A DISCUSSION OF POWER COUPLING BANDWIDTH LIMITATIONS OF PLANAR SCHOTTKY DIODES AT SUBMILLIMETER WAVELENGTHS

J.L. Hesler and B. Gelmont

Department of Electrical Engineering
University of Virginia
Charlottesville, VA 22903
e-mail: hesler@virginia.edu

Abstract

Planar Schottky diodes can be used to build sensitive, rugged and reproducible mixers that operate at room temperature. Discrete-chip surface-channel planar Schottky barrier diodes have been used in the development of receivers at frequencies up to 700 GHz with performance comparable with that of the best whisker contacted diodes [1]. However, the packaging parasitics of the planar diode geometry have limited the development of such mixers at higher frequencies. The planar diode chip can be modeled as a set of lumped capacitors (representing fringing fields around the diode) and inductors (representing the metal connection lines) that affect the coupling of power to the diode junction. Bode [2] and Fano [3] showed that there is an inherent limit to the bandwidth over which power can be coupled to a complex load of this sort using a lossless passive coupling circuit. We have extended the Bode-Fano theory to circuits with three or more elements, and have used this theory to examine limitations to mixer design caused by planar diode packaging at submillimeter wavelengths. In particular, we have examined the coupling bandwidth limitation for a discrete-chip surface-channel planar Schottky barrier diode mounted in a microstrip channel [4]. The simulations indicate that for diodes chips with semi-insulating GaAs support substrates at frequencies around 500 GHz the coupling of power to the anode is not significantly limited by the chip parasitics. However, at frequencies above approximately 1000 GHz, the packaging parasitics impose tight limits on the power coupling bandwidth, thus complicating mixer design. The bandwidth limitation condition derived here is general and can be used to explore power coupling limitations for a variety of devices and structures.

Introduction

Fig. 1(a) shows a simple equivalent circuit of the whisker-contacted Schottky barrier diode. The capacitance $C_{j,avg}$ is an average value for the junction capacitance over the local oscillator (LO) pump cycle. $R_{d,opt}$ is the RF and LO source resistance that provides optimum mixer performance. Bode showed that there is a fundamental limitation to the bandwidth over which a reasonable transfer of power can be achieved for a parallel RC circuit, given by [2]

$$\int_0^\infty \ln\frac{1}{|\rho|}\,d\omega \leq \frac{\pi}{RC} \tag{1}$$

where ρ is the reflection coefficient looking toward the diode through a lossless matching network. Fig. 2 shows the frequency profile of the reflection coefficient that yields maximum bandwidth given some maximum reflection ρ_a within the band. Using this profile allows the

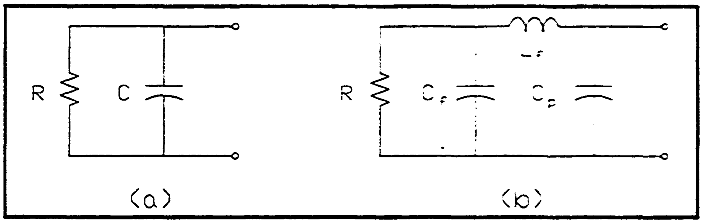

Fig. 1. Simple equivalent circuits of the (a) whisker-contacted and (b) planar Schottky barrier diode.

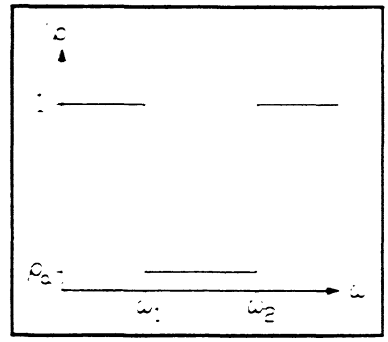

Fig. 2. Impedance matching profile for optimum bandwidth.

evaluation of the integral in (1),

$$(\omega_2 - \omega_1)\ln(\frac{1}{\rho_a}) \le \frac{\pi}{RC} \quad , \tag{2}$$

that illustrates the tradeoff between power-coupling bandwidth and ρ_a for a parallel RC circuit. As an example, for the UVa-1T15 whisker-contacted diode (epitaxial layer doping $1\cdot10^{18}$ cm^{-3} and anode diameter 0.25 μm), some reasonable estimates for $R_{d,opt}$ and $C_{j,avg}$ are $R_{d,opt}$=150 Ω and $C_{j,avg}=2\cdot C_{j0}$ =0.5 fF. For a center frequency of 3 THz and an in-band reflection coefficient ρ_a of 0.2, the largest obtainable fractional bandwidth for the 1T15 is 140%. Thus, for the whisker-contacted diode, the junction capacitance does not inherently limit the power coupling bandwidth, and the design is instead limited by other issues.

