A Superconducting Mixer Tuned By The Quantum Susceptance
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Abstract
The high frequency quasiparticle tunneling current in a superconducting tunnel junction has been recently measured. The ratio of the reactive component of this current to the rf voltage is the quantum susceptance. We have successfully employed this susceptance as the tuning element for a SIS mixer.

Introduction
SIS (superconductor-insulator-superconductor) mixers are now the dominant technology for receivers in radioastronomy for observations in the millimeter-wave band. There is substantial effort to extend their operation into the sub-mm range [1]. It seems most likely that future millimeter-wave observational instruments will use focal plane arrays of SIS mixers [2]. At these high frequencies, the admittance due to the geometric capacitance of the tunnel junction is significant. A finite capacitance is necessary in order to shunt noise sidebands generated by the Josephson pair currents. At 100 GHz a good compromise is achieved with a $\omega R_n C$ product of about 4, where $R_n$ is the normal state resistance [4]. For $\omega R_n C \geq 1$ one must resonate the junction capacitance $C$ at the frequency of operation. Waveguide backshorts have been used successfully to tune out the capacitance at high frequencies, but are impractical for array applications [2]. On-chip thin film inductors have also been successful in resonating the junction capacitance. However these inductors are not variable, so one cannot adjust the resonant frequency [3,5]. Thus there is a need for a variable integrated tuning element at millimeter-wavelengths for array applications. We demonstrate in the present work a novel electronic on-chip tuning element.
Quantum Susceptance

In this paper we will concern ourselves only with the quasiparticle currents and assume the Josephson current is suppressed by a magnetic field. The quantum admittance can be calculated from the quasiparticle density of states which determines the dc I-V curve [6]. For an applied voltage:

\[ V(t) = V_0 + V_\omega \cos(\omega t), \]  

where \( V_0 \) is the dc bias voltage and \( V_\omega \) is the rf signal voltage at a fixed frequency \( \omega \), the admittance is:

\[ Y_Q(V_0,V_\omega) = G_Q(V_0,V_\omega) + iB_Q(V_0,V_\omega). \]  

The real part of this admittance is the quantum conductance, \( G_Q \), and the imaginary part of the admittance is the quantum susceptance, \( B_Q \).

The quasiparticle dc I-V curve of a SIS tunnel junction and its Kramers-Kronig transform are shown in Figure 1, along with the calculated quantum conductance and susceptance. These are calculated for a 90 GHz rf signal with an available power of \(-3\) pW.

The quasiparticle susceptance and conductance have recently been experimentally measured [7,8]. It has also been demonstrated that a lumped circuit model is accurate in describing the voltage dependence of these two quantities, even for a relatively large input power (\( \alpha > 1 \)) [8]. For a small rf power, and within one photon energy of the gap voltage, the quasiparticle admittance can be modeled as a voltage-variable susceptance, \( B_Q(V_0) = -j/\omega L(V_0) \), in parallel with a constant conductance which is about three times larger than the normal state conductance at 90 GHz.

Mixer Design

For proper tuning, the SIS junction capacitance needs to be resonated by a tuning element which is placed in rf-parallel with the mixer without shorting the dc bias current. In our design we use a series array of four tunnel junctions as the mixing element. The inductive tuning element consists of a separate series array of seven tunnel junctions which is referred to as the tuning array. This tuning array is independently dc-biased and is in rf-parallel with the mixing array. The junctions of the tuning array have the same current density and area as the junctions of the mixing array. In order to decrease the absorption of photons from the input signal by the tuning array, we have used about twice as many tuning junctions as mixing junctions. This ensures that the dominant mixing effect is from the mixing array. The use of a series array for tuning adds
less capacitance than a single junction.

