A CAD Tool for the Design and Optimization of Schottky Diode Mixers up to Terahertz Frequencies

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Abstract—Emerging applications at millimeter and submillimeter-wave spectral bands demand reliable CAD tools to be employed in the design of receivers up to Terahertz frequencies. In this paper we present a CAD tool for the design and optimization of Schottky diode frequency mixers at millimeter and submillimeter-wave spectral bands. This tool couples a diode physical numerical model with the circuit simulator using a multi-tone harmonic balance technique based on the Almost Periodic Fourier Transform (APFT). Contrary to approximate approaches, as matrix conversion techniques, no restrictions are considered regarding LO and RF pump powers. The mixer CAD tool allows a complete analysis of mixer circuits up to Thz frequencies taking into account both the external circuit and the Schottky diode structure.

I. INTRODUCTION

Terahertz spectral bands are one of the least explored regions of the electromagnetic spectrum with applications in radio-astronomy, planetary science, security, medicine, etc. The necessity for Terahertz circuit development makes essential to have accurate simulation tools to be employed in the design and optimization of these circuits as a previous step to the fabrication process [1].

In this paper we present a novel CAD tool that couples a Schottky diode physical numerical model with a circuit simulator using appropriate harmonic-balance techniques (Fig. 1). This allows the concurrent design of circuits (mixers, detectors and multipliers) taking into account both the device structure (doping and length of the epitaxial layer, and area of the device) and the embedding circuit (bias, available power, and loads at different harmonics and intermodulation frequencies). A similar CAD for Schottky multipliers design was already presented in [2], [3], [4].

The Schottky diode model consists of a physics-based numerical device simulator which incorporates accurate boundary and interface conditions for self-consistent treatment of tunnelling transport, image-force effects, impact ionization, and non-constant recombination velocity. This physics-based simulator accounts for limiting mechanisms such as avalanche breakdown, velocity saturation, and increase in the series resistance with the input power.

II. TECHNIQUES FOR MULTI-TONE HARMONIC BALANCE

There are different methods available for transforming signals between time and frequency domains that are suitable for use with multi-tone signals. The most important are:

- **FFT (Fast Fourier Transform):** Only applicable to mixer analysis when LO and RF frequencies are commensurable. In this case, a base frequency for the FFT can be selected as the great common divider of LO and RF.

Fig. 1. Schematic of the analyzed circuit for mixers simulation.
• **MFFT (Multidimensional Fast Fourier Transform):** A generalization of the FFT to analyze circuits whose base frequencies are incommensurable. It is the most general algorithm with no additional assumptions to the ones assumed for the traditional harmonic balance for periodic signals, but it requires models described by algebraic equations [5].

• **APFT (Almost Periodic Fourier Transform):** APFT is based on a generalization of the matrix form of the Fourier transform without any restriction regarding the frequencies to be analyzed and the sampling instants. These approaches are slower than MFFT and mapping techniques because APFTs do not employ the FFT. However, APFTs support incommensurable frequencies and are easy to implement and very flexible.

• **Mapping Techniques:** Actual base frequencies are replaced by artificially selected base frequencies, so that the original spectrum is mapped onto an equivalent periodic and dense spectrum. Waveforms transformed through the mapping become periodic and its Fourier coefficients can be efficiently calculated by the one-dimensional FFT.

Aspects such as underlying assumptions, limitations, flexibility, dynamic range and time consumption in the calculations have been considered before selecting the algorithms to be implemented in the simulator. After an in-depth study of the different approaches, the most appropriate algorithm are the APFTs. This is because **MFFT** and **Mapping Techniques** are only valid for non-linear systems described by algebraic relationships, i.e., memoryless systems. FFT does not support incommensurable frequencies and the computational cost highly depends on $\text{gcd}(f_{LO}, f_{RF})$.

### A. Introduction to the APFT

APFT is based on a generalization of the matrix form of the Fourier transform without any restriction regarding the frequencies to be analyzed and the sampling instants.

By considering only a finite number of frequencies $\Delta_k = \{w_0, w_2, \ldots, w_{K-1}\}$, it is possible to sample a waveform at a finite number of time points and calculate its Fourier coefficients.

If $X(k) = X^C_k + j \cdot X^S_k$ represents the Fourier coefficients at frequencies $w_k$ of a certain waveform $x(t)$, and assuming a certain truncation error, $x(t)$ can be expressed as:

$$x(t) = \sum_{w_k \in \Delta_k} \left( X^C_k \cdot \cos(w_k \cdot t) + X^S_k \cdot \sin(w_k \cdot t) \right)$$  \hspace{1cm} (1)

By sampling $x(t)$ at $S$ time points, Eq. 1 can be rewritten as a matrix product,

$$x = A \cdot X$$  \hspace{1cm} (2)

where,

$$x = [x(t_1) \ x(t_2) \ x(t_3) \ldots \ x(t_S)]^T$$  \hspace{1cm} (3)

$$X = \begin{bmatrix} X_0^C & X_1^C & \ldots & X_{K-1}^C \end{bmatrix}^T$$  \hspace{1cm} (4)

$$A = \begin{bmatrix} 1 \cos(w_1 t_1) \sin(w_1 t_1) & \ldots & \cos(w_{K-1} t_1) \sin(w_{K-1} t_1) \\ 1 \cos(w_1 t_2) \sin(w_1 t_2) & \ldots & \cos(w_{K-1} t_2) \sin(w_{K-1} t_2) \\ \vdots & \ddots & \vdots \\ 1 \cos(w_1 t_S) \sin(w_1 t_S) & \ldots & \cos(w_{K-1} t_S) \sin(w_{K-1} t_S) \end{bmatrix}$$  \hspace{1cm} (5)

Fig. 2 shows a general flowchart for APFT methods, including the three steps mentioned above.

