An Active Zone Small Signal Model for Hot Electron Bolometric Mixers

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Abstract—In this paper, the influence of contact resistance between the NbN film and the Au pads on the formation of a hot spot in superconducting hot electron bolometers (HEB) is discussed. Based on a one-dimensional heat transport equation for electrons and phonons for the steady state, a distributed small signal model is derived. One finds that the mixing is limited to a small portion of the HEB, where the derivative of the resistivity with temperature is significantly nonzero. It is this small active zone which yields the electrothermal feedback term, not the whole device resistance as in traditional small signal models. Predicted IF bandwidth, receiver noise and conversion gain coincide very well with experimental findings.

Index Terms—hot electron bolometer, hot spot, electrothermal feedback, small signal models

I. INTRODUCTION

One-dimensional hot spot mixer models have been proposed recently for diffusion-cooled [1] and for the phonon-cooled hot-electron bolometers (HEB) [2]. In addition two-dimensional models have been proposed assuming RF skin depth as a major source of frequency dependent losses [3,4]. Here an one-dimensional large signal model including contact resistance is presented. From this large signal model, a distributed small signal model is derived which indicates, that only the zones, where the local resistivity of the bolometer has a significant temperature derivative contribute to the conversion gain. As discussed in [3], hot-spot models provide reasonable predictions of the IV curves - at least for moderate and high bias voltages. There are discrepancies for small bias voltages, where the IV curves in the hot spot model have a tendency to be too resistive. A possible explanation is, that the absorbed radio-frequency (RF)-heating power is not used efficiently for small bias voltages. This can possibly be explained by taking the contact resistance of the NbN layer to the gold antenna pads into account: The superconducting NbN film in devices made at Chalmers extends under the whole antenna structure including ground planes and CPWs. So it is well justified to assume that IF- and DC current flows exclusively in the superconducting NbN layer. This stands in contrast to the RF current with a frequency close to the quasiparticle bandgap (for 600GHz and 4.2K operation temperature) or far above the gap (for 1.6THz). Almost the whole RF current flows in the thick Au top layer except in the vicinity of the NbN bridge where the RF current has to be squeezed into the NbN layer. Given a certain contact resistance and contact capacitance, a transfer region is defined in which the current crosses over from the Au pads to the NbN film. The region to be taken for the integration of the heat balance relations must therefore extend under the Au pads. Adding terms for the thermal capacity of the electrons and assuming a small time-dependent change of the temperature around an operating point results in a time-dependent, one-dimensional integro-differential small signal model. This model depends on the local temperature as well as on the electrothermal feedback where the element as a whole interacts with the load resistance. In previous papers this cumbersome step was avoided and small signal parameters were derived based on the total differential of the resistance with respect to heating powers in the operating point. These traditional small signal models show a conversion gain of the order 10dB too high [7] and various countermeasures have been proposed to explain or avoid this discrepancy ranging from empirical correction factors of the RF heating efficiency or the influence of RF skin effect [4]. For 600 GHz, RF skin depth is far too weak to explain the measured gain data [3]. In this paper, focussing on data at 600GHz, an one-dimensional linearized small signal relation for a small temperature change profile is set up. Transforming this differential equation from a small temperature to a resistance change, allows to restrict the differential equation to parts of the HEB bridge close to the critical temperature, where a small heating power change results in a significant resistance change. Hence only regions, where the derivative of the (local) resistivity with electron temperature is nonzero contribute in conversion gain. These regions are called “active zones”. The remaining part of the bolometer bridge is a passive. It acts merely as a constant parasitic series resistance. The solution of the small signal differential equation can be restricted to the active zones only. The active zones in a HEB are then treated as lumped elements in series with a passive series resistance. It is important to note, that there are two inherently different modes of operation for the HEB – for small heating powers, the center temperature does not exceed the critical temperature significantly giving rise to a single active zone on...
the center of the HEB. For larger heating powers, a normal conducting hot spot forms acting as a passive series resistance: At the boundary between the normal conducting hot spot and the superconducting rest of the bridge, two independent active zones are formed. This splitting of the bolometer bridge in a small active part and a parasitic series resistance is also reflected in the form of the expressions for electrothermal feedback: Instead of the "usual" term proportional to \((R_L- R_B)/(R_L+R_B)\) where \(R_L\) stands for the load resistance and \(R_B\) for the resistance of the HEB in its operating point, the electrothermal feedback term becomes proportional to \((R_L+ R_B -r)/(R_L+R_B+r)\) where \(r\) is the resistance in the active zone. This agrees much better with measured results than the traditional relation. Another consequence of the Active Zone model is in the mechanisms of RF- and bias power absorption: For the large signal model it is obvious that the dc bias power (oscillating at the IF frequency at maximum) is only absorbed where the film is normal conducting or whenever the critical current of the film is exceeded. (The latter is usually not the case in the operating points of interest, critical current effects are therefore not treated in this model.) RF power is absorbed uniformly across the bridge – its frequency is above the local quasiparticle bandgap. This led to the definition of different heating efficiencies \(C_{dc}\) and \(C_{RF}\) in former Hot-Spot models. Now there is no difference in the small signal heating efficiency for RF and DC signals in the active zones – the active zone being close to the critical temperature provides always a very low quasiparticle bandgap which allows uniform RF absorption (in the active zone), for DC absorption, the local critical current is always exceeded. Nevertheless, treating the HEB as a "black" box, different heating efficiencies for RF and DC will be found: In the case of RF power, only the fraction of the power is used for mixing, which is absorbed by the active zone. The same applies to DC power – only the power absorbed in the active zone is relevant for mixing. If there is only a single active zone (surrounded by superconducting material) the whole DC power has to be taken into account. For two active zones, the situation is different, there DC power is also absorbed (and wasted) in the hot spot core, the relevant DC power is only the fraction given by the ratio of the integrated resistance of the active zones divided by the whole resistance in the operating point.