Figure 2 shows the waveguide mixer mount and the microstrip circuitry [6]. The local oscillator (lo) is directed through a WR-10 cross-coupler into the input waveguide. A four-step Chebychev single ridge transformer at the end of this waveguide provides a broad-band waveguide-to-microstrip transition. The waveguide impedance is transformed to 50Ω at the last ridge section which makes rf and dc contact to the 50Ω input microstrip line on the chip (position A in Fig. 2). This input section is \( \lambda/4 \) long at 100 GHz. The rf impedance transformation by the ridge enables the use of low impedance SIS junctions. This eases matching on the intermediate frequency (IF) side since we do not employ an IF impedance transformer. The SIS mixing array is placed between the end of this microstrip-line (pos. B) and a 90° radial stub (pos. C). The 90° radial stub, like an unterminated quarter wave microstrip line, provides an rf ground at a well defined point with a broad bandwidth and transmits the dc-bias and IF-signal. The rf signal is thus shorted to the ground plane. A second 90° radial stub, next to the one terminating the mixing array, provides a separate rf termination for the tuning array in parallel with the SIS element. The radial stubs are in close proximity relative to the wavelength and are also capacitively coupled via a 2μm wide gap between them. Thus we have two dc- and IF-isolated broadband rf-ground terminations at nearly the same geometrical location, and the tuning array can be independently dc biased. The 1.4 GHz IF signal is then filtered with a bias-tee. A similar filter is used for the tuning array, except with a larger contact pad. The dc and IF grounds are provided by the waveguide walls.

The local oscillator signal is provided by a YIG tuned oscillator which is tripled, and fed into the mixer chip along with a test signal from a cryogenic hot/cold load [9]. By measuring the IF output for different hot/cold load temperatures, the receiver noise and gain are determined. The receiver includes an electromechanical switch and various ports to test the noise, match, and gain of the IF system. With these we can determine the mixer noise and mixer gain. The Nb/Al-O\textsubscript{x}/Nb trilayer tunnel junctions used in this work each have an area of 4(μm)\textsuperscript{2} and a resistance of 11.5Ω [10].

**Receiver Results and Discussion**

The performance of this mixer is evaluated using our test receiver. The results are compared quantitatively with the quantum theory of mixing [6]. Initially the success of the tuning approach is evaluated from the pumped I-V
curves. In the presence of rf radiation, a SIS junction I-V curve exhibits photon assisted tunneling steps. Usually a SIS mixer is biased in the middle of the first step below the gap voltage. The if output resistance of the mixer is the differential resistance of the dc I-V curve in the middle of the first photon step. The input impedance of the if amplifier should match this differential resistance for best if coupling efficiency. Thus the shape of the tunneling steps is important in determining receiver performance.

The shape of the tunneling steps depends on the amplitude of the rf signal and the admittance presented to the device by the embedding circuitry. In our design, for optimum receiver performance the rf conductance of the mixer needs to be close to 20 mmhos while the capacitance is resonated [5,11].

Fig. 3 shows the measured dc I-V curve, and two pumped I-V curves with photon assisted tunneling steps. The step-like proximity effect feature above the gap rise is caused by the slight non-uniformity of the junction areas. These curves are taken with a 90 GHz local oscillator signal. Tunneling steps are shown for two different biasing conditions for the tuning array. These bias voltages on the tuning array are chosen to separately analyze the effects of the susceptance and the conductance. There is a distinct difference between the shapes of the pumped I-V curves. The dynamic resistance of the mixing array in the middle of the first photon step is a factor of two larger for curve b than curve a. Thus, we have altered the embedding admittance seen by the mixing array by changing the bias voltage of the tuning array.

The embedding admittance seen by the mixing array can be calculated from the shape of the tunneling steps in Figure 3. This is done using the voltage-match method in the three frequency approximation [12]. This approximation introduces an error in mixer calculations done for devices with $\omega R_n C < 4$ [2]. The mixing array modeled here is estimated to have a $\omega R_n C$ product of about 1.5.