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Fig. 2. General flowchart of APFT techniques.
proposed in 1988 a time-selection algorithm to choose an adequate set of time points in order to guarantee a well-conditioned square matrix $A$. In this case, the time-selection algorithm can be seen as an orthogonalization process. The vector $X$ of Fourier coefficients can be directly calculated just by computing the ordinary inverse of $A$, and yields Eq. 7.

$$X = A^{-1} \cdot x$$  \hspace{1cm} (7)

If $A$ is not an square matrix, that is $S > 2K - 1$, the problem of computing Fourier coefficients consists of finding the shortest vector $X$ that minimizes the distance $\rho(X)$ (Eq. 8). This is a typical least-square problem that can be solved by methods published in literature: [7] - [10]. This APFT scheme was firstly suggested by Zhang and Hong in 1990 [11]. Another time-selection algorithm, different from the one employed in [6] was applied to obtain $S$ time points 2 or 3 times the $2K-1$ points required. By combining the orthogonalization in the time-selection algorithm with the solving of the overdetermined system by least-squares accuracy is improved with respect to previous methods.

$$\rho(X) = ||x - A \cdot X||$$  \hspace{1cm} (8)

Other implementations of the APFT have been published since [11]. The orthogonal APFT by Rodrigues [12] achieves the best possible conditioning for matrix $A$ by making it exactly orthogonal. Unfortunately, this method has a big inconvenient when applied to device models that are not described by analytical equations. This inconvenient is related to the highest time instant $t_{\text{max}}$ where the time domain waveform must be evaluated, which in Rodrigues’ APFT depends on the desired significant digits to be taken into account for representing the frequency values in the analysis. As the numerical model of the Schottky diode is described by partial differential equations, and considering that a high sampling frequency (low time steps) is mandatory to reduce the error in the time domain resolution of such a system, then a high number of points would be necessary to reach the mentioned $t_{\text{max}}$.

### B. APFT implementation in the mixers CAD tool

To summarize, a good APFT method should solve two important problems. First, the need for accuracy in the estimation of Fourier coefficients, which is directly proportional to the condition number of matrix $A$. Second, the computation time that is required, which depends not only on the number of frequencies and the number of time points to be considered but also on the time-selection algorithm. Thus, for the mixers CAD tool we have implemented an APFT method, conceptually similar to the one by Zhang and Hong but with the following simplifications and considerations:

- The time-selection algorithm is simplified to a random selection of $S$ time-points. As a consequence, a poor-conditioned matrix $A$ is generated.
- Fourier coefficients vector $X$ is estimated by $X = A^T \cdot x$, where $A^T$ is the Moore-Penrose pseudo-inverse of matrix $A$ (also known as generalized inverse) [9]. The motivation for employing pseudo-inverse lies on the fact that this method proportionates the solution that minimizes the distance given by Eq. 6. In [7] it is discussed the strategy of using the Moore-Penrose generalized inverse to solve poorly-conditioned systems.

Table I shows a comparison between several APFTs and the one we have proposed and employed in the multi-tone harmonic balance tool for mixer analysis. The test has been performed by applying the APFT to the time domain current signal obtained by pumping a Schottky diode with a 2-tone ($P_{LO}=2$ mW @ 100 GHz and $P_{RF}=0.1$ mW @ 105 GHz) voltage waveform. A box truncation scheme (box $H_1XH_2$) has been employed, where $H_1$ and $H_2$ indicate that only those frequency components corresponding to $H_1$ LO harmonics, $H_2$ RF harmonics and their intermodulation products have been taken into account. $M$ is the number of points employed in the time-selection algorithms to choose the $S$ points that will be used to form matrix $A$.

The random component inherent to the time points selection in APFT techniques makes necessary to perform a Monte Carlo analysis in order to evaluate the statistical goodness of the results. An unbiased estimation of conversion loss (taking as reference the results obtained by the FFT) together with a low standard deviation are desired. Results are shown in table I, where $E/L$ represents the mean conversion loss and $\sigma_L$ is the standard deviation. Computation times correspond to a single execution of the APFT. The Monte Carlo analysis has been done using Mathworks MATLAB 7.0 running on a Pentium IV platform with a 2.8 GHz clock frequency and 1 GB of available RAM.

![Fig. 3](image_url)  
**Fig. 3.** Influence of truncation on APFT performance
III. MIXERS DESIGN AND OPTIMIZATION

It has been already commented that our CAD tool allows the concurrent analysis and optimization of the external circuit and the Schottky diode. In this section we present how the different design parameters affect to the performance of the mixer circuit.