Fig. 3 shows the planar diode chip geometry and the location of the significant parasitic elements near the diode's anode region, which we have reduced to the simple equivalent circuit of Fig. 1(b) for this analysis. C_f represents the parallel combination of $C_{j,avg}$ and the parasitic capacitance from the finger to the ohmic-contact pad, C_{fp}. The other parasitics are the finger inductance L_f and the pad-to-pad capacitance C_p that represents the fringing capacitance between the pads. These parasitics can potentially reduce the achievable bandwidth from that of the whisker-contacted geometry. In order to determine the significance of the planar diode parasitics on the high frequency performance, this article derives an equation similar to (2) for the planar diode equivalent circuit.

Statement of the Problem

The basic setup of the problem is shown in Fig. 4. Z_L represents the load whose power-coupling bandwidth limit is to be determined. Throughout this analysis, the impedances are assumed to be normalized to the source impedance. The goal of the analysis is to determine the fundamental limits to the performance of the matching network that are imposed by the load impedance Z_L. The reflection coefficient looking toward the matching network is taken as the figure of merit by which the bandwidth is judged.

Darlington showed that any physically realizable load can be represented by a purely reactive two-port network with a resistance terminating one of the ports [5]. This

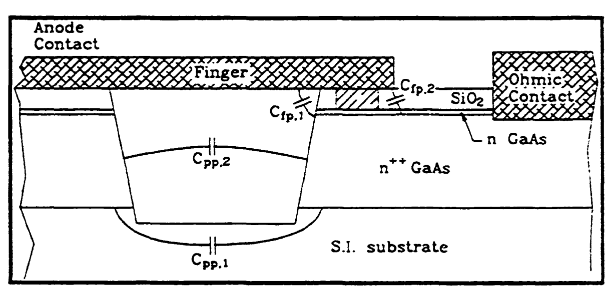

Fig. 3. Schematic of the diode chip near anode indicating physical location of fringing capacitances.

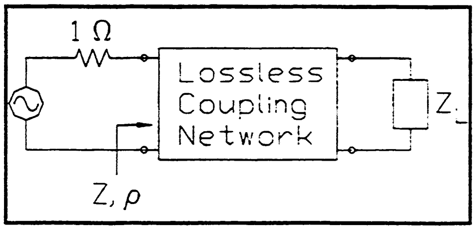

Fig. 4. Basic setup of Bode-Fano problem

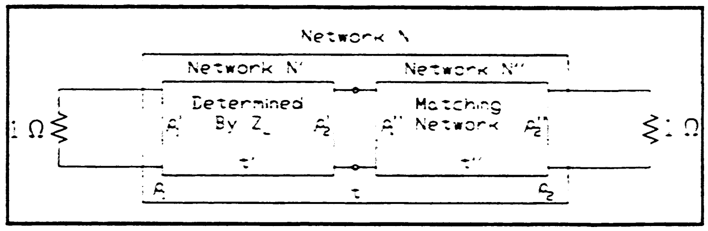

Fig. 5. Division of Bode-Fano circuit into two subcircuits N′ and N″

terminating resistance can be made arbitrary by the inclusion of an ideal transformer within the reactive two-port network. Using a Darlington representation of the load impedance allows the problem to be redrawn as shown in Fig. 5.

