On using the shot noise in SIS tunnel junctions for characterizing IF amplifiers

David Woody, Steve Padin
California Institute of Technology
Owens Valley Radio Observatory, Big Pine, CA 93513 USA

Henry LeDuc and Jeff Stern
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109 USA.

Abstract—The IF amplifier in SIS heterodyne mixers can represent a major contribution to the receiver noise. Achieving quantum limited performance of SIS receivers will require accurate measurement and characterization of this noise contribution. A standard method for in situ characterization of the IF amplifier uses the shot noise from the SIS tunnel junction biased above the gap voltage as a calibrated noise source. This provides an accurate measurement of the IF amplifier noise and gain for source impedances equal to the normal state resistance of the junction, $R_n$. This paper describes an extension of this technique to determine the 2-port noise and gain parameters of the IF amplifier by using the junction as a noise source over its full dc bias range. Measuring the IF output power over the full bias range (with no LO applied) samples the IF amplifier response for source impedances ranging from zero to $>10R_n$. The IF amplifier characteristics are obtained by fitting the measurements to a six parameter model of the IF amplifier system. Although there is insufficient data to accurately determine the standard noise and gain parameters, the contribution from the IF system to the receiver noise is accurately determined.

This method of characterizing the IF system references the noise and gain parameters to the plane of the tunnel junction and includes the effect of the circuitry between the junction and the IF amplifier. This is particularly useful for receivers which don't use standard 50 ohm amplifiers with cooled isolators. This paper describes the IF noise and gain model together with the fitting procedure. Examples of this technique applied to the receivers used on the Owens Valley Radio Observatory millimeter array are presented. These receivers include mixers with integrated IF transformers and mixers with integrated HEMT amplifiers.

I. Introduction

An important step in improving the performance of low noise radio astronomy receivers is identifying and quantifying the various noise contributions. Although many of the contributions can be estimated from calculations or laboratory measurements of individual components it is important to be able to make in situ measurements of the noise sources. The IF amplifier is a major source of noise in SIS heterodyne receivers and its contribution must be determined before the mixer noise and other noise sources can be evaluated. Several techniques have been developed for characterizing the IF system using specialized laboratory instrumentation such as that employed by Kerr et al. [1]. But these laboratory instruments often are not appropriate for use on operating telescopes either because of inconvenience or some sacrifice in performance necessitated by installation of special test devices. Thus it is important to develop methods for accurately measuring the IF noise in receivers using a minimum of extra equipment.

*This work was supported by the NSF under Grant AST 91-00306 and by NASA Grant NAGW 2196.
The noise in a heterodyne receiver can be written as the sum of two basic components,

\[ T_{\text{rec}} = T_{\text{rf}} + T_{\text{rf}}^* \]  

(1)

\( T_{\text{rf}} \) includes the noise associated with the optical losses in front of the mixer and the mixer noise. \( T_{\text{rf}}^* \) is the noise present at the mixer’s IF-port even in the absence of any LO or mixing but referred to the receiver’s input under normal operating conditions. It is given by

\[ T_{\text{rf}}^* = L(T_{\text{if}} + T_{10}) \]  

(2)

where \( L \) is the mixer’s conversion loss, \( T_{\text{if}} \) is the noise temperature of the IF amplifier chain and \( T_{10} \) is the noise arising from the linear component of the leakage current in the SIS tunnel junction.

One method for estimating \( T_{\text{rf}}^* \) is to use lab measurements of the IF system. The conversion loss can be determined from the standard hot and cold load receiver measurements used to measure \( T_{\text{rec}} \) if the gain of the amplifier system is known and \( T_{\text{if}} \) can be measured separately using a noise test fixture. These characterizations usually apply to 50 ohm source impedances and corrections based upon a circuit model must be made for non-50 ohm impedances. These corrections can be significant and accurate values for the full set of 2-port noise and gain parameters are often not available. In addition the IF circuit inside the mixer block is often poorly characterized making it difficult to properly include in the circuit model.

Another method is to use the shot noise from the SIS tunnel junction biased above the gap voltage as a calibrated noise source [2]. This yields \( T_{\text{rf}} \) and IF gain for a source impedance equal to the normal state resistance of the junction, including the effects of any IF circuitry in the mixer block. But you still have to estimate the change in IF parameters in going from \( R_n \) to actual IF impedance.