A. External circuit parameters

In contrast to multiplier design, mixers are limited less by the properties of the Schottky diode than by those of the circuit, especially the practical impossibility of achieving optimum terminations at a large number of mixing frequencies. Some ways to optimize the external circuit are the use of DC bias, optimization of LO power, image enhancement, and the use of matched source and load impedances [13]. Of course, IF impedance must be also optimized to minimize conversion loss. In the CAD tool, this task is done by an optimization algorithm based on the gradient descent method.

As can be seen in Fig. 4, there is a trade-off between LO power and bias [13]. Furthermore, conversion loss is minimized and gets constant beyond a certain LO power, consequently, there is a linear relationship between IF and LO powers. Fig. 5 shows the variation in the real part of the conjugate-matched LO impedance as a function of LO power. As in multipliers [3], Re[Z(f_{LO})] increases with bias as a consequence of the higher electric fields inside the Schottky diode that reduce the electron mobility.

The sensitivity of conversion loss with IF impedance is analyzed in Fig 6. It can be depicted that the IF impedance is not a limiting factor due to the wide range of values where minimum loss are achieved.

One of the best known methods of reducing the conversion loss of a mixer is to terminate the diode in a reactance at the image frequency. Thus, power that would be dissipated in the image termination is converted to the IF [13]. It can be noticed in Fig 7 the reduction of the conversion loss through a correct choice of the image terminating reactance in the 100-105 GHz mixer circuit. Although the minimum conversion loss occurs for a 50Ω reactance, it is convenient to choose a higher value for it in order to avoid the high sensitivity region.

Another well-known characteristic of mixers performance is illustrated in Fig. 8. When the RF power is much lower than the LO power, which is the general regime in space applications receivers, conversion loss are determined by LO power and the influence of RF power can be neglected. This is the fundamental assumption of
those simulators based on approximate approaches as the impedance matrix conversion techniques. Fig. 8 also demonstrates that our mixer CAD tool allows the analysis of mixers when they are operating well into the non-linear region for RF.

![Fig. 8. 100-105 GHz Mixer. Influence of RF power on conversion loss.](image)

**B. Schottky diode parameters**

The influence on performance of the Schottky diode parameters (anode area, epilayer length and doping, temperature, ...) is analogous to the multipliers performance (see [2]). For instance, as it occurs in multipliers, dividing by 2 the anode area of the Schottky diode, the curve of conversion loss shifts 3 dBm to the left so minimum losses can be achieved with half LO power (Fig. 9). Fig. 10 shows the reduction in conversion loss when the length of the epitaxial layer is reduced, as a consequence of a lower series resistance.

![Fig. 9. 100-105 GHz Mixer. Influence of anode area on conversion loss](image)

Influence of ambient temperature (self-heating has not been considered) in the mixer performance is also presented in Fig. 12. Conversion loss is reduced as a consequence of higher electron mobilities at low temperatures.

![Fig. 10. 100-105 GHz Mixer. Influence of epilayer length on conversion loss](image)

**IV. VALIDATION**

The validation of the numerical simulator for mixers design, as for multipliers, requires a great quantity of data from Schottky mixer measurements. Unfortunately, only a few data have been published in literature [14] - [20], which is not enough to afford a complete validation of the simulation tool. Anyway, the numerical simulator was already validated for multipliers design and this can be extended to the mixer design just by taking into account that the only change in the simulator lies on the time-frequency conversion techniques that are employed.

Fig. 12 shows simulation results for the 585 GHz mixer described in [18]. A conversion loss of 8 dB, achieved in the range of 0.2 to 1 mW LO power is reported for this mixer. According to harmonic balance simulations, the 8 dB conversion loss is obtained at around 0.6 mW of LO power. Since no information is given in [18] regarding the termination impedances at the image frequency, three cases have been simulated: $Z_Y = Z_{LO, matched}$ and $Z_Y = 0 + j300 \, \Omega$ (image enhancement).

A 100 GHz mixer has been also simulated with similar parameters to those corresponding to the mixer circuit reported in [14]. The conversion loss (5.3 dB) is analogous to the obtained by the numeric mixer CAD tool.
V. CONCLUSION

A novel CAD tool has been presented for the design and optimization of mixer circuits at millimeter and submillimeter-wave bands. One of the advantages of this tool is that no assumptions are made regarding LO and RF powers and frequencies. The mixer CAD tool represents a complement to a previous existent simulation tool for multipliers. As a result, a complete simulation tool is available for the design of receivers up to Terahertz frequencies.

The degree of freedom that arises as a consequence of the coupling of a numerical model for Schottky diodes with the harmonic balance circuit simulator has enabled us to study the different operation regimes and the physical limitations of mixers from the circuit point of view: bias, input power, loads at different frequency components, etc.

Mixer design and optimization aspects have been presented, which show the potential of the CAD tool to perform different analysis of mixer circuits taking into account all possible design parameters. Some comparisons between simulations and measurements have been presented. Further efforts to validate the mixer CAD tool are in progress.

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