The goal of this analysis, then, is to determine the limitations to the performance of the entire circuit N caused by the load circuit N′ independent of the matching circuit N″. The relationship between the overall circuit N and the load circuit N′ was explored by Fano [3]. Fano showed that at an n^{th} order zero of transmission of N′ in the right half of the complex frequency plane (RHP), ρ_1 and its first $2n$-1 derivatives are equal to ρ_1' and its first $2n$-1 derivatives and are thus independent of the circuit N″. Using this basic insight, Fano showed that by performing a contour integration around the RHP involving the function $F=\ln(1/\rho_1)$, an integral relation can be determined between the frequency response of N and the physical circuit parameters of N′.

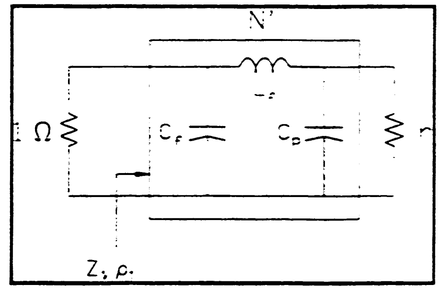

Fig. 6. Equivalent circuit model of planar diode used during derivation of Bode-Fano equations.

One caveat to this analysis is that under certain circumstances the adjacent elements of N′ and N″ are of the same type and orientation. The network N is then called degenerate because both N′ and N″ have zeros of transmission at the same location. For this degenerate case, only the first $2n$-2 derivatives of ρ_1' are independent of the circuit N″, and the bandwidth can be improved by increasing the value of the final element of N′. The bandwidth limitation determined by the circuit with n-1 elements then determines the behavior, as will be discussed later.

The next section uses the integral relations developed by Fano to derive the relations for the planar diode equivalent circuit. However, Fano's theory was derived for the case of low-pass matching. For many circuits, the low-pass theory can be simply extended to the band-pass case using standard transformation techniques. For the circuit under consideration, this transformation is not applicable because the circuit elements are not accessible for the connection of the requisite parallel elements. Kerr extended Fano's theory to the case of a bandpass matching without the use of this lowpass to bandpass transformation [6]. In addition, Fano's analysis was only valid for circuits with two elements. In [7], we extended the analysis of Fano to the three-element circuit of Fig. 1(b), and we will now use the results of this analysis in deriving the bandwidth limitation equations for this circuit.

Derivation of Equations

The circuit N′ in Fig. 6 shows the key parasitics for a typical planar diode chip. The resistor r is an arbitrary resistance that eventually drops out of the calculations. The first step of the analysis is to determine the coefficients of the Taylor series for the function $F(s) = \ln(1/\rho_1(s))$. The circuit N′ has three zeros of transmission at infinity, and it is thus convenient to define the variable $\xi \equiv 1/s$. Because of the three zeros of transmission, the first 5 coefficients of the Taylor series of ρ_1, and therefore F as well, are determined entirely by the circuit N′ for the non-degenerate case. The Taylor series for F is given by

$$F(s) = \ln(1/\rho_1) = jA_0 + A_1\xi + A_2\xi^2 + A_3\xi^3 + \cdots \tag{3}$$

where A_0 is either 0 or π depending upon the sign of ρ_1, and A_n are

$$A_n = \frac{1}{n!}\left.\frac{d^nF(\xi)}{d\xi^n}\right|_{\xi=0} \tag{4}$$

The impedance looking into the circuit N′ is given by

$$Z_1' = \frac{s^2 rL_fC_p + sL_f + r}{s^3 rL_fC_fC_p + s^2L_fC_f + sr(C_f + C_p) + 1} \tag{5}$$

Using (5) to calculate the inverse of the reflection coefficient yields

$$\frac{1}{\rho_1} = \frac{Z_1'+1}{Z_1'-1} = -\frac{s^3rL_fC_fC_p + s^2(L_fC_f + rL_fC_p) + s(L_f + r(C_f + C_p)) + (1+r)}{s^3rL_fC_fC_p + s^2(L_fC_f - rL_fC_p) - s(L_f - r(C_f + C_p)) + (1-r)} \tag{6}$$

Finally, the function F(s) in terms of ξ is

$$F(s) = \ln\left[\frac{1}{\rho_1}\right] = \ln\left[-\frac{\xi^3(1+r) + \xi^2(L_f + r(C_f + C_p)) + \xi(L_fC_f + rL_fC_p) + rL_fC_fC_p}{\xi^3(1-r) - \xi^2(L_f - r(C_f + C_p)) + \xi(L_fC_f - rL_fC_p) + rL_fC_fC_p}\right] \tag{7}$$