Blundell et al. have developed a method for determining \( T_{\text{rf}} \) using hot and cold load measurements at low LO levels [3]. This method is based upon the graphical method of determining \( T_{\text{rec}} \) by plotting the IF output power, \( P_{\text{if}} \), as a function of load temperature, \( T_{\text{load}} \). The hot and cold load data provide two points through which a straight line is drawn. The line intersects the \( P_{\text{if}}=0 \) line at \(-T_{\text{rec}}\). They found that the set of lines for a series of low LO measurements crossed at a common point. The \( T_{\text{load}} \) coordinate of this point is \(-T_{\text{rf}}\). This is referred to as the "intersecting lines" technique. By including the LO off case (a horizontal line) it is seen that this technique is equivalent to subtracting the \( P_{\text{if}} \) value measured with the LO off from the raw hot and cold data. The mixer noise contribution to \( T_{\text{rf}} \) is a function of the LO power and thus \( T_{\text{rf}} \) under normal operating conditions will differ from the value derived from the intersecting lines technique derived using low LO levels.

Both the junction shot noise and intersecting lines technique are easy to apply and have been successfully used for characterizing SIS receivers. This paper extends the junction shot noise calibration technique to incorporate the IF response for the junction biased over the full range from zero to above the gap voltage. The method described in this paper was used as part of a technique for separating the optical loss and mixer noise contributions to \( T_{\text{rf}} \) [4]. The theory of 2-port noise and gain characterization is applied to the IF system and a generalized formulation of the IF output noise is developed in section II. Section III describes how to derive the IF parameters from the measured data and gives examples using the 1mm receivers used by the Owens Valley Millimeter Array (OVMA). Section IV discusses how accurately these parameters are determined and in particular it is shown why the intersecting lines technique works so well. It is also shown that the LO dependence of \( T_{\text{rf}} \) can be determined using this technique.
II. Generalized Formulation of the IF output Power

A complete characterization of an IF amplifier requires measuring the four real noise parameters and the four complex S-parameters or their equivalents [5]. The typical IF amplifier chain has several amplifiers in series ending in a power detector. For such a system the load termination is fixed and the reverse gain, S12, and the phase of the forward gain, S21, can be set to zero. Thus the IF system is characterized by seven real parameters which are functions of frequency; four noise parameters, input conductance and susceptance and gain for a matched load. We will use the following standard formulation for the IF system noise temperature [6,7],

\[ T_y = T_{\text{min}} + T_d \frac{(G_{\text{opt}} - G_s)^2 + (B_{\text{opt}} - B_s)^2}{G_s G_{\text{opt}}}, \]  

where \( Y_s = G_s + jB_s \) is the source admittance. The IF system output power is given by

\[ P_{\text{out}} = 2G_{11}g_0 \frac{\{\langle i_s^2 \rangle + 4kB_s \left[ T_{\text{min}} + T_d \left[((G_s - G_{\text{opt}})^2 + (B_s - B_{\text{opt}})^2) \right]\right] \}}{(G_s + G_{11})^2 + (B_s - B_{11})^2}. \]

where \( \langle i_s^2 \rangle \) is the mean square source noise current and B the predetection bandwidth. \( Y_{11} = G_{11} + jB_{11} \) is the input admittance for the IF chain and \( g_0 \) is the net IF gain for a matched load. The second term in the numerator is the equivalent noise current from the IF system noise.

We will define the mixer IF-port for the SIS mixer as being at the tunnel junction. The junction capacitance and other parasitics associated with the junction chip and mixer block will be included as part of the IF system. The IF-port admittance of an SIS junction will in general be real and equal to the dc differential conductance for low IF’s. This is the case in the absence of RF or LO power and for double sideband receivers. At higher frequencies, determined by the sharpness of the current vs. voltage characteristics, there is a quantum susceptance which peaks at the gap voltage [8,9]. The quantum susceptance is negligible at frequencies much below \( \delta \)V\( / \)h, where \( \delta \) is the width of the voltage transition from the "off" to the "on" state at the gap voltage. The devices used for millimeter and submillimeter receivers have transition widths of \( \sim 50\mu \)V, corresponding to \( \delta \)V\( / \)h \( \sim 10\)GHz. Thus the effect of the quantum susceptance at the IF-port can be neglected for SIS heterodyne receivers with IF’s below \( \sim 5\)GHz.

