A TECHNIQUE FOR ACCURATE NOISE TEMPERATURE MEASUREMENTS FOR THE SUPERCONDUCTING QUASIPARTICLE RECEIVER

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ABSTRACT

We propose an extremely simple new technique to determine the noise temperature of the rf input section of a superconducting quasiparticle heterodyne receiver. Our method uses standard hot/cold-load measurements of the receiver driven by local oscillator power levels below optimum. We present both analytic and computed results based on Tucker's quantum theory of mixing (no experimental data are at present available) to assess the accuracy and the precision of this new technique.

INTRODUCTION

Lossy components in the rf input section of a heterodyne receiver contribute to the receiver noise temperature. For cryogenic receivers this rf input section noise has proven extremely difficult to measure. This is because the noise is the aggregate of thermal radiation arising from quite small losses at various temperatures ranging from room temperature down to the cryogenic operating temperature. The magnitude of each loss, and the temperature at which it is incurred, cannot be accurately estimated. This is an important problem for practical superconductor-insulator-superconductor (SIS) quasiparticle receivers, because this rf input section noise can dominate the entire receiver noise temperature.

For example, in Ref [1] the detailed sources of noise in a 114 GHz SIS receiver were quantitatively analyzed. The input section noise temperature was determined by two separate methods. In spite of careful and elaborate measurements, both methods yielded an uncertainty in the input section noise of about 16 K, which was by far the greatest
source of uncertainty in inferring the mixer noise temperature. This problem can be much more severe for submillimeter wavelength receivers: quasi-optical components are generally used, measurement equipment and technique are more rudimentary, and material properties are less favorable and often less well determined at the higher frequencies.

We propose a new technique to determine the input section noise temperature of an SIS receiver. Our method uses standard hot/cold-load measurements of a receiver driven by local oscillator power levels less than the receiver optimum. (The possibility of using such measurements as a diagnostic was suggested in Ref. [2].) The measurement is so simple that it may be performed during each experiment, to make sure that the input section noise temperature (i.e. beam-pattern matching, junction placement, etc.) has not changed. We present both analytic and computed results based on Tucker's quantum theory of mixing (no experimental data are at present available) to assess the accuracy and the precision of this new technique.

THE INTERSECTING LINES TECHNIQUE

The noise temperature of a microwave receiver is generally determined, quite accurately, by the so-called Y-factor method: Hot and cold matched loads, at temperatures $T_h$ and $T_c$, are alternately placed at the receiver input and the total receiver IF output powers, $P_h$ and $P_c$ respectively, are measured. The receiver noise temperature $T_R$ is then given by the equation

$$T_R = \frac{(T_h - Y T_c)}{(Y - 1)} ,$$

where $Y = \frac{P_h}{P_c}$. This procedure can be performed graphically: The receiver output power is plotted against the load temperature, and the straight line connecting the points $(P_h, T_h)$ and $(P_c, T_c)$ is extrapolated back to intersect the load temperature axis. The temperature given by that intersection point is the negative of $T_R$.

Blundell, Miller, and Gundlach [2] recently described a remarkable and curious property of such hot/cold-load measurements of SIS receivers. They showed that if this graphical procedure is performed for a variety of local oscillator (LO) power levels less than the receiver optimum, the hot/cold-load straight lines for each LO level all intersect at a single point. Figure 1, copied from Ref. [2], is an example of this. The measured receiver output power is plotted for $T_h = 295$ K and $T_c = 77$ K and a straight line is drawn connecting these two points, for each of five different LO levels. These five lines all quite precisely intersect.
Fig. 1. The output power from an SIS receiver is plotted for hot and cold loads and a straight line is drawn connecting these two points, for each of five different LO levels. This figure is copied from Ref. [2].

In the remainder of this paper we will examine the equations describing the SIS receiver to understand the reason for this intersection. We will show that the load temperature corresponding to this intersection point, which we call $-T_x$, is the (negative of the) equivalent input noise temperature of the rf input section of the receiver: $T_{RF} = T_x$. Thus $T_{RF}$ is for the first time amenable to easy measurement.