Taking the appropriate derivatives of F and evaluating them at $\xi = 0$ yields the coefficients of the Taylor series:

$$\begin{aligned} &A_0 = \pi\,, \quad A_1 = \frac{2}{C_f}\,, \quad A_2 = 0\,, \quad A_3 = \frac{2}{3}\frac{L_f - 3C_f}{L_fC_f^3}\,, \\ &A_4 = 0\,, \quad A_5 = \frac{2}{5}\frac{5C_f^3 + 5C_f^2C_p - 5C_fC_pL_f + C_pL_f^2}{C_f^5C_pL_f^2}\,. \end{aligned} \tag{8}$$

In Table 1 of Fano's paper, he gives the integral conditions for physical realizibility for a circuit with n zeros of transmission at infinity as [3, p. 68]

$$\int_0^\infty \omega^{2k}\ln\left(\frac{1}{|\rho_1|}\right)d\omega = (-1)^k\frac{\pi}{2}F_{2k+1} \tag{9}$$

where k runs from 0 to 2 and F_{2k+1} is defined as

$$F_{2k+1} = A_{2k+1} - \frac{2}{2k+1}\sum_i \lambda_{ri}^{2k+1} \tag{10}$$

and where the A_{2k+1} are the coefficients of the Taylor series for $F(\xi)$ and λ_{ri}^{2k+1} are the zeros of ρ_1 in the right hand plane, which are determined by the matching network. Using the reflection coefficient profile shown in Fig. 2, (9) can be simplified to

$$K\Omega_{2k+1} = (-1)^k(2k+1)F_{2k+1} \tag{11}$$

where Ω_{2k+1} is defined as

$$\Omega_{2k-1} = \omega_2^{2k-1} - \omega_1^{2k-1} \tag{12}$$

and K is given by

$$K = \frac{2}{\pi} \ln\left(\frac{1}{|\rho_a|}\right) \tag{13}$$

Substituting the values of k and expanding F_{2k-1} we arrive at a series of three equations

$$\begin{aligned} A_1 - K\Omega_1 &= 2\sum \lambda_{ri} \\ 3A_3 + K\Omega_3 &= 2\sum \lambda_{ri}^3 \\ 5A_5 - K\Omega_5 &= 2\sum \lambda_{ri}^5 \end{aligned} \tag{14}$$

All that remains is to determine the right-half-plane (RHP) zeros λ_{ri} that maximize the coefficient K within the matching band, thus minimizing ρ_a. The zeros λ_{ri} must be real or appear in complex conjugate pairs and must have a real part greater than zero. Fano showed that for a *CL* circuit in the non-degenerate case, the bandwidth can be maximized by choosing a single real root [3, p. 72]. However, as Fano mentions, his proof for the two element *CL* circuit could not be extended to circuits with larger numbers of elements [3, p. 73]. In [7], we give a proof for the three element *CLC* circuit and show that, except for the degenerate case, *K* is maximized by a single pair of either real or complex conjugate roots. Thus, choosing a pair of roots $\lambda_{r1} = x + \sigma$ and $\lambda_{r2} = x - \sigma$ where $\sigma = y$ for two real roots and $\sigma = iy$ for a pair of complex-conjugate, then the summation of the zeros in (14) becomes

$$\begin{aligned} \sum \lambda_{ri} &= 2x \\ \sum \lambda_{ri}^3 &= 2(x^3 + 3x\sigma^2) \\ \sum \lambda_{ri}^5 &= 2(x^5 + 10x^3\sigma^2 + 5x\sigma^4) \end{aligned} \tag{15}$$

Substituting (15) into (14) yields

$$\begin{aligned} A_1 - K\Omega_1 &= 4x \\ 3A_3 + K\Omega_3 &= 4(x^3 + 3x\sigma^2) \\ 5A_5 - K\Omega_5 &= 4(x^5 + 10x^3\sigma^2 + 5x\sigma^4) \end{aligned} \tag{16}$$