The embedding admittance for the mixer, $Y_{emb}$, is obtained by fitting the pumped I-V curves for various bias voltages on the tuning array. Values of $B_t$ and $G_t$ are inferred from $Y_{emb}$ by subtracting other components of the embedding admittance. These values are given in Table 1. Using this method the change in the embedding susceptance between curves a and b is inferred to be $\Delta B_t = 42$ mmhos. The theoretical value is $\Delta B_t = 49$ mmhos. This is in good agreement with the experimentally derived value.

The double-sideband receiver noise temperature, $T_R$ of this mixer at 90
GHz is measured as a function of the dc-bias voltage of the tuning array. The mixer gain $g_m$ is obtained by testing the if system independently. The receiver noise temperature depends on the mixer gain as

$$T_R = T_m + T_{if} / g_m,$$

where $T_m$ is the mixer noise temperature and $T_{if}$ is the if amplifier noise temperature which is around 8K.

The negative susceptance of the tuning array increases as the tuning voltage, $V_t$ approaches the gap voltage. At this voltage it resonates the array capacitance. Resonating this capacitance (at $V_t - V_g$) gives the largest mixer gain and the lowest receiver noise temperature. The experimental results for $g_m$ and $T_R$ for three specific values of $V_t$ are given in Table I.

The theoretical mixer gain is calculated using the embedding admittances inferred from the pumped I-V curves. The experimental value of $\Delta B_t$ agrees well with the theoretical value. This verifies the validity of the lumped element circuit model we have employed, in which the quasiparticle admittance is modeled as a voltage variable susceptance in parallel with a conductance. The tuning array acts as a separate passive tuning element, and the mixer gain is significantly improved by the tuning array.

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References


Table I. Values of $B_t$ and $G_t$ calculated by fitting the pumped I-V curves in Fig. 3, experimental and theoretical double-sideband (DSB) mixer gain and experimental receiver noise temperature for three different bias voltages of the tuning array. The bias voltages are shown in the inset of Fig. 3. The last column is for no tuning array. The conductance $G_t$ of the tuning array does not appear to play a major role in determining the mixer gain, as seen by comparing columns b and c.

<table>
<thead>
<tr>
<th>Bias Point</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_t$</td>
<td>$V_{gs}$</td>
<td>$(V_{gs} - (\hbar\omega)/e)$</td>
<td>$(V_{gs} - (\hbar\omega)/e)$</td>
<td>-</td>
</tr>
<tr>
<td>$B_t$ (mmho)</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>$G_t$ (mmho)</td>
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<td>35</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>$g_m$ (dB)-Calc.</td>
<td>-0.6</td>
<td>-5.3</td>
<td>-5</td>
<td>-3.3</td>
</tr>
<tr>
<td>$g_m$ (dB)-Expt.</td>
<td>-1.1</td>
<td>-5.1</td>
<td>-4.3</td>
<td>-</td>
</tr>
<tr>
<td>$T_R$ (K)-Expt.</td>
<td>47</td>
<td>76</td>
<td>65</td>
<td>-</td>
</tr>
</tbody>
</table>
Fig. 1: DC I-V curve of a single tunnel junction and its Kramers-Kronig transform (top). The calculated quasiparticle conductance and susceptance (bottom) for a small input power at 90 GHz.
Fig. 2: The waveguide mount and the mixer chip. a) Side views of the mixer mount showing the waveguide ridge. b) The mixer chip layout (top) and the rf equivalent circuit (bottom). The test signal along with the local oscillator signal is fed into the input microstrip (A). Both arrays are terminated in $90^\circ$ radial stubs providing two dc-isolated rf grounds near locations B and C, where the junctions connect. The if and dc signals are extracted at D for the mixer while the tuning array is biased through E.
Fig. 3: Measured pumped I-V curves of the mixing array for two different bias voltages on the tuning array; also shown is the unpumped dc I-V curve. The inset shows the conductance and the susceptance of the tuning array at bias voltages a, b, and c on the tuning array.