**EXPLANATION OF THE TECHNIQUE**

Our proposed technique relies on the fact that the SIS mixer output noise temperature [3] is largely independent of mixer gain for low local oscillator power. Figure 2 shows a standard diagram of a heterodyne receiver. The receiver consists of three blocks: the rf input section, the mixer, and the IF amplifier; with respective gains $G_{RF}$, $G_M$, and $G_{IF}$; and with respective equivalent input noise temperatures $T_{RF}$, $T_M$, and
Cascading these blocks, the total output power $P_T$ from the receiver when a matched load at temperature $T$ is placed at the receiver input is given by

$$P_T = [(T+T_{RF})G_RF G_M + T_{out} + T_{IF}] k B G_{IF},$$

where we define the equivalent output noise temperature of the mixer

$$T_{out} = T_M G_M.$$ 

In functional form, Eq. 2 simply states that $P_T(T)$ is a straight line. Equation 2 could also be written $P_T = (T+T_R) k B G_{RF} G_M G_{IF}$, so that $P_T = 0$ for $T = -T_R$, which is a restatement of the graphical Y-factor method of determining $T_R$.

Let us now hypothesize that $T_{out}$ is independent of the LO power, $P_{LO}$, for low LO power levels. (We will discuss the accuracy and the range of validity of this hypothesis for SIS mixers, below.) Then in Eq. 2 only $G_M$ will depend upon $P_{LO}$, and so $P_T$ is independent of $P_{LO}$ for some (negative) input load temperature $T = -T_X = -T_{RF}$. Therefore our hypothesis implies that we can read $T_{RF} = T_X$ directly from the intersection point on a graph like Fig. 1. Contributions to $P_T$ are sketched in Fig. 3.

![Fig. 3. Contributions to the receiver output power for interpreting the intersecting lines technique are sketched.](image)

What if our hypothesis is not completely true? Consider the possibility that $T_{out}$ also has a component which is proportional to $G_M$. Call this component $\tau G_M$. Then Eq. 2 shows that the $P_T(T)$ lines for various $P_{LO}$ still intersect at a point, but in this case $T_X$ is no longer equal to $T_{RF}$ but to $T_{RF} + \tau$, which can be as large as $T_{RF} + T_{M}$. (We take $G_{RF} = 1$ for convenience.) Consider further the possibility that $T_{out}$ has a component with some more complicated dependence upon $G_M$ as $P_{LO}$ is changed. Then Eq. 2 shows that the...
PT(T) lines for various PLO will not intersect at a point, and TX is not defined. This happens in fact for SIS mixers at higher PLO levels.

It is instructive to apply the intersecting lines technique to the ideal exponential Schottky diode mixer receiver [4]. In this case T_{out} = (1 - 2G_m)T_D, where the diode noise temperature T_D is constant [5]. Equation 2 then remarkably predicts that all PT(T) lines will intersect at a point, for every value of PLO. The intersection point, however, gives TX = TRF - 2T_D, which can be considerably smaller than TRF. Nevertheless, Schottky receiver engineers are likely to find the intersecting lines technique helpful in characterizing their receivers.

ANALYSIS

The LO power of an SIS mixer is characterized by the parameter \( \alpha = eV_{LO}/h\omega \), where \( V_{LO} \) is the LO voltage across the SIS junction and \( \omega \) is the LO frequency. We are interested in PLO levels less than the receiver optimum so it is appropriate to expand the equations from Tucker's quantum mixer theory [6] which describe the SIS mixer in a power series in \( \alpha \). The analysis presented in this section is based on such a small-\( \alpha \) limit expansion [7]. All of the conclusions are confirmed and delineated by numerical computations from the quantum mixer theory. The techniques used and the approximations made for the numerical computations are presented in Ref. [3].

\( G_m \) is proportional to \( \alpha^2 \) to lowest order. \( T_{out} \) has a more complicated dependence. \( T_{out} \) consists of three types of noise [3]. First, there is the "uncorrelated" noise arising from the leakage current shot noise of the SIS junction; this is independent of \( \alpha \). Second, there is the "correlated" LO-induced shot noise, which is conventionally represented by correlated noise sources placed at the mixer's signal, image, and IF terminations; this is proportional to \( \alpha^4 \) to lowest order. Third, \( T_{out} \) can include quantum noise, but for now we consider a double-sideband mixer which is perfectly matched to the LO source, so quantum noise is entirely included in the factor \( (T+T_{RF}) \) in Eq. 2 and does not appear as a component of \( T_{out} \).