Eliminating the variables x and σ yields the equation

$$\begin{aligned} 144(A_1 - K\Omega_1)(5A_5 - K\Omega_5) &+ (A_1 - K\Omega_1)^6 \\ - 20(A_1 - K\Omega_1)^3(3A_3 + K\Omega_3) &- 80(3A_3 + K\Omega_3)^2 = 0 \end{aligned} \tag{17}$$

Note that this equation is valid for both the real root and complex-conjugate root solutions. If we introduce the fractional bandwidth b and center frequency ω_0, related to ω_1 and ω_2 by the equations $\omega_1 = \omega_0(1-b/2)$ and $\omega_2 = \omega_0(1+b/2)$, then the Ω_{2k+1} can be rewritten as

$$\begin{aligned} \Omega_1 &= b\omega_0 \\ \Omega_3 &= \frac{b(12 + b^2)\omega_0^3}{4} \\ \Omega_5 &= \frac{b(80 + 40b^2 + b^4)\omega_0^5}{16} \end{aligned} \tag{18}$$

Finally, defining the variables $B_{C_f} \equiv \omega_0 C_f$, $B_{C_p} \equiv \omega_0 C_p$, and $X_{L_f} \equiv \omega_0 L_f$ and substituting (8) and (18) into (17) yields a polynomial in K and b,

$$
\begin{aligned}
0 = 2880 &+ K^6 b^6 B_{C_f}^{\ 3} B_{C_p} X_{L_f}^{\ 2} \\
&- K^5 12 b^5 B_{C_f}^{\ 2} B_{C_p} X_{L_f}^{\ 2} \\
&+ K^4 (60 b^4 B_{C_f} B_{C_p} X_{L_f}^{\ 2} + 60 b^4 B_{C_f}^{\ 3} B_{C_p} X_{L_f}^{\ 2} + 5 b^6 B_{C_f}^{\ 3} B_{C_p} X_{L_f}^{\ 2}) \\
&+ K^3 (-120 b^3 B_{C_f} B_{C_p} X_{L_f} - 120 b^3 B_{C_p} X_{L_f}^{\ 2} - 360 b^3 B_{C_f}^{\ 2} B_{C_p} X_{L_f}^{\ 2} \\
&\qquad - 30 b^5 B_{C_f}^{\ 2} B_{C_p} X_{L_f}^{\ 2}) \\
&+ K^2 (720 b^2 B_{C_p} X_{L_f} + 720 b^2 B_{C_f} B_{C_p} X_{L_f}^{\ 2} + 60 b^4 B_{C_f} B_{C_p} X_{L_f}^{\ 2} \\
&\qquad + 240 b^4 B_{C_f}^{\ 3} B_{C_p} X_{L_f}^{\ 2} + 4 b^6 B_{C_f}^{\ 3} B_{C_p} X_{L_f}^{\ 2}) \\
&+ K (-1440 b B_{C_f} - 1440 b B_{C_p} + 2880 b B_{C_f} B_{C_p} X_{L_f} + 240 b^3 B_{C_f} B_{C_p} X_{L_f} \\
&\qquad - 440 b B_{C_p} X_{L_f}^{\ 2} - 120 b^3 B_{C_p} X_{L_f}^{\ 2} - 1440 b B_{C_f}^{\ 2} B_{C_p} X_{L_f}^{\ 2} \\
&\qquad - 720 b^3 B_{C_f}^{\ 2} B_{C_p} X_{L_f}^{\ 2} - 18 b^5 B_{C_f}^{\ 2} B_{C_p} X_{L_f}^{\ 2})
\end{aligned}
\qquad (19)
$$

Equation (19) can then be used to generate curves showing the relationship between ρ_a and the fractional bandwidth b for different system parameters. Once we have solved (19) for a given set of values, we can then use (16) to determine whether we have two real roots or a pair of complex-conjugate roots.

