A DISCUSSION OF POWER COUPLING BANDWIDTH LIMITATIONS OF PLANAR SCHOTTKY DIODES AT SUBMILLIMETER WAVELENGTHS

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Abstract

Planar Schottky diodes can be used to build sensitive, rugged and reproducible mixers that operate at room temperature. Discrete-chip surface-channel planar Schottky barrier diodes have been used in the development of receivers at frequencies up to 700 GHz with performance comparable with that of the best whisker contacted diodes [1]. However, the packaging parasitics of the planar diode geometry have limited the development of such mixers at higher frequencies. The planar diode chip can be modeled as a set of lumped capacitors (representing fringing fields around the diode) and inductors (representing the metal connection lines) that affect the coupling of power to the diode junction. Bode [2] and Fano [3] showed that there is an inherent limit to the bandwidth over which power can be coupled to a complex load of this sort using a lossless passive coupling circuit. We have extended the Bode-Fano theory to circuits with three or more elements, and have used this theory to examine limitations to mixer design caused by planar diode packaging at submillimeter wavelengths. In particular, we have examined the coupling bandwidth limitation for a discrete-chip surface-channel planar Schottky barrier diode mounted in a microstrip channel [4]. The simulations indicate that for diodes chips with semi-insulating GaAs support substrates at frequencies around 500 GHz the coupling of power to the anode is not significantly limited by the chip parasitics. However, at frequencies above approximately 1000 GHz, the packaging parasitics impose tight limits on the power coupling bandwidth, thus complicating mixer design. The bandwidth limitation condition derived here is general and can be used to explore power coupling limitations for a variety of devices and structures.

Introduction

Fig. 1(a) shows a simple equivalent circuit of the whisker-contacted Schottky barrier diode. The capacitance $C_{j, avg}$ is an average value for the junction capacitance over the local oscillator (LO) pump cycle. $R_{d, opt}$ is the RF and LO source resistance that provides optimum mixer performance. Bode showed that there is a fundamental limitation to the bandwidth over which a reasonable transfer of power can be achieved for a parallel $RC$ circuit, given by [2]

$$\int_0^\infty \ln \left( \frac{1}{|\rho|} \right) d\omega \leq \frac{\pi}{RC}$$

(1)

where $\rho$ is the reflection coefficient looking toward the diode through a lossless matching network. Fig. 2 shows the frequency profile of the reflection coefficient that yields maximum bandwidth given some maximum reflection $\rho_a$ within the band. Using this profile allows the
Fig. 1. Simple equivalent circuits of the (a) whisker-contacted and (b) planar Schottky barrier diode.

evaluation of the integral in (1),

$$\omega_2 - \omega_1 \ln \left( \frac{1}{\rho_s} \right) \leq \frac{\pi}{RC},$$

that illustrates the tradeoff between power-coupling bandwidth and \( \rho_s \) for a parallel \( RC \) circuit. As an example, for the UVa-1T15 whisker-contacted diode (epitaxial layer doping \( 1 \times 10^{18} \text{ cm}^{-3} \) and anode diameter \( 0.25 \mu \text{m} \)), some reasonable estimates for \( R_{dop} \) and \( C_{j,avg} \) are \( R_{dop}=150 \Omega \) and \( C_{j,avg}=2-C_{p}=0.5 \text{ fF} \). For a center frequency of 3 THz and an in-band reflection coefficient \( \rho_s \) of 0.2, the largest obtainable fractional bandwidth for the 1T15 is 140%. Thus, for the whisker-contacted diode, the junction capacitance does not inherently limit the power coupling bandwidth, and the design is instead limited by other issues.

