Compact Optical Assemblies for Large-Format Imaging arrays

G. Yassin, S. B. Sørensen, and P. K. Grimes

Abstract—Compact telescopes with ~1m diameter primary, have recently been employed in the Cosmic Microwave Background (CMB) investigations. Some of these telescopes are designed to measure the CMB polarization at an angular scale of a few arcminutes. Considering that the polarization signal is extremely weak, the radiation pattern needs to be circular, with very low cross-polarization. In this paper, we compare the performance of two compact dual-reflector systems, fed by a relatively large focal plane array, at millimetre wavelengths. We consider the case where the apertures of the feeds are forced to lie on a planar surface, allowing only the central feed to be exactly on focus. This geometry is chosen in order to make the fabrication of the feed array block easier. Our simulation show that Compact Range Antenna (CRA) has a much better performance than the more commonly employed offset Gregorian configuration. We illustrate that the CRA provides a circular beam with extremely low cross-polarisation, even when the antenna is fed by more than 200 feeds, at 90 GHz.

I. INTRODUCTION

The effort to increase the sensitivity of astronomical receivers is an important on-going research activity and considerable effort has already been invested to improve the performance of the single pixel detector. Recent advances in detector technology led to the development of detectors with background-limited sensitivity. Additional sensitivity therefore can only be achieved by increasing the number of detectors.

A recent motivation for further performance improvement, came from the emergence of new Cosmology instruments that attempt to measure the CMB polarization. The sensitivity of these instruments needs to be two orders of magnitude better than existing CMB anisotropy telescopes in order to detect the B-mode component, which is the signature of primordial gravitational waves [1]. Several instruments are now being designed or built including PLANCK, BICEP, EBEX, QUAD and CLOVER [2]. Although the design strategy and frequency coverage of these instruments differ substantially, they all have a key common feature: they employ a focal plane detector array feeding a compact optical assembly. Consequently, a lot of care is needed in designing the optics since the beams formed by the feeds at the edges of the array can exhibit high sidelobes, cross polarization and distortion by aberrations.

In this paper, we compare the performance of two offset compact optical assemblies that have been considered for large format imaging arrays, at millimetre and submillimetre wavelengths. The first is the Compact Range Antenna (CRA), which has a concave hyperboloidal subreflector and the second is the Gregorian that has an ellipsoidal subreflector. Each assembly has a parabolic primary and is fed by approximately 200 corrugated feeds array at 90 GHz.

The radiation pattern of each antenna will be analysed employing the rigorous Physical Optics (PO) method as implemented in the software package GRASP. In each case, the design parameters will be optimised for minimum cross-polarization and beam distortion by forcing the Mizuguchi condition.

II. METHOD OF DESIGN AND ANALYSIS

A. The Physical Optics Method

The Physical Optics method is a powerful tool in computing scattering from reflectors of finite conductivity. The method is now commonly employed in both the analysis and design of antennas and complex optical systems that comprise several reflectors. Calculation of the radiated fields is done in two steps:

1. Find the equivalent currents induced on the reflector surface.
2. Calculate the fields radiated by those currents.

The total field at any point may be written as the sum of incident and scattered fields:

$$ E = E^i + E^s $$

The induced current in PO, are found assuming that only the geometrically bright region is illuminated by the sources.

For a perfect conductor, the tangential component of the electric field vanishes on the reflector surface hence the induced current at any point is given by:

$$ K^{PO} = \begin{cases} n \times H = 2n \times H^i & \text{on illuminated surface} \\ 0 & \text{on the Shadow surface} \end{cases} $$

$$ K^{*PO} = 0 \quad \text{Everywhere} $$
where \( \mathbf{K}^{E0} \) and \( \mathbf{K}^{M0} \) are respectively the electric and magnetic current densities, \( \mathbf{H}' \) is the incident magnetic field on the reflector and \( \mathbf{n} \) is a unit vector normal to the surface. The above expression was derived by assuming a tangential plane at the point of interest and using the images method.

Following the calculation of the current at any point on the geometrically illuminated surface, the fields at any point can be found by re-writing the fields from Maxwell equations using the equivalence method:

\[
\begin{align*}
\mathbf{E} &= \mathbf{E} + i\omega \mathbf{A} - \frac{\nabla \times \mathbf{A}}{i\omega \epsilon_0} \\
\mathbf{H} &= \mathbf{H}' + \frac{1}{\mu} \nabla \times \mathbf{A}
\end{align*}
\]

where \( \mathbf{A} \) is the electric vector potential given by:

\[
\mathbf{A} = \frac{\mu}{4\pi} \int \mathbf{K}^{M0} e^{i\mathbf{k} \cdot \mathbf{R}} \frac{dS}{R}
\]

in the above expression \( R = |\mathbf{r} - \mathbf{r}'| \) where \( \mathbf{r} \) is the observation point, \( \mathbf{r}' \) is the integration variable over the surface and the integration is carried out over the illuminated surface.

It is worthwhile noting that while the radiated fields by the currents are calculated exactly, the induced currents themselves are found using two approximations, first by assuming that the geometrical shadow region does not radiate (which is clearly untrue) and then by neglecting the current non-uniformity near the edges. It turns out however that for apertures of several wavelengths across, the Physical Optics method is extremely reliable in both far- and near-field calculations [3]. Moreover, for very small apertures, the PO method can be supplemented by the Physical Theory of Diffraction correction (PTD) [4], to yield very accurate results in practical systems.