Application of the Theory to a Discrete Planar Schottky Diode

Table 1 gives the equivalent circuit values for the SC1T5 type planar diode for 5 μm and 20 μm finger lengths (indicated by -S5 and -S20 respectively). These parameters were generated by matching the equivalent circuit of Fig. 1(b) to the results of finite-element modeling of the mounted diode performed using Hewlett Packard's High Frequency Structure Simulator over frequencies from 450–700 GHz [8]. The capacitance C_f includes both the finger-to-pad capacitance C_{fp} and the time averaged junction capacitance $C_{j,avg}$. The capacitance $C_{j,avg}$ was found by averaging the capacitance waveform (determined using harmonic balance simulations) over an LO cycle. The optimum RF and LO diode resistance, $R_{d,opt}$, was estimated to be 100 Ω based on the results of harmonic balance simulations. These circuit values were then used with (19) to calculate curves of ρ_a versus fractional bandwidth for various planar diode geometries at different frequencies.

Fig. 7 shows the relation between ρ_a and the fractional bandwidth b for an SC1T5 type planar diode [4] with a 5 μm finger length plotted at frequencies from 500 GHz to 2000 GHz. The bandwidth limitation is not a practical limitation at the lowest frequency, and it is only above about 1000 GHz that the bandwidth limitation significantly effects the coupling of power to the

Table 1. Equivalent circuit values used during bandwidth modeling for the SC1T5 diode for various finger lengths.

Diode Type	$C_{pp} \equiv C_p$ (fF)	C_{fp} (fF)	$C_{j,avg}+C_{fp} \equiv C_f$ (fF)	L_f (pH)
SC1T5-S5	2.8	0.4	4.4	20
SC1T5-S20	2.0	0.8	4.8	29

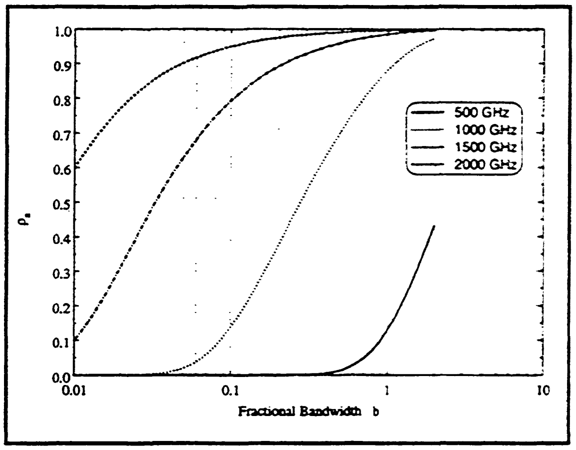

Fig. 7. ρ_a versus fractional bandwidth for the SC1T5-S5 diode for frequencies from 500 GHz to 2000 GHz.

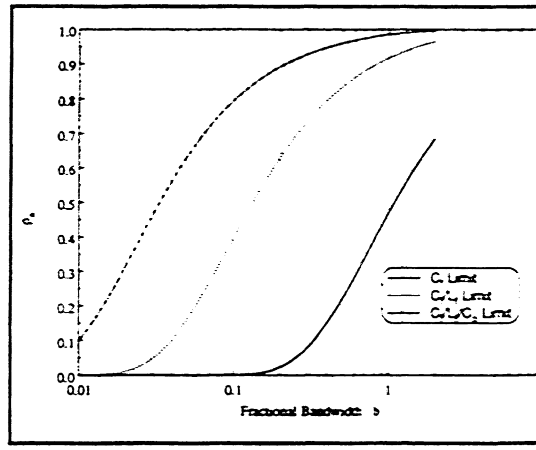

Fig. 8. ρ_a versus fractional bandwidth for the SC1T5-S5 diode at 1500 GHz for *C*, *CL* and *CLC* circuits.

diode. Note that this is a power coupling bandwidth, not a predicted mixer bandwidth. In order to illustrate the relative importance of each circuit element, Fig. 8 compares the bandwidth limitation for an SC1T5-S5 diode at 1500 GHz considering first the effect of C_f only, then both C_f and L_f, and finally for the full three-element circuit. Fig. 9 shows a set of bandwidth limitation curves comparing different finger lengths and different junction capacitances. As the finger length is reduced, L_f decreases while C_p increases; also, the bandwidth increases, thus indicating the importance of L_f in relation to C_p. Also shown is the improvement in bandwidth as the junction capacitance is reduced from 2 fF to 1 fF.