Fig. 3 shows the planar diode chip geometry and the location of the significant parasitic elements near the diode’s anode region, which we have reduced to the simple equivalent circuit of Fig. 1(b) for this analysis. \( C_f \) represents the parallel combination of \( C_{j,avg} \) and the parasitic capacitance from the finger to the ohmic-contact pad, \( C_{fp} \). The other parasitics are the finger inductance \( L_f \) and the pad-to-pad capacitance \( C_p \) that represents the fringing capacitance between the pads. These parasitics can potentially reduce the achievable bandwidth from that of the whisker-contacted geometry. In order to determine the significance of the planar diode parasitics on the high frequency performance, this article derives an equation similar to (2) for the planar diode equivalent circuit.

**Statement of the Problem**

The basic setup of the problem is shown in Fig. 4. \( Z_L \) represents the load whose power-coupling bandwidth limit is to be determined. Throughout this analysis, the impedances are assumed to be normalized to the source impedance. The goal of the analysis is to determine the fundamental limits to the performance of the matching network that are imposed by the load impedance \( Z_L \). The reflection coefficient looking toward the matching network is taken as the figure of merit by which the bandwidth is judged.

Darlington showed that any physically realizable load can be represented by a purely reactive two-port network with a resistance terminating one of the ports [5]. This
The terminating resistance can be made arbitrary by the inclusion of an ideal transformer within the reactive two-port network. Using a Darlington representation of the load impedance allows the problem to be redrawn as shown in Fig. 5.

The goal of this analysis, then, is to determine the limitations to the performance of the entire circuit N caused by the load circuit N' independent of the matching circuit N". The relationship between the overall circuit N and the load circuit N' was explored by Fano [3]. Fano showed that at an n\textsuperscript{th} order zero of transmission of N' in the right half of the complex frequency plane (RHP), \( \rho_i \) and its first 2n-1 derivatives are equal to \( \rho_i \)' and its first 2n-1 derivatives and are thus independent of the circuit N". Using this basic insight, Fano showed that by performing a contour integration around the RHP involving the function \( F=\ln(1/\rho_i) \), an integral relation can be determined between the frequency response of N and the physical circuit parameters of N'.

One caveat to this analysis is that under certain circumstances the adjacent elements of N' and N" are of the same type and orientation. The network N is then called degenerate because both N' and N" have zeros of transmission at the same location. For this degenerate case, only the first 2n-2 derivatives of \( \rho_i \)' are independent of the circuit N", and the bandwidth can be improved by increasing the value of the final element of N'. The bandwidth limitation determined by the circuit with n-1 elements then determines the behavior, as will be discussed later.

The next section uses the integral relations developed by Fano to derive the relations for the planar diode equivalent circuit. However, Fano's theory was derived for the case of low-pass matching. For many circuits, the low-pass theory can be simply extended to the band-pass case using standard transformation techniques. For the circuit under consideration, this transformation is not applicable because the circuit elements are not accessible for the connection of the requisite parallel elements. Kerr extended Fano's theory to the case of a bandpass matching without the use of this lowpass to bandpass transformation [6]. In addition, Fano's analysis was only valid for circuits with two elements. In [7], we extended the analysis of Fano to the three-element circuit of Fig. 1(b), and we will now use the results of this analysis in deriving the bandwidth limitation equations for this circuit.

**Derivation of Equations**

The circuit N' in Fig. 6 shows the key parasitics for a typical planar diode chip. The resistor r is an arbitrary resistance that eventually drops out of the calculations. The first step of
the analysis is to determine the coefficients of the Taylor series for the function \( F(s) = \ln(1/\rho_1(s)) \). The circuit \( N' \) has three zeros of transmission at infinity, and it is thus convenient to define the variable \( \xi = 1/s \). Because of the three zeros of transmission, the first 5 coefficients of the Taylor series of \( \rho_1 \), and therefore \( F \) as well, are determined entirely by the circuit \( N' \) for the non-degenerate case. The Taylor series for \( F \) is given by

\[
F(s) = \ln(1/\rho_1) = jA_0 + A_1\xi + A_2\xi^2 + A_3\xi^3 + \cdots
\]

where \( A_0 \) is either 0 or \( \pi \) depending upon the sign of \( \rho_1 \), and \( A_n \) are

\[
A_n = \left. \frac{1}{n!} \frac{d^n F(\xi)}{d\xi^n} \right|_{\xi=0}
\]