Ideally, in the absence of LO or RF power, the noise current associated with quasiparticles tunneling through an SIS junction is just the shot noise associated with the dc current [8]. This has been quantitatively verified by Dubash et al. for the types of devices used for low noise heterodyne receivers [10, 11]. Extra noise can arise from the ac Josephson effect in high current density (low capacitance) junctions at low bias voltages. Another possible deviation from shot noise can occur in devices which have pin-hole shorts across the tunnel barrier. The junctions used for this work show no indications of such micro-shorts or excess noise when biased above \( \sim 1.5\)mV. Thus the mean square current noise is given simply by \( \langle i_s^2 \rangle = 2eI_{\text{dc}}B \).

We are initially interested in the output power from the IF system in the absence of any LO or RF power. For this purpose we can set the source susceptance, \( B_s \), equal to zero and use the shot noise formula for source current noise, then equ. 4 can be written as
\[ P_{\text{out}}(G_s, I_{dc}) = g \frac{2eI_{dc} + aG_s^2 + bG_s + c}{(G_s + G_{11})^2 + B_{11}^2}. \] (5)

The bandwidth, B, has been incorporated into \( g \), e.g. \( g = 2G_{11}g_0B \). The coefficients \( a, b, \) and \( c \) have the following dependence on the noise parameters:

\[ a = \frac{4kT_d}{G_{\text{opt}}}, \] (6)

\[ b = 4k(T_{\text{min}} - 2T_d), \] and

\[ c = \frac{4kT_d(G_{\text{opt}}^2 + B_{\text{opt}}^2)}{G_{\text{opt}}}. \] (8)

We hope to determine the six parameters \( G_{11}, B_{11}, g, a, b, \) and \( c \) by using the SIS tunnel junction as a variable conductance and variable noise source. In high quality junctions, the differential conductance varies by more than a factor of a thousand and the dc current changes by more than a factor of fifty as the bias is increased from the "off" state below the gap voltage to the "on" state above the gap voltage. The junction also serves as a perfect short when biased at zero voltage and zero current. The large range of conductance and noise current should make the SIS tunnel junction an excellent tool for probing the IF system gain and noise parameters. Measuring the IF output power as the dc bias voltage is varied from zero to well above the gap voltage should allow us to determine the six parameters by using equ. 5 to fit the data. Once the parameters are determined, then the IF noise contribution and gain can be determined for any real IF-port admittance for the SIS heterodyne receiver. The case of negative admittance can be handled with proper definition of the noise parameters, but will not be dealt with here since such mixers are notoriously noisy [12].

The three equations 6, 7 and 8 are insufficient to uniquely determine the four IF system noise parameters from the values for the coefficients \( a, b, \) and \( c \). But physical constraints can be used to limit the range of possible values. It can be shown that \( T_{\text{min}} < \approx 4T_d \) [13] and if we force \( G_{\text{opt}} > 0 \) then the range of solutions to this set of equations are

\[ 0 \leq B_{\text{opt}} \leq \sqrt{\frac{c}{a}}. \] (9)

\[ \frac{b}{2a} \leq G_{\text{opt}} \leq \sqrt{\frac{c}{a}}. \] (10)

\[ \frac{b}{8k} \leq T_d \leq \frac{\sqrt{ac}}{4k}, \] and

\[ \frac{(b + |b|)}{4k} \leq T_{\text{min}} \leq \frac{b + 2\sqrt{ac}}{4k}. \] (12)

It is also required that \( a > 0, c > 0, \) and \( b^2 < 4ac \) for a physically a realizable IF system. Using a priori information to limit any one of the noise parameters will also limit the range of the others. You can treat \( B_{\text{opt}} \) as a free parameter and write \( G_{\text{opt}}, T_d, \) and \( T_{\text{min}} \) as functions of \( a, b, c \) and \( B_{\text{opt}} \). Then the allowed
solutions for a given set of a, b, and c is a parametric line in the 3-dimensional space of \((G_{opt}, T_d, T_{min})\).