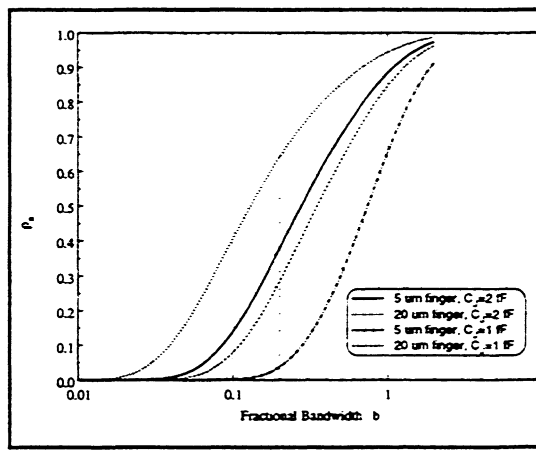

Fig. 9. ρ_a versus fractional bandwidth for various diode at 1500 GHz.

Conclusions

In this paper we have derived the bandwidth limitation equation for the coupling of power to a resistor through a three-element *CLC* circuit. This derivation was made possible by our extension of the Bode-Fano theory to circuits with three or more zeros of transmission at infinity. The results of this analysis were then used to examine the performance of a fundamental mixer built using discrete planar diode chips with semi-insulating GaAs support substrates. The simulations indicate that at frequencies around 500 GHz the couplıng of power to the diode is not significantly limited by the chip parasitics. However, at frequencies above approximately 1000 GHz, the packaging parasitics impose tight limits on the power coupling bandwidth for this diode and mixer geometry, thus complicating mixer design. The bandwidth limitation condition derived here is general, and can be used to explore power coupling limitations for a variety of devices and structures.

Acknowledgments

The authors would like to thank Dr. Anthony Kerr and Dr. John Granlund of the National Radio Astronomy Observatory for their invaluable advice, comments and criticisms of this work.

This research has been supported by the U.S. Army National Ground Intelligence Center through contract DAHC90-91-C-0030.

References

1. J.L. Hesler, W.R. Hall, T.W. Crowe, R.M. Weikle, II, B.S. Deaver, Jr., R.F. Bradley, and S.-K. Pan, "Fixed-Tuned Submillimeter Wavelength Waveguide Mixers Using Planar Schottky Barrier Diodes," *IEEE Trans. Microwave Theory Tech.*, Vol. 45, pp. 653-658, May 1997.

2. H.W. Bode, *Network Analysis and Feedback Amplifier Design*, New York, Van Nostrand, 1945.

3. R.M. Fano, "Theoretical Limitations on the Broadband Matching of Arbitrary Impedances," J. Franklin Inst., vol. 249, pp. 57-83, 139-154, Jan./Feb. 1950.

4. W.L. Bishop, E. Meiburg, R.J. Mattauch, T.W. Crowe and L. Poli, "A Micron-Thickness, Planar Schottky Diode Chip For Terahertz Applications With Theoretical Minimum Parasitic Capacitance," 1990 IEEE MTT-S Int. Mic. Symp. Digest, pp. 1305-1308, May 1990.

5. S. Darlington, "Synthesis of Reactance 4-Poles Which Produce Prescribed Insertion Loss Characteristics", *Journal of Mathematics and Physics*, vol. 18, pp. 275-353, September 1939.

6. A.R. Kerr, "Some Fundamental and Practical Limits on Broadband Matching to Capacitive Devices, and the Implications for SIS Mixer Design," IEEE-MTT43, No. 1, pp. 2-13, Jan. 1995.

7. J.L. Hesler, V. Celli, and B.L. Gelmont, "An Extension to Fano's Theory of Broadband Matching of Arbitrary Impedances," submitted to *IEEE Trans. on Circuits and Systems: I - Fund. Theory and Applic.*, Feb. 1998.

8. J.L. Hesler, *Planar Schottky Diodes In Submillimeter-Wavelength Waveguide Receivers*, Ph.D. Dissertation, University of Virginia, Jan. 1996.