The impedance looking into the circuit \( N' \) is given by

\[
Z' = \frac{s^2rL_fC_f + sL_f + r}{s^3rL_fC_fC_p + s^2L_fC_f + sL_f + r(C_f + C_p) + 1}
\]

Using (5) to calculate the inverse of the reflection coefficient yields

\[
\frac{1}{\rho_1} = \frac{Z'_0 + 1}{Z'_0 - 1} = \frac{s^3rL_fC_fC_p + s^2(L_fC_f + rL_fC_p) + s(L_f + r(C_f + C_p)) + (1 + r)}{s^3rL_fC_fC_p + s^2(L_fC_f - rL_fC_p) - s(L_f - r(C_f + C_p)) + (1 - r)}
\]

Finally, the function \( F(s) \) in terms of \( \xi \) is

\[
F(s) = \ln\left[ \frac{1}{|\rho_1|} \right] = \ln\left[ -\frac{\xi(1 + r) + \xi^2(L_f + r(C_f + C_p)) + \xi(L_fC_f + rL_fC_p) + rL_fC_fC_p}{\xi(1 - r) - \xi^2(L_f - r(C_f + C_p)) + \xi(L_fC_f - rL_fC_p) + rL_fC_fC_p} \right]
\]

Taking the appropriate derivatives of \( F \) and evaluating them at \( \xi = 0 \) yields the coefficients of the Taylor series:

\[
A_0 = \pi, \quad A_1 = \frac{2}{C_f}, \quad A_2 = 0, \quad A_3 = \frac{2}{3} \frac{L_f - 3C_f}{L_fC_f^2}, \quad A_4 = 0, \quad A_5 = \frac{2}{5} \frac{5C_f^3 + 5C_f^2C_p - 5(C_fC_pL_f + C_pL_f^2)}{C_f^2C_pL_f^2}
\]

In Table 1 of Fano's paper, he gives the integral conditions for physical realizibility for a circuit with \( n \) zeros of transmission at infinity as [3, p. 68]

\[
\int_0^\infty \omega^{2k} \ln\left( \frac{1}{|\rho_1|} \right) d\omega = (-1)^k \frac{\pi}{2} F_{2k+1}
\]

where \( k \) runs from 0 to 2 and \( F_{2k+1} \) is defined as

\[
F_{2k+1} = A_{2k+1} - \frac{2}{2k+1} \sum_{i} J_{2k+1}^{2k+1}
\]

and where the \( A_{2k+1} \) are the coefficients of the Taylor series for \( F(\xi) \) and \( J_{2k+1}^{2k+1} \) are the zeros of \( \rho_1 \) in the right hand plane, which are determined by the matching network. Using the reflection coefficient profile shown in Fig. 2, (9) can be simplified to