A particularly powerful constraint is to require \(Y_{opt} = Y_{11}\) as is the case for systems employing cooled isolators with 50ohm transmission lines ending at the junction. The noise from an isolator cooled to 4K can be significant compared to the extremely low noise IF amplifiers currently available [14] and the IF circuitry in the mixer block can result in the power match no longer being equal to the noise match, e.g. \(Y_{opt} \neq Y_{11}\).

The usefulness of this method for determining the standard noise parameters, as opposed to just the noise contribution as a function of source admittance, will depend upon the actual values of the three coefficients and what additional IF system data is available. At this point we have not dealt with how accurately the six parameters in equ. 5 can be determined from real data. These issues will be discussed in the following section.

III. Measurements on Radio Astronomy Receivers

Three examples of the measured and calculated output power vs. bias voltage are shown in fig. 1. The first example, mixer block R4/2A, is one of the standard 1mm band receivers in use on the OVMA [15]. It utilizes a backshort tuner and incorporates a 160ohm to 50ohm IF transformer [16] before feeding a 1-2GHz balanced amplifier [17]. The other two examples are from mixer block M3 which is a tunerless mixer based upon the design of Blundell et al. [18] with a 0.5-4.5GHz HEMT amplifier integrated into the block [19]. The middle data set is for the 1.0-1.2GHz section of the IF band while the last data set is for the 1.84-1.96GHz section of the IF band. These mixers operate in the 200-270GHz band and utilize scalar feed horns with transitions to .98x.49mm waveguide. The SIS devices are single – ultum Al-AlOx-Al tunnel junctions [20]. The device in mixer R4/2A is a simple junction without any on chip RF or IF circuitry while the junction in M3 incorporates an RF matching circuit and an RF blocking filter for the IF-port on the chip [18].

The data in fig. 1 were obtained in an automated receiver test fixture which measures the output power for ambient and liquid nitrogen temperature loads as a function of the bias voltage. Although no LO was applied there was a measurable direct detection dc current and IF response below the voltage gap at 2.9mV. The IF response is most likely a result of "self-mixing" of the thermal noise power in these broad band mixers. The absorbed power from the thermal loads can approach 1nW for mixers with large instantaneous RF bandwidths which is sufficient to produce measurable mixing of the noise with itself. This mechanism would produce a response proportional the square of the noise power. Although the effect is small, it can influence the deduced IF system parameters and the data shown in fig. 1 has been extrapolated to an effective load temperature of 0K.

A hybrid procedure is employed to fit equ. 5 to the measured IF power. The input admittance \((G_{11}, B_{11})\) is determined from a raster search over the Smith Chart with a linear least squares fit for g, a, b, and c at each raster point. At each raster point the data yield unique well defined values for g, a, b, and c but these values and the RMS residual vary as you move across the Smith Chart. Fig. 2 shows the contour plots of the RMS residuals for the same three data sets displayed in fig. 1. The minimum RMS residuals are 2-3\% of the average IF power. The values of \(G_{11}, B_{11}\), g, a, b, and c at the minimums are used in equ. 4 to calculate the best fit IF power plotted in fig. 1.

The fit faithfully reproduces the details in the \(P_{IF}\) vs. \(V_{dc}\) data over the full bias range, including the superconducting short at \(V_{dc} = 0\) and the structure at \(V_{pop}\). The measurements cover more than three decades of conductance and more than a factor of fifty in source noise current. The largest errors in the technique are a result of inaccuracies in determining the differential conductance, \(dI_{dc}/dV_{dc}\), from the discretely sampled current and voltage data. The \(I_{dc}\) data have been smoothed to reduce the errors in calculating \(dI_{dc}/dV_{dc}\).
Table 1
Valid ranges for IF system parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>R4/2A</th>
<th>M3 1.84-1.96GHz IF</th>
<th>M3 1.0-1.2GHz IF</th>
</tr>
</thead>
<tbody>
<tr>
<td>G11 [1/ohm]</td>
<td>.0006-.008</td>
<td>.01-.08</td>
<td>.008-.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B111 [1/ohm]</td>
<td>0-.008</td>
<td>0-.06</td>
<td>0-.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g$ [Hz/(Aohm)$^2$]</td>
<td>.0002-.0004</td>
<td>.003-.007</td>
<td>.0003-.0006</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Goptim [1/ohm]</td>
<td>.001-.003</td>
<td>.006-.06</td>
<td>.007-.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tn [K]</td>
<td>2.4-25</td>
<td>1.1-13</td>
<td>7-32</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tm [K]</td>
<td>0-9.1</td>
<td>0-4.7</td>
<td>7-23</td>
</tr>
</tbody>
</table>