The paper is structured as follows: First, a large signal, steady state hot spot model will be set up. This model serves as a base for the derivation of a time-dependent distributed small signal model. The latter is approximated by the active zone model. Expressions for the heating efficiencies, the electrothermal feedback term, the conversion gain, the IF gain bandwidth based on the new model are derived. Finally, theoretical findings and experiment are compared.

II. STEADY STATE LARGE SIGNAL MODEL

The steady state large signal model consists of two parts: the first part solves for the RF and DC current distribution in the NbN film and the Au pads. It results in a concentrated RF heating in the HEB bridge and a decaying "tail" under the Au pads. Therefore even above the quasiparticle bandgap the RF power absorption in the NbN film is not homogenous. Its distribution is then inserted in the heat balance relation which is solved for the electron temperature profile along the HEB bridge. It is important to know, that for "very bad" contacts (with very low electron transparency) the RF current transfer region becomes large up to several micrometers. At the same time, the parameters for the electron conduction term are not influenced by the top layer. The situation is different for "good" contacts: Now the electron transparency is high, shortening the current transfer region to some nanometers and increasing the electron cooling under the antenna pads. This process is associated with a proximity effect reducing the critical temperature under the pads. This effect is easily seen as a "two step transition" in the \(R(T)\) curve \([1]\) which has not been observed for NbN HEB. Hence the NbN-Au contacts seem to be of the "bad" type and the model focuses on this case assuming the electron conductivity to be constant and assigning a constant critical temperature to the NbN film all over the modeled structure.

1) Contact resistance

In addition to the resistance of the Au pads and the resistance of the NbN film, there is a detrimental contact resistance between these two metals. In order to enhance contact adhesion, a Ti layer is sputtered on top of the NbN film. This film acts in the worst case as a capacitive layer. Discretising the structure in a lumped element equivalent circuit one arrives at the circuit problem depicted in Figure 1:

![Fig. 1: Equivalent lumped element circuit for the contact resistance problem.](diagram)