\[
KQ_{2k+1} = (-1)^k (2k+1) F_{2k+1}
\]
where $\Omega_{2k+1}$ is defined as
\[
\Omega_{2k+1} = \omega_2^{2k+1} - \omega_1^{2k+1}
\]  
and $K$ is given by
\[
K = \frac{2}{\pi} \ln \left( \frac{1}{|\rho_s|} \right)
\]  
Substituting the values of $k$ and expanding $F_{2k-1}$ we arrive at a series of three equations
\[
\begin{align*}
A_1 - K\Omega_1 &= 2 \sum \lambda_{ri} \\
3A_3 + K\Omega_3 &= 2 \sum \lambda_{3ri} \\
5A_5 - K\Omega_5 &= 2 \sum \lambda_{5ri}
\end{align*}
\]  
All that remains is to determine the right-half-plane (RHP) zeros $\lambda_{ri}$ that maximize the coefficient $K$ within the matching band, thus minimizing $\rho_s$. The zeros $\lambda_{ri}$ must be real or appear in complex conjugate pairs and must have a real part greater than zero. Fano showed that for a CL circuit in the non-degenerate case, the bandwidth can be maximized by choosing a single real root [3, p. 72]. However, as Fano mentions, his proof for the two element CL circuit could not be extended to circuits with larger numbers of elements [3, p. 73]. In [7], we give a proof for the three element CLC circuit and show that, except for the degenerate case, $K$ is maximized by a single pair of either real or complex conjugate roots. Thus, choosing a pair of roots $\lambda_{r1} = x + \sigma$ and $\lambda_{r2} = x - \sigma$ where $\sigma = y$ for two real roots and $\sigma = iy$ for a pair of complex-conjugate, then the summation of the zeros in (14) becomes
\[
\begin{align*}
\sum \lambda_{ri} &= 2x \\
\sum \lambda_{3ri}^3 &= 2(x^3 + 3x \sigma^2) \\
\sum \lambda_{5ri}^5 &= 2(x^5 + 10x^3 \sigma^2 + 5x \sigma^4)
\end{align*}
\]  
Substituting (15) into (14) yields
\[
\begin{align*}
A_1 - K\Omega_1 &= 4x \\
3A_3 + K\Omega_3 &= 4(x^3 + 3x \sigma^2) \\
5A_5 - K\Omega_5 &= 4(x^5 + 10x^3 \sigma^2 + 5x \sigma^4)
\end{align*}
\]  
Eliminating the variables $x$ and $\sigma$ yields the equation
\[
144(A_1 - K\Omega_1)(5A_5 - K\Omega_5) + (A_1 - K\Omega_1)^6
- 20(A_1 - K\Omega_1)^3(3A_3 + K\Omega_3) - 80(3A_3 + K\Omega_3)^2 = 0
\]  
Note that this equation is valid for both the real root and complex-conjugate root solutions. If we introduce the fractional bandwidth $b$ and center frequency $\omega_0$, related to $\omega_1$ and $\omega_2$ by the equations $\omega_1 = \omega_0(1-b/2)$ and $\omega_2 = \omega_0(1+b/2)$, then the $\Omega_{2k+1}$ can be rewritten as
\[
\begin{align*}
\Omega_1 &= b\omega_0 \\
\Omega_3 &= \frac{b(12 + b^2)}{4} \omega_0^3 \\
\Omega_5 &= \frac{b(80 + 40b^2 + b^4)}{16} \omega_0^5
\end{align*}
\]  
Finally, defining the variables $B_{Cf} = \omega_0 C_f$, $B_{Cs} = \omega_0 C_s$, and $X_{Lj} = \omega_0 L_j$ and substituting (8) and (18)
into (17) yields a polynomial in $K$ and $b$.

$$0 = 2880 + K^6 b^6 C_j^3 C_s X_L^2$$

$$- K^3 12 b^5 C_j^2 C_s X_L^2$$

$$+ K^4 (60 b^4 C_j C_s X_L^2 + 60 b^4 C_j^3 C_s X_L^2 + 5 b^6 C_j^3 C_s X_L^2)$$

$$+ K^3 (-120 b^3 C_j C_s X_L^2 - 120 b^3 C_j^3 C_s X_L^2 - 360 b^3 C_j^2 C_s X_L^2)$$

$$- 30 b^5 C_j^2 C_s X_L^2$$

$$+ K^2 (720 b^2 C_j X_L^2 + 720 b^2 C_j C_s X_L^2 + 60 b^4 C_j^3 C_s X_L^2$$

$$+ 240 b^4 C_j^3 C_s X_L^2 - 4 b^6 C_j^3 C_s X_L^2)$$

$$+ K (-1440 b C_j^3 C_s X_L^2 + 1440 b C_j C_s X_L^2 + 2880 b C_j C_s X_L^2 + 240 b^3 C_j C_s X_L^2$$

$$- 440 b C_j C_s X_L^2 - 120 b^3 C_j C_s X_L^2 - 1440 b C_j^3 C_s X_L^2$$

$$- 720 b^3 C_j^2 C_s X_L^2 - 18 b^5 C_j^2 C_s X_L^2)$$

Equation (19) can then be used to generate curves showing the relationship between $\rho_a$ and the fractional bandwidth $b$ for different system parameters. Once we have solved (19) for a given set of values, we can then use (16) to determine whether we have two real roots or a pair of complex-conjugate roots.