This discussion shows that \( T_{out} \) is independent of \( \alpha \) and hence independent of PLO for low LO power levels for a matched SIS mixer. There is no \( \alpha^2 \) term. We conclude that the intersecting lines technique works perfectly.

This is not precisely true. The separation between "uncorrelated" and "correlated" noise is not perfect because they are coupled by the SIS mixer's nonlinearity. This produces a small \( \alpha^2 \) term in \( T_{out} \), which causes a small error in the equation \( T_{RF} = T_X \). But this is negligible; numerical computations show that the error in the estimation of \( T_{RF} \) is always less than 0.1 K for reasonable mixer parameters.
If the SIS mixer is not perfectly matched to the LO source there are two other processes producing small $\alpha^2$ terms in $T_{\text{out}}$ and hence small errors in the intersecting lines technique. We will discuss this for a resistive mismatch, and assume that a reactive mismatch gives the same result. We define the photon point currents $I_n = I_{dc}(V_n)$ and slopes $G_n = dI_{dc}(V_n)/dV$, where $I_{dc}(V)$ is the unmodulated dc I-V curve of the SIS junction, $V_n = V_0 + n\hbar\omega/e$, and $V_0$ is the dc bias voltage. The mismatch is characterized by the parameter $g = 2\hbar\omega G_s/e I_1$, where $G_s$ is the mixer's LO source conductance. The mixer is perfectly matched to the LO source when $g = 1$.

First, the leakage current shot noise which is coupled into the IF amplifier depends upon the output admittance of the mixer, and this is affected by the LO level. This produces an $\alpha^2$ term in $T_{\text{out}}$ with an effective input temperature

$$\tau_1 = \frac{I_0}{I_1} \frac{2G_1}{G_L+G_0} \frac{1-g^2}{4g} T_Q,$$

where $T_Q = \hbar\omega/k$ and $G_L$ is the IF load conductance seen by the mixer. Second, the quantum noise which appears as a component of $T_{\text{out}}$ is to lowest order $\alpha^2$ and gives an effective input temperature

$$\tau_2 = \frac{(g-1)^2}{4g} T_Q.$$  

The error (in degrees K) in the equation $T_{\text{RF}} = T_X$ is the sum of Eqs. 4 and 5. It is clear that these two terms can become large if $g$ is very large or very small, but in general these terms are not a serious problem. Our conclusion remains that the intersecting lines technique works satisfactorily.

AN EXAMPLE

Figure 4 shows a blow-up of the intersection region for a 235 GHz receiver using a six-junction array SIS mixer. The data, the output power from the receiver using hot and cold matched loads, for optimum LO power (line 1) and subsequently decreasing LO power (lines 2 to 6), was supplied by S.-K. Pan of the National Radio Astronomy Observatory. Line 1 gives $T_R = 73.0$ K. We see that lines 2 to 6 intersect in a rather narrow region, and that line 1 gives somewhat larger output power, as expected. Because the uncertainty in each power measurement was ± 0.5 $\mu$W, Fig. 4 gives $T_{\text{RF}} = 48.3 \pm 1.7$ K. More extensive data should narrow this uncertainty to ± 0.6 K, the same as the uncertainty of the Y-factor measurement of $T_R$. Systematic errors should also be the same as for the Y-factor measurement of $T_R$. Unfortunately, the rf input section noise of this receiver is not otherwise known, so the accuracy of the intersecting lines technique cannot be experimentally verified at present.
CONCLUSION

We have presented an extremely simple new technique to determine the noise temperature of the rf input section of a superconducting quasiparticle heterodyne receiver. The accuracy and the precision of this new technique are supported by analytic and computed results based on Tucker's quantum theory of mixing.

Acknowledgment: We are indebted to A.R. Kerr and S.-K. Pan for sharing unpublished experimental data with us, and for considerable contributions to the subject matter of this paper. This work was supported by NSF Grant #AST-8922301.
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