This equivalent circuit must be solved for RF currents and DC current separately due to difference in the capacitive coupling being negligible for DC and IF and very important for RF. IF and DC currents flow only in the gold layer, when the NbN film is heated strongly and turned normal conducting locally. The RF current "sees" a normal conducting NbN film since the RF frequency is far above the quasiparticle bandgap. As a consequence, far away from the HEB bridge, the whole RF current is transported by the thick Au film providing a series resistance two orders of magnitude lower than the thin NbN film. Nevertheless, the RF current must enter the NbN film at a certain place - at least in the bridge, all currents have to flow in the NbN film. This current transfer takes
place in a characteristic length, referred to as “transfer length”. Integration of the model differential equation has to extend at least over the bridge and these transfer lengths. It is most convenient to solve the above current problem in terms of the fraction of current flowing in the NbN called \( n_i \) where \( i \) denotes the index of the current loop. In total there are \( N \) such current loops, \( i \) located at the beginning of the metal pad, \( N \) deep under the pad. With a contact impedance \( Z_c \), a gold pad \( (R_{Au}) \) and NbN \( (R_{NbN}) \) impedance per length segment \( \Delta \), one has to solve the following recurrence relation for all \( n_i \):

\[
n_i = n_{i+1} + n_N - 1 - \frac{\sum_{j=i+2}^{N} R_{nbN}(1 - n_j) - n_j R_{nbN}(T)}{Z_c(\omega)} \quad (1)
\]

The values inserted for the resistance of the gold layer for RF calculations take the RF skin depth (30nm at 1THz at 200nm total thickness) into account by assuming the total current to flow in a layer of twice the RF skin depth. A typical result of the current distribution for several contact resistances for 600GHz to 2.5THz).

\[\text{Fig. 2: Fraction of the RF current flowing in the NbN layer for contact resistances ranging from 5p}\Omega/m^2 \text{ to 5n}\Omega/m^2 \text{ for 600GHz. The calculations have been performed for a 35Å thick NbN films covered by a 100nm thick Au layer and a 5nm thick Ti layer.}\]

The above current distribution (and resulting RF heating distribution) is used in the subsequent heat balance:

\[\text{2) Heat balance}\]

For an one dimensional HEB, the heat balance equations for the quasiparticles and for phonons becomes:

\[-\frac{\partial}{\partial x} \lambda(T_c) \frac{\partial}{\partial x} T_c(x,y) + \sigma_{\text{electron}} \left[T_e(x,y)^{3.6} - T_{\text{ph}}(x,y)^{3.6}\right] = \frac{P_{LO}(y)}{L \cdot W \cdot H} + \rho(T_c) j(x,y)^2 \quad (2)\]

\[\sigma_{\text{electron}} \left[T_e^{3.6} - T_{\text{ph}}^{3.6}\right] = \sigma_{\text{phonon}} \left[T_{\text{ph}}^4 - T_{\text{bath}}^4\right] \quad (3)\]

Here \( T_c, T_{\text{ph}} \) and \( T_{\text{bath}} \) are the electron, phonon and bath temperature. The HEB bridge width is denoted by \( W \), the thickness is \( T \) and \( L\) stands for the length of the analyzed structure and \( x \) is the coordinate in length direction. The current density in the bridge is denoted by \( j \) and the resistivity is \( \rho \). In (3) phonon diffusion is neglected. The electron-phonon cooling efficiency \( \sigma_{\text{electron}} \) and phonon escape \( \sigma_{\text{phonon}} \) are found in [5]. The electron diffusion data are taken from [3,4,6]. Eliminating the phonon temperature approximately, one is left with a nonlinear ordinary integro-differential equation (in the case of voltage biasing):

\[-\frac{\partial}{\partial x} \lambda(T_c) \frac{\partial}{\partial x} T_c + \sigma \left[T_e^{3.6} - T_{\text{bath eff}}^{3.6}\right] = \frac{P_{LO}(x)}{L \cdot W \cdot H} + \rho(T_c) \left[\int_{\chi=0}^{L} j(T_c(\chi))d\chi\right]^2 \quad (4)\]

In the above relations, \( \lambda(T_c) \) denotes the electron thermal conductivity, \( \sigma \) is the effective electron-phonon cooling efficiency. \( L \) is the length of the whole HEB structure (including the RF current transfer zones under the pads), its width is given by \( W \) and the film thickness by \( H \). The bias voltage is \( V_0 \), \( \rho(T_c) \) denotes the film resistivity.

