal axis is the center opacity of the thinnest line; the vertical axis is N_v (equal to the product $N_{tot}\frac{v_0}{\sigma}\frac{{r_0}^2}{B}$). The thick contours are lines of constant BR (brightness ratio), and the thin contours are lines of constant SR (smoothness ratio); particular values are marked on the contours. Contours of constant WR (FWHM width ratio) are also marked with dashed lines.