**Application of the Theory to a Discrete Planar Schottky Diode**

Table 1 gives the equivalent circuit values for the SC1T5 type planar diode for 5 $\mu$m and 20 $\mu$m finger lengths (indicated by -S5 and -S20 respectively). These parameters were generated by matching the equivalent circuit of Fig. 1(b) to the results of finite-element modeling of the mounted diode performed using Hewlett Packard’s High Frequency Structure Simulator over frequencies from 450–700 GHz [8]. The capacitance $C_j$ includes both the finger-to-pad capacitance $C_{fp}$ and the time averaged junction capacitance $C_j^{avg}$. The capacitance $C_j^{avg}$ was found by averaging the capacitance waveform (determined using harmonic balance simulations) over an LO cycle. The optimum RF and LO diode resistance, $R_{dop}$, was estimated to be 100 $\Omega$ based on the results of harmonic balance simulations. These circuit values were then used with (19) to calculate curves of $\rho_a$ versus fractional bandwidth for various planar diode geometries at different frequencies.

Fig. 7 shows the relation between $\rho_a$ and the fractional bandwidth $b$ for an SC1T5 type planar diode [4] with a 5 $\mu$m finger length plotted at frequencies from 500 GHz to 2000 GHz. The bandwidth limitation is not a practical limitation at the lowest frequency, and it is only above about 1000 GHz that the bandwidth limitation significantly effects the coupling of power to the

<table>
<thead>
<tr>
<th>Diode Type</th>
<th>$C_{pp}$</th>
<th>$C_{fp}$</th>
<th>$C_{j,avg}+C_{fp}$</th>
<th>$L_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC1T5-S5</td>
<td>2.8</td>
<td>0.4</td>
<td>4.4</td>
<td>20</td>
</tr>
<tr>
<td>SC1T5-S20</td>
<td>2.0</td>
<td>0.8</td>
<td>4.8</td>
<td>29</td>
</tr>
</tbody>
</table>

Table 1. Equivalent circuit values used during bandwidth modeling for the SC1T5 diode for various finger lengths.
diode. Note that this is a power coupling bandwidth, not a predicted mixer bandwidth. In order to illustrate the relative importance of each circuit element, Fig. 8 compares the bandwidth limitation for an SC1T5-S5 diode at 1500 GHz considering first the effect of $C_f$ only, then both $C_f$ and $L_p$, and finally for the full three-element circuit. Fig. 9 shows a set of bandwidth limitation curves comparing different finger lengths and different junction capacitances. As the finger length is reduced, $L_f$ decreases while $C_p$ increases; also, the bandwidth increases, thus indicating the importance of $L_f$ in relation to $C_p$. Also shown is the improvement in bandwidth as the junction capacitance is reduced from 2 fF to 1 fF.

Conclusions

In this paper we have derived the bandwidth limitation equation for the coupling of power to a resistor through a three-element CLC circuit. This derivation was made possible by our extension of the Bode-Fano theory to circuits with three or more zeros of transmission at infinity. The results of this analysis were then used to examine the performance of a fundamental mixer built using discrete planar diode chips with semi-insulating GaAs support substrates. The simulations indicate that at frequencies around 500 GHz the coupling of power to the diode is not significantly limited by the chip parasitics. However, at frequencies above approximately 1000 GHz, the packaging parasitics impose tight limits on the power coupling bandwidth for this diode and mixer geometry, thus complicating mixer design. The bandwidth limitation condition derived here is general, and can be used to explore power coupling limitations for a variety of devices and structures.
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References