\[\text{III. TIME DEPENDENT LINEARISED SMALL SIGNAL MODEL}\]

Calculating the temperature profile in a one-dimensional HEB bridge during mixing, the RF heating term will consist of a large constant LO heating part and a small time varying “beat term” at the intermediate frequency. This heating power is fed into a time dependent heat balance relation taking thermal capacitance effects into account.

\[+c \frac{\partial}{\partial t} T_c - \frac{\partial}{\partial x} \lambda(T_c) \frac{\partial}{\partial x} T_c + \sigma \left[T_e^{3.6} - T_{\text{bath eff}}^{3.6}\right] = \frac{P_{LO}(x)+2\sqrt{P_{LO}(x)}P_{S}e^{i\omega t}}{L \cdot W \cdot H} + \rho(T_c) \left[\int_{\chi=0}^{L} j(T_c(\chi))d\chi\right]^2 \quad (5)\]

Knowing a temperature distribution which fulfils the large signal model \( T_{\delta}(x) \), heated by the a time averaged LO power, a small variation \( \bar{\sigma}(t) \) around this “operating point”
temperature profile oscillating at the IF frequency \( \omega \) is modeled as:

\[
T(x, t) = T_0(x) + \tau(t) e^{i\omega t}
\]  

(6)

Inserting this relation in the above differential equation and linearizing

\[
T^{3.6} = T_0^{3.6} + 3.6T_0^{2.6} \tau(t) e^{i\omega t}
\]  

(7)

a first order small signal model is obtained in Fourier space where \( p_{\text{DC}} \) denotes the small signal DC power absorbed in the HEB and the thermal conductivity \( \lambda \) and the resistivity \( \rho \) depend still on the large signal temperature profile:

\[
+ i\alpha \tau - \frac{\partial}{\partial x} \lambda - \frac{\partial}{\partial x} \tau + 2.6\sigma T_e^{3.6} \tau = 2\sqrt{P_{\text{LO}}(x)} P_f^{2} + \frac{p_{\text{DC}}}{L \cdot W \cdot H}
\]  

(8)

Formally multiplying (8) with the derivative of the local resistivity change profile \( \rho(\tau) = \frac{\partial \rho}{\partial T} \bigg|_{\tau} = R^\text{K} \tau \) and after some manipulations one obtains a differential equation for the small signal resistivity change profile

\[
\rho(\tau) = \frac{\partial \rho}{\partial T} \bigg|_{\tau} = R^\text{K} \tau
\]  

(9)

IV. THE ACTIVE ZONE MODEL

Further insight in the above relation is obtained by replacing the small signal temperature profile by the small signal resistivity change profile \( \rho(\tau) \) of the structure:

\[
\rho(\tau) = \frac{\partial \rho}{\partial T} \bigg|_{\tau} = R^\text{K} \tau
\]  

(10)

In the above relation, \( r \) denotes the resistance of the active zone, \( R_b \) is the resistance of the HEB structure in its steady state operating point and \( R_N \) is the normal resistance, seen by the RF current. It has been taken into account, that the RF beating term is absorbed along the whole structure and only the part actually absorbed in the active zone plays a role for the small signal approach and that the DC power (also at IF frequency) is absorbed only, where the NbN film becomes normal conducting. The rest of the HEB not belonging to the active zone acts as a passive load and the power absorbed there does not affect the small signal conductivity and is therefore wasted. For the resistivity as a function of the IF frequency one obtains:

\[
\rho(\omega) = R_{\text{K max}} \left[ \frac{r}{R_N} \cdot 2\sqrt{P_{\text{LO}}(x)P_f^{2}} + \frac{r}{R_b} \cdot \frac{p_{\text{DC}}}{L \cdot W \cdot H} \right]
\]  

(11)