The fits are equally good over vertical strips in the Smith Chart which extend almost to the edges of the chart. Thus the parameter values are not accurately determined by the data. Table 1 lists the ranges of gain and noise parameter values consistent with the data without using any a priori information. The ranges given in Table 1 incorporate equs. 9-12 along with the range of parameter values within the lowest contours in fig. 2. The parameters in table 1 are consistent with the expected values derived from the IF circuit designs, in particular, the IF transformer in mixer R4/2A was designed to give a $G_{opt} = G_{11} = 0.006$ for $G_1$ and 1.9GHz is predicted by the MMICAD circuit model of the GaAs HEMT integrated into the mixer block. The parameters cannot be separately varied within these ranges since they are interdependent, but it is clear that the standard IF gain and noise parameters are poorly determined using this method without another method for constraining some of the parameters.

IV. Discussion

The poor determination of the standard noise parameters limits the usefulness of this technique for locating minor problems in the IF circuitry but it can point to gross errors in design or fabrication. The situation is considerably improved for determining the IF system’s contribution to the receiver noise. This just requires determining what fraction of the IF power originates from the IF chain. Fig. 3 plots the calculated output power as a function of $G_s$ for several different parameter sets spanning the allowed range of parameter values. The gain and IF noise temperature have the expected dependence on $G_s$. But the net output power which is the product of the gain and noise temperature is relatively flat. This is a consequence of $G_{opt} G_1$ as is the case for many IF amplifiers. Also notice that the fractional spread in net power is less than the spread in gain or noise temperature. Thus the output power from the IF system as a function of $G_s$ is well determined despite the poor determination of the standard amplifier noise and gain parameters. The errors in determining the conversion loss and IF amplifier noise separately can be larger than a factor of two for low IF-port conductances. But their net contribution to the receiver noise can be determined to better than 20%.

The itemized contributions to the receiver noise as a function of the LO induced dc current for mixer M3 operating at 230GHz is shown in fig. 4. At each LO level $G_s$ was obtained from the differential dc conductance at the bias voltage used for measuring $T_{rec}$. The IF output power from the leakage noise, $P_{10}$, was calculated from equ. 5 using the best fit parameter values for $G_{11}$, $B_{11}$ and $g$, $a=b=c=0$, and the value for $I_{dc}$ at the same bias voltage with the LO off. This was normalized by the measured hot and cold load response to convert $P_{10}$ to an equivalent temperature at the receiver input, e.g. $T_{10}$ in equ. 2. The IF amplifier contribution, $LT_{IF}$ in equ. 2, was similarly obtained using the same $G_{11}$, $B_{11}$ and $g$ plus the best fit parameter values for $a$, $b$, and $c$ and $I_{dc}=0$. $T_{IF}$ was obtained by subtracting the sum of these contributions from $T_{rec}$. 
The receiver noise temperature initially decreases with increasing LO level as the conversion efficiency improves, as expected for a receiver dominated by IF noise. $T_{\text{rf}}$ approaches a constant value at low LO levels as expected. The optical losses are independent of LO level and the mixer noise which is inversely proportional to the quantum efficiency reaches a maximum limiting value as the LO approaches zero [21]. $T_{\text{rf}}$ degrades as the LO level increases and the quantum efficiency (the efficiency with which incident photons are converted to quasiparticle carriers) decreases. Note that the effect of increasing mixer noise shows up before the conversion loss, $L$, begins to degrade and increase the contributions from the leakage current and IF amplifier noise. As seen from the flatness of the leakage and IF amplifier contributions, the conversion loss is nearly constant over a large LO range. Thus it is the mixer noise and hence the quantum efficiency which sets the upper limit on the LO for low noise performance.