**B. Design Hints**

Offset parabolic systems normally have poor cross polarization performance and high aberrations. In dual reflector systems, therefore it is common to compensate for that by forcing the Mizuguchi condition:

\[
\tan(\psi_f) = M_0 \tan\left(\frac{\alpha}{2}\right)
\]

where \( \psi_f \) is the angle between the feed and subreflector axes, \( \alpha \) is the angle between the axes of the two reflectors and \( M_0 \) is given by

\[
M_0 = \frac{\epsilon + 1}{\epsilon - 1}
\]

Where \( \epsilon \) is the eccentricity of the subreflector.

Another constraining quantity is the effective focal length given by:

\[
f_e = f M_0 \frac{1 + \tan^2\left(\frac{\alpha}{2}\right)}{1 + M_0^2 \tan^2\left(\frac{\alpha}{2}\right)}
\]

Where \( f \) is the focal length of the main reflector. In what follows we shall describe our effort to design a CRA for CLOVER. Here, the feeds in the array are all pointing orthogonal to the telescope beam imposing the condition:

\[
\alpha + \psi_f = 90^\circ
\]

The actual choice of the two angles depends on the system design and an example of that is given in Fig. 1 with the requirements of primary diameter of 1.6 m at 90 GHz.

*Figure 1* GRASP ray tracing of a CRA illuminated by three feeds in the plane of asymmetry (x-z plane).

Taking a ratio \( f_e / D = 2 \) which is typical for this system, we have \( \alpha = -65^\circ \) and \( \epsilon = -2.06748 \). The design of the system was then taken from the optimised ratios: \( f / D = 4.5, 2c / D = 6.3 \).

The final design parameters are given in Table 1.

<table>
<thead>
<tr>
<th>Table 1: Design parameters of the CRA system</th>
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<tbody>
<tr>
<td>Primary diameter (D)</td>
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<td>Primary focal length (f)</td>
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<tr>
<td>Eccentricity of subreflector (( \epsilon ))</td>
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<tr>
<td>Distance between foci of subreflector (2c)</td>
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<tr>
<td>Angle between feed and subreflector axes (( \psi_f ))</td>
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<tr>
<td>Angle between primary and subreflector axes</td>
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III. SIMULATED RESULTS

The following simulations were made for a hexagonal array of 216 corrugated horns illuminating the CRA described in Table 1. Each feed is assumed to have a Gaussian beam with an edge taper of 14.4° at -12 dB. This corresponds to a feed aperture diameter of about 18 mm. Assuming a feed wall thickness of 1.25 mm, the outer feed centre will be located at 164 mm away from the centre of array. Below, is a GRASP simulation pattern of the central feed showing the E-plane and the H-plane cuts, as well as the cross polarization.

Next we compare the radiation patterns resulting from the outer feeds along the axis of symmetry (y-axis) and asymmetry (x-axis). The results are shown in Fig. 3. Notice that the cross polarisation level has now risen to just below -40 dB and that the main beam is no longer circular below approximately -20 dB. However, the level of cross polarisation and beam distortion remain acceptable considering that the feed apertures are not exactly located in the focal plane. We should emphasize however that the shift of the centres in the top patterns of Fig. 3 is the result of misalignment in the cut coordinate system orientation and hence should be ignored.

Finally we compare the performance of the CRA with the Offset Gregorian antenna, which is employed in PLANCK. A ray diagram of the system is shown in Fig. 4. The design parameters of this antenna were taken from “GRASP Technical Manual” and are shown in Table 2.
We recall that in the CRA, the outer feed scanned the beam by 2.8 degrees. Since the F/D ratio for the GRA is 2.96 the outer feed needs to be located at 231 mm from the centre in order to have the same scan angle. Another way of looking at it is to require that the two antennas have the same beamwidth. For that to happen, we need to under-illuminate the GRA by the effective focal lengths ratio. This requires horns of larger aperture diameter by approximately the same ratio, and hence the outer feed will be further displaced by the distance quoted above. The feed opening angle towards the subreflector is now smaller, giving a taper of -12 db at 9.7 degrees. The simulated pattern of this system is shown in Fig. 5. Here we can easily see that the circularity of beam is no longer satisfactory and that the cross polarization level has risen to -35 db. This is despite the fact that the outer feeds were only shifted by 173.2 mm instead of 231 mm (to have the same number of feeds as the CRA) and that those feeds were also tilted towards the centre in order to have a reasonable spill-over (as can be seen from Fig. 4).

We investigated the performance of compact optical assemblies when fed by a large focal plan arrays. Our simulations show that for a typical CMB telescope operating at 90 GHz and a -3db full beamwidth of 8.4 arcminutes, the Compact Range Antenna gives excellent performance even when the array is not exactly located on the focal surface. The Gregorian system however, does not seem to be suitable for large beam scanning.

REFERENCES

Figure 5 (a): Simulated beam patterns for the offset Gregorian system with the feed shifted 173.2 mm in the x direction

Figure 6 (b): Simulated beam patterns for the offset Gregorian system with the feed shifted 173.2 mm in the y direction