Inspection of the three diffusion-related terms in the LHS of (10) one finds a differential diffusion term (which has the same form as the large signal diffusion term). This term is reduced by a term depending on the x-derivative of \( R^\text{K} \). Assuming a single active zone, the x-derivative of the (large signal) temperature will be very small allowing to neglect this term. For two distinct hot spots, there will be indiffusion of electrons into the active zone from the “hot” side which is in principle outbalanced by outdiffusion towards the “cold” side of the active zone. This becomes obvious by assuming a constant (large signal) temperature slope across the active zone and taking into account that \( R^\text{K} \)s at first order symmetric to the hot-spot boundary. The last diffusion-related term accounts for electrons being directly removed to the antenna pads leaving reducing the contribution of diffusion cooling close to the pads. This term is very small in NbN bolometers but may be important in Nb or Al HEBs. Neglecting the small diffusion terms in (9) and assuming the local resistivity derivative \( R^\text{K} \) to be constant \( (R_{\text{K max}}/2) \) within the active zone with length \( A \) and assuming that the small signal resistivity change profile \( \rho(\tau) \) is limited to the active zone one arrives at an algebraic equation for the small signal resistivity:

\[
\rho = R_{\text{K max}} \left[ \frac{r}{R_N} \cdot 2\sqrt{P_{\text{LO}}(x)P_f^{2}} + \frac{r}{R_b} \cdot \frac{p_{\text{DC}}}{L \cdot W \cdot H} \right]
\]  

(11)

In the above relation, \( r \) denotes the resistance of the active zone, \( R_b \) is the resistance of the HEB structure in its steady state operating point and \( R_N \) is the normal resistance, seen by the RF current. It has been taken into account, that the RF beating term is absorbed along the whole structure and only the part actually absorbed in the active zone plays a role for the small signal approach and that the DC power (also at IF frequency) is absorbed only, where the NbN film becomes normal conducting. The rest of the HEB not belonging to the active zone acts as a passive load and the power absorbed there does not affect the small signal conductivity and is therefore wasted. For the resistivity as a function of the IF frequency one obtains:

\[
\rho(\omega) = R_{\text{K max}} \left[ \frac{r}{R_N} \cdot 2\sqrt{P_{\text{LO}}(x)P_f^{2}} + \frac{r}{R_b} \cdot \frac{p_{\text{DC}}}{L \cdot W \cdot H} \right]
\]  

(11)
For the active zone model, the small signal equivalent circuit is depicted in Figure 3:

![Equivalent Circuit Diagram]

From this equivalent circuit, the dc power dissipated in the active zone as a function of the small signal resistance change is obtained

\[ P_{DC} = \left[ \frac{R_L + R_B - r}{R_L + R_B + r} \right] \cdot A \cdot \tilde{\rho} \cdot I^2 \]

Here \( r \) denotes the power dependent resistance of the active zone, \( A \) is the length of the active zone and \( \tilde{\rho} \) is the small signal change of the resistivity in the active zone as discussed before.

This relation is inserted in (12) and yields the small signal resistance response of the HEB under the influence of electrothermal feedback from which all other small signal parameters are calculated.

In order to be able to compare this model with traditional small signal models, the heating efficiencies (with respect to DC power and RF power on an active zone with length \( A \) ) are obtained as partial derivatives from (12):

\[ C_{RF} = A \cdot \frac{\partial \rho}{\partial \rho_{DC}} = \frac{r}{R_N} \cdot \frac{A \cdot 2}{L \cdot W \cdot H} + i \alpha + \frac{\lambda}{A^2} \cdot 2.6 \sigma T_e^{3.6} \]  

(14)

For such a circuit, the well known “electrothermal feedback” term (ETF) for IF frequencies far above the RC-time constant set up by the load resistance and the DC block capacitance becomes:

\[ ETF = \frac{1}{1 - C_{DC} I_0^2 \frac{R_L + R_B - r}{R_L + R_B + r}} \]  

(15)

As expected, it is only the active zone resistance \( r \) which fed back by the sum of the passive zones' and the load resistance. Taking care about the changed circuit topology, one obtains somewhat changed relations for the conversion gain, the IF gain bandwidth and as a consequence even for the noise properties. Nevertheless, the noise relations being already based on a one-dimensional approach are taken directly from [7]. The mixer time constant is prolonged by electrothermal feedback (15):

\[ \tau_{\text{mixer}} = \frac{\tau_{\text{electron energy relaxation}}}{1 - C_{DC} I_0^2 \frac{R_L + R_B - r}{R_L + R_B + r}} \]  

(16)