The limiting value for $T_{\text{rf}}$ is the same value obtained using the intersecting lines technique [3]. The flatness of the net IF power curves in fig. 3 explains why this technique works so well. To first order, the IF amplifier's noise contribution to the IF power is independent of the IF-port conductance. Also the IF power contributed by the shot noise in the leakage current will scale as the gain divided by $G_s$ and thus will be relatively flat for the low IF-port conductances seen at low LO levels or with the LO off. The contribution of the IF system noise plus the leakage shot noise can be obtained by simply measuring the IF output power with the LO off. Calculating the receiver noise after subtracting this LO off power from the raw hot and cold load measurements for low LO levels then yields $T_{\text{rf}}$ directly.

V. Summary

The gain and noise contributions of the IF system have been formulated as a function of the IF-port conductance with six free parameters. The relationship between these parameters and the standard noise and gain parameters is also derived. A procedure is described for determining the values for the six parameters from measurements of the IF output power vs. bias voltage with the LO off. The technique does not use any special equipment or setup and can be applied to any SIS heterodyne receiver. It also does not rely on the often incorrect assumption that the IF-port conductance is equal to the normal state conductance nor does it assume that the IF system contribution to the noise is the same as the noise with the LO off at the same $V_{\text{dc}}$.

Several examples of the procedure are given for the millimeter band receivers used on the OVMA. It is shown that although the standard noise and gain parameters are poorly determined, the net contribution of the IF amplifier noise and leakage current, $T_{\text{if}}$, to the receiver noise is accurately determined. This allows the sum of the optical loss and mixer noise, $T_{\text{rf}}=T_{\text{rec}}+T_{\text{if}}$, to be determined even for the case of large LO power.

References


Fig. 1. $I_{dc}$ (dashed line), measured $P_{if}$ (solid line), and calculated $P_{if}$ (dotted line) as a function of $V_{dc}$. (a) is mixer R4/2A with best fit parameter values of $G_{11} = .0035$, $|B_{11}| = .0063$, $g = .00029$, $a = 70.$, $b = .19$ and $c = .0034$. (b) is mixer M3 1.84-1.96GHz IF, with best fit parameter values of $G_{11} = .044$, $|B_{11}| = .036$, $a = 65.$, $b = .72$ and $c = .11$. (c) is mixer M3 1.0-1.2GHz IF, with best fit parameter values of $G_{11} = .013$, $|B_{11}| = .0013$, $a = 65.$, $b = 1.2$ and $c = .016$. The circles are the measured and '+'s are the calculated $P_{if}$ with the dc bias off, e.g. $V_{dc} = 0$ and $I_{dc} = 0$.

Fig. 2. Contour plots over the Smith Chart for IF system input impedance, $Z_{11}/50$ohm, of the RMS residuals for fitting equ. 5 to the measured $P_{if}$ data in fig. 1. The contour intervals are .5% of the average IF power and the conjugate pair of points with the smallest residuals are indicated by '+'s.
Fig. 3. Plot of the calculated gain, $T_{if}$, and $P_{if}$ for the same mixers displayed in fig. 1. In each plot the lower set of the lines are the gain, middle set $T_{if}$ and top set $P_{if}$. The parameter sets used for calculating the lines span the range of values consistent with the measured data in fig. 1 and listed in table 1.

Fig. 4. Contributions to $T_{rec}$ for mixer M3 in the 1.84-1.96GHz IF band at 230 GHz as a function of LO induced dc current. Filled squares: $T_{rec}$ measured using hot and cold loads. Open squares: leakage current noise contribution, $LT_{p}$, and triangles: IF amplifier noise contribution, $LT_{if}$. These values were obtained using eqn. 5 with the best fit parameter values and the measured $G_{s}$. Diamonds: $T_{if}$ calculated by subtracting $T_{if}^{\star}=(LT_{p}+LT_{if})$ from $T_{rec}$.