This time constant is required for the conversion gain as a function of IF frequency:

\[ G = \frac{P_L}{P_s} = 2 \cdot \frac{R_L}{(R_B + R_L + r)^2} \left[ \frac{1}{1 + i \omega \tau_{\text{mixer}}} \right]^2 \times \frac{1}{1 - C_{DC} \cdot \left[ \frac{R_L + R_B - r}{R_L + R_B + r} \right] \cdot I^2} \]  

(17)

In the following, IV curves obtained by solving a large signal model containing contact resistance and gain, noise and bandwidth curves obtained using the active zone model are summarized and compared with measurements.
V. COMPARISON WITH EXPERIMENT

Assuming a contact resistance of $25 \text{p}\Omega/\text{m}^2$ one obtains a (90\%) current transfer zone extending 1µm under the antenna pad. Solving the large signal model under these conditions, one obtains IV curves for different RF heating power levels. Figure 4 summarizes measured IV curves and calculated ones for a device with the same geometry.

Based on these results, the RF heating efficiency $C_{RF}$ is obtained by solving the small signal relation for the resistivity shown in Figure 5. One observes a sharp peak for the RF heating efficiency where a hot spot is formed.

Fig. 4 Calculated and measured IV curves at 0.6THz for device M2-1. The measured curves are dots connected with solid lines; the calculated curves are dashed.

Fig. 5 RF heating efficiency for LO heating powers ranging from 100nW to 250nW in steps of 25nW for device M2-1. The calculated points correspond to the points from Fig.4 and 5.

The calculated and experimentally obtained conversion gain at 1.5GHz is shown in Fig. 7. For the experiments, the optical losses have been assumed to be 3.5 dB.

For an intrinsic bandwidth of 5.5GHz one obtains an IF bandwidth as a function of bias voltage and LO power as follows. Observe the “peaks” of the bandwidth, where the active zone splits.

Fig. 6 Fraction of RF power absorbed in the active zone(s) LO heating powers ranging from 100nW to 250nW in steps of 25nW for device M2-1. The calculated points correspond to the points from Fig.4 through 6.

Fig. 7 Conversion loss as a function of bias current and LO power for the device M2-1. The measured and calculated points correspond to the points from Fig. 4 through 6.

Fig. 8 IF bandwidth for device M2-1. The measured points are obtained for the points with the best noise, the best gain and the best IF gain bandwidth. The measured receiver noises (including optics losses) are indicated.
For the device output noise one assumes the HEB to “see” a room temperature load at its input. This is not really true, since at least a fraction $\alpha$ of the optical losses are cold losses. For the output noise temperature one obtains where $T_{\text{noise}}^{\text{out}}$ denotes the output noise, $T_{\text{TF}}$ the Thermal fluctuation noise at the output and $T_{\text{Jn}}^{\text{out}}$ denotes the Johnson noise at the output:

$$T_{\text{noise}}^{\text{out}} = T_{\text{TF}}^{\text{out}} + T_{\text{Jn}}^{\text{out}} + 2 \times [295\alpha + 10(1 - \alpha)] \times G \quad (18)$$

The (overestimated) output noise assuming 295K at the input and $\alpha=1$ is shown in Figure 9.

The DSB receiver noise temperature is then calculated using:

$$T_{\text{RX,DSB}} = T_{\text{TF,n,DSB}}^{\text{in}} + T_{\text{Jn,DSB}}^{\text{in}} + \frac{T_{\text{IF}}^{\text{out}}}{2G} \quad (19)$$

where $T_{\text{IF}}$ is the noise contribution from the low noise IF amplifier (7 K) and $G$ is the conversion gain. The results are indicated in Fig 10:

VI. CONCLUSION

Besides the influence of contact resistance between the Au pads and the NbN film, a new small signal model for a hot-electron bolometric mixer is presented. The model uses the fact, that only a small part of the HEB bridge actually takes part in the conversion and mixing process. Therefore the small signal power absorbed in other regions of the HEB is wasted reducing the obtained conversion gain. For measurements at 600GHz, theoretical and experimental results show excellent coincidence improving the prediction capability of hot spot models considerably.

VII